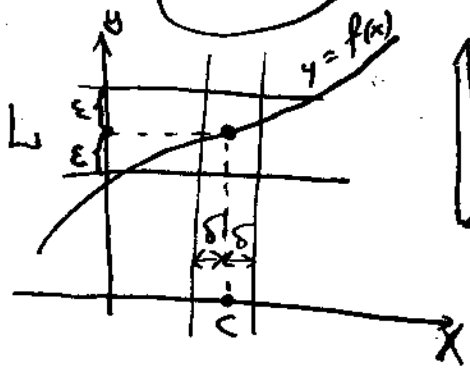
Limit and Continuity in 2-D

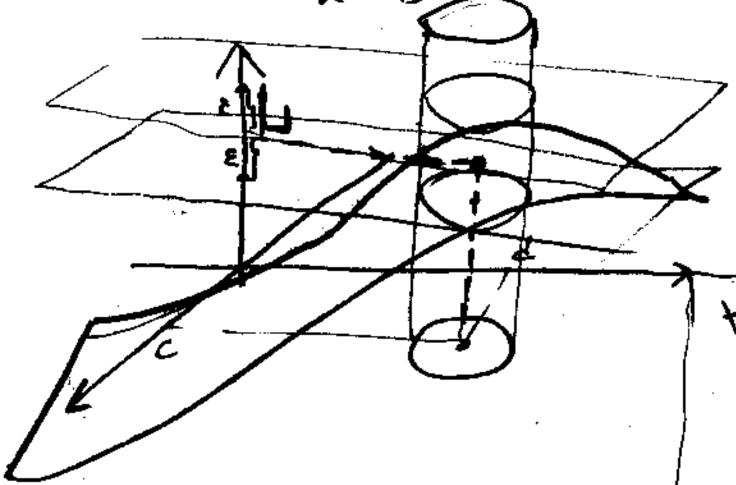
1-D



As x approaches c
f(x) approaches L

$\forall \varepsilon > 0 \ \exists \delta > 0 \ / \ |x-c| < \delta \Rightarrow |f(x) - L| < \varepsilon$

2 - D



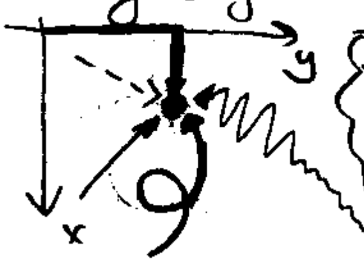
As (x,y) approaches (c,d)
f(x,y) approaches L

$\forall \varepsilon > 0 \ \exists \delta > 0 \ / \ d((c,d),(x,y)) = \sqrt{(x-c)^2 + (y-d)^2} < \delta$

$\Rightarrow |f(x,y) - L| < \varepsilon$

New IN 2-D

① Many ways to approach the point



The limit must be Independent of the way we approach the point

This gives us a tool of disproving limits:
IF TWO Different ways give different answers
$\Rightarrow$ No limit

This gives us a tool for evaluating limits in practice

② Relationship with 1-D

$\lim_{(x,y)\to(c,d)} f(x,y) \Rightarrow$ sometimes $\Rightarrow \lim_{x\to c} \lim_{y\to d} f(x,y) \Rightarrow \lim_{y\to d} \lim_{x\to c} f(x,y)$