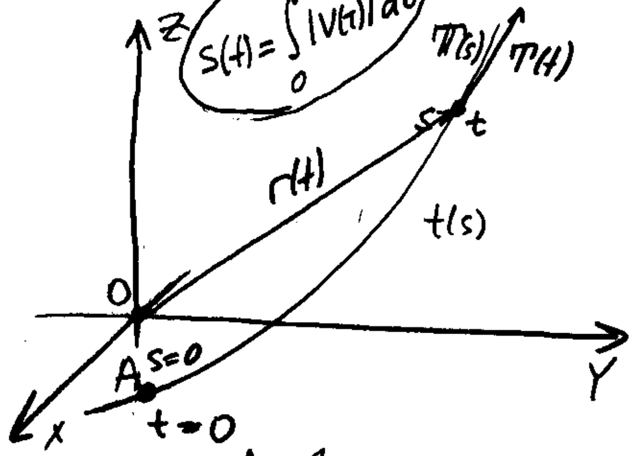
The Curvature

$$s(t)=\int_0^t |V(t)|\,dt$$



$t \Leftrightarrow$ point $\Leftrightarrow s$
(time)   (distance from point A)

Many ways to parameterize curve:

1) $\overline{r}(t)$, t is time $\Rightarrow \begin{cases} x=f(t) \\ y=g(t) \\ z=h(t) \end{cases}$

2) $\overline{\mathbb{r}}(s)$, s is the distance along the curve $\Rightarrow \begin{cases} x=\mathbb{f}(s) \\ y=\mathbb{g}(s) \\ z=\mathbb{h}(t) \end{cases}$

different functions, but the same curve

Q: Is there a quantity that is independent on the choice of parameter

$\Rightarrow$ Consider the rate of change of the unit tangent vector with respect to the distance travelled, or how the trajectory curves:

$\Rightarrow$ Curvature (vector) $= \dfrac{d\overline{\mathbb{T}}(s)}{ds}$

Scalar Curvature $= \left|\dfrac{d\mathbb{T}(s)}{ds}\right| = k(s)$

Computing Curvature in terms of "t"

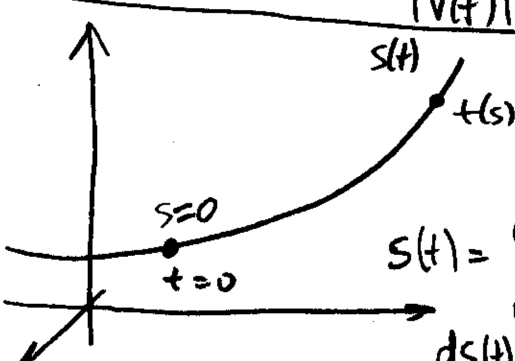
$$\frac{d\mathbb{T}(s)}{ds}=\frac{dT(t(s))}{ds}=\frac{d\overline{T}(t(s))}{dt}\cdot\frac{dt}{ds}=\overline{T}'(t)\cdot\frac{1}{|V(t)|}=\frac{\overline{T}'(t)}{|V(t)|}$$

$$k=\left|\frac{\mathbb{T}(s)}{ds}\right|=\frac{|T'(t)|}{|V(t)|}$$

Also, Normal vector $\overline{N}=\dfrac{\frac{d\mathbb{T}(s)}{ds}}{\left|\frac{d\mathbb{T}(s)}{ds}\right|}=\dfrac{1}{k}\dfrac{d\mathbb{T}(s)}{ds}$
(Vector curvature is NORMAL to the curve)

Binormal vector

$$\overline{B}=\overline{T}\times\overline{N}$$



$$s(t)=\int_0^t |V(t)|\,dt$$

$$\frac{ds(t)}{dt}=|V(t)| \Rightarrow \frac{dt}{ds}=\frac{1}{|V(t)|}$$