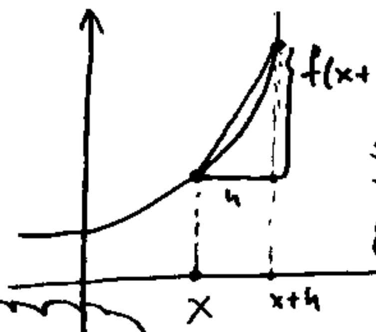
Differentiability versus Derivative

I-D

Derivative

velocity, slope of the tangent



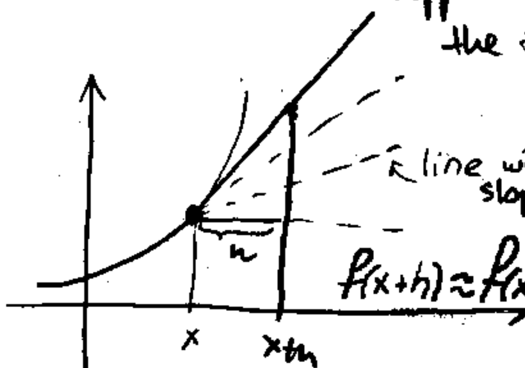
slope of the secant $= \dfrac{f(x+h)-f(x)}{h}$

SECANT $\Rightarrow$ TANGENT as $h \to 0$

The Derivative $f'(x) =$ slope of the tangent $= \lim\limits_{h\to 0} \dfrac{f(x+h)-f(x)}{h}$

Differential

Approximating the function



$f(x+h) \approx f(x) + m\cdot h$

The best prediction is when Differential

$(*)$ $f(x+h) - f(x) = m\cdot h + O(h)$

$O(h) =$ "O" little is the quantity of the order of magnitude smaller then h, that is

$$\lim_{h\to 0} \frac{O(h)}{h} = 0$$

Differential is $= m\cdot h$ where m is a number such that $(*)$ holds. This number m is unique

Find m:

$(*) \Rightarrow \underbrace{\dfrac{f(x+h)-f(x)}{h}}_{\to f'(x)} = m + \underbrace{\dfrac{O(h)}{h}}_{\to 0}$ as $h \to 0$

$f'(x) = m$

$\Leftarrow$

Function is differentiable

that is $\exists\, m$ such that $f(x+h) - f(x) = m\cdot h + O(h)$

IF and only if

Function has derivative

$f'(x) = \lim\limits_{h\to 0} \dfrac{f(x+h)-f(x)}{h}$

and differential is

$df(x) = f'(x)\cdot dx$

$m = f'(x)$, $h = dx$

$\Rightarrow$ **Differential** is the principal linear part of the increment of the function, this is the matematical way to say that differential is the best linear approximation to the funct.

$\Rightarrow$ IF linear approximation is not enough, consider quadratic, cubic, etc...

$$f(x+h) \cong f(x) + \underset{\substack{\| \\ f'(x)}}{m}\cdot h + \underset{\substack{\| \\ \frac{f''(x)}{2}}}{l}\cdot h^2 + \underset{\substack{\| \\ \frac{f'''(x)}{3!}}}{k}\cdot h^3 + \dots + \underset{\substack{\| \\ \frac{f^{(n)}(x)}{n!}}}{s}\cdot h^{(n)} + O(h^{(n)})$$

Taylor Expansion