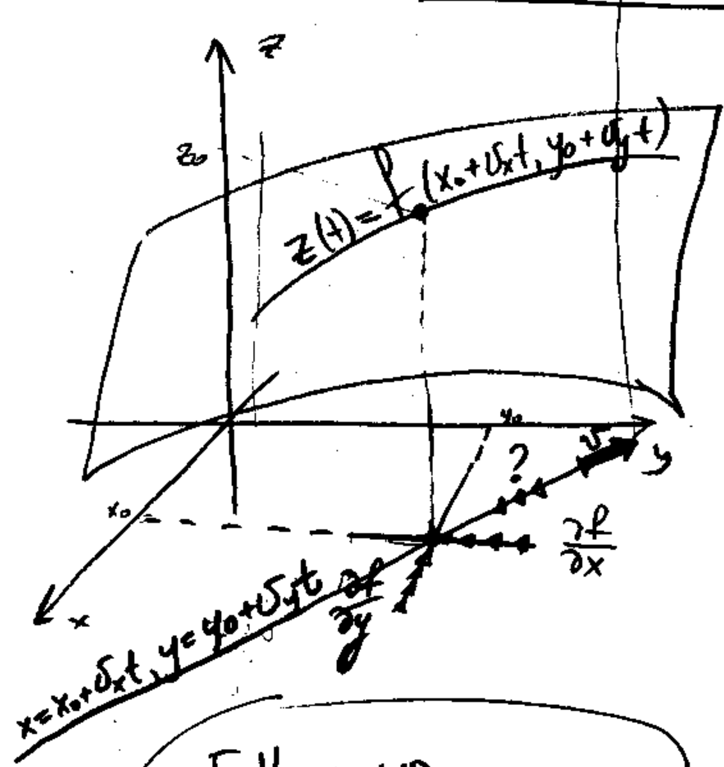
Directional Derivatives.



Motivation

For a functions of 2 variables $z = f(x,y)$, at a point (x_0,y_0)

$\frac{\partial f}{\partial x}(x_0,y_0)$ is the derivative in X-direction

$\frac{\partial f}{\partial y}(x_0,y_0)$ is the derivative in Y-direction

??? would be in AN ARBITRARY direction?

Follow up

Consider vector $\overline{v}$ in XY-plane of unit lenght, $|v| = 1$;

v defines a direction.

$\begin{cases} x = x_0 + v_x t \\ y = y_0 + v_y \cdot t \end{cases}$ Defines the line passing through (x_0,y_0) in direction $\overline{v}$.

Plug $x = x_0 + v_x t$, $y = y_0 + v_y \cdot t$ into $f(x,y) \Rightarrow z(t) = f(x_0 + v_x t, y_0 + v_y t)$

Consider the derivative of $z(t)$ at $t = 0$:

$$z'(0) = \lim \frac{z(t) - z(0)}{t}$$

Directional Derivative

$$\lim_{t \to 0} \frac{f(x_0 + v_x t, y_0 + v_y t) - f(x_0,y_0)}{t} = D_v f(x_0,y_0)$$

$z'(0)$ = $z(t)$ is $f(x,y)$ Restricted to the line = we found derivative of $f(x,y)$ along the line

IN VECTOR notations:

$$D_v f(p) = \frac{f(p + vt) - f(p)}{t}$$

Formula for $D_v f$:

If $f(x,y)$ is differentiable at $(x,y) \Rightarrow$

$$D_v f(x,y) = \frac{\partial f}{\partial x}(x,y)\, v_x + \frac{\partial f}{\partial y}(x,y)\, v_y$$