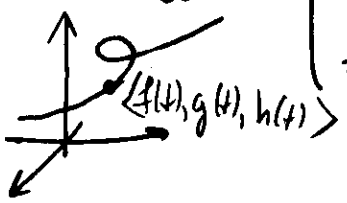
Surfaces in 3-space

Curve = a set of points that CAN be characterized by one parameter

Surface = a set of points that CAN BE characterized by two parameters

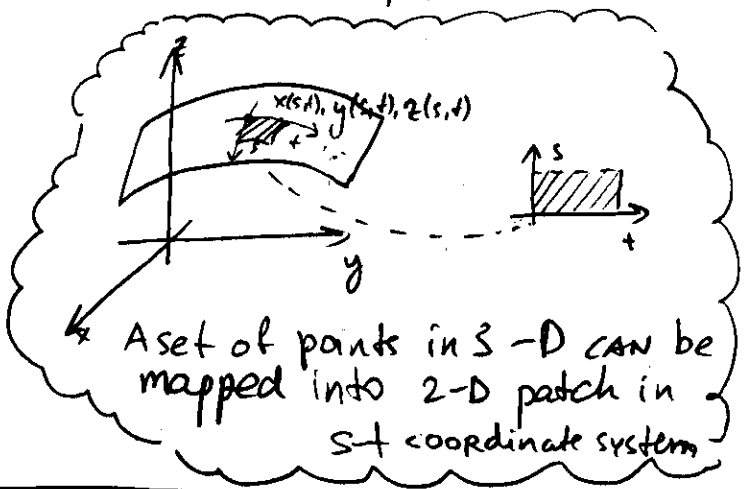
3-D

Curve
$$\begin{cases} x = f(t) \\ y = g(t) \\ z = h(t) \end{cases}$$



3-D

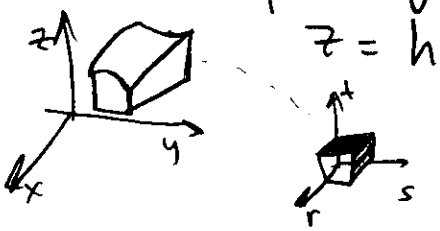
Surface
$$\begin{cases} x = f(s,t) \\ y = g(s,t) \\ z = h(s,t) \end{cases}$$



Solid = a set of points that CAN be characterized by three parameters

3-D

Solid =
$$\begin{cases} x = f(r,s,t) \\ y = g(r,s,t) \\ z = h(r,s,t) \end{cases}$$



Naive Illustration (3-D) = (x, y, z) = three parameter

Curve : 2 equations (Fix two from three)

Surface : 1 equations (Fix one from three)

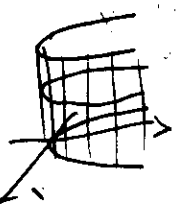
Solid : 0 equations (All three are free)

Some Classification: Surfaces

Cylinders

(A surface obtained by a parallel transport of a curve along a line)

Ex:



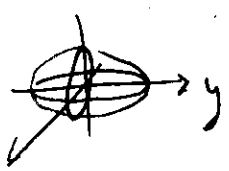
$y - x^2 = 0$

quadratic surfaces

A surface described by a polynomial of degree 2

$$Ax^2 + By^2 + Cz^2 + Dxy + Eyz + Fxz + Gx + Hy + Iz = J$$

EX: Ellipse $\frac{x^2}{2} + \frac{y^2}{4} + \frac{z^2}{2} = 1$



All Other

(Everything Else)

Ex: $\sin^2 x + \cos^2 y - z = 0$