

All limit laws that apply in 1-D apply in 2-D.

A 2-D limit converts to a bunch of regular 1-D limits by using limit laws:

Continuity =  $\lim_{(x,y) \rightarrow (c,d)} xy = \left( \lim_{(x,y) \rightarrow (c,d)} x \right) \left( \lim_{(x,y) \rightarrow (c,d)} y \right)$

$= \lim_{x \rightarrow c} x \cdot \lim_{y \rightarrow d} y$

1-D

$y = f(x)$  is continuous at  $x=c$  if and only if

$\lim_{x \rightarrow c} f(x) = f(c)$

2-D

$z = f(x,y)$  is continuous at  $(x,y) = (c,d)$  if and only if

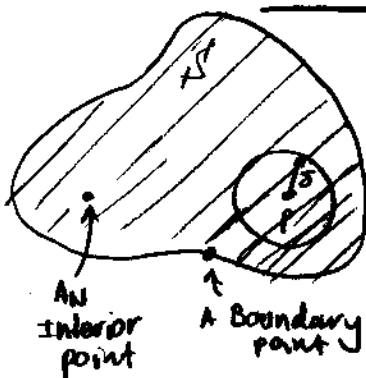
$\lim_{(x,y) \rightarrow (c,d)} f(x,y) = f(c,d)$

- 1)  $f$  is defined
- 2)  $\lim f$  exists
- 3) Results in 1) and 2) are the same

As usual,

- 1) Continuous on a set = continuous at every point of the set
- 2) Sum, difference, product, composition, Ratio of continuous functions is continuous (subject to regular restrictions)

Some definitions from Analysis



1) Neighborhood of radius  $\delta$  of a point  $P$  = set of all points  $Q$  such that  $d(P,Q) < \delta$

2) A point  $P$  is called an interior point if there exists a neighborhood contained in  $S$

3) A point  $P$  is called a boundary point if every neighborhood of  $P$  contains both points from  $S$  and not from  $S$

4) Boundary = set of all boundary points (not necessarily belongs to  $S$ )

5) A set is closed if it contains all its boundary pts

6) A set is open if all its points are interior pts.