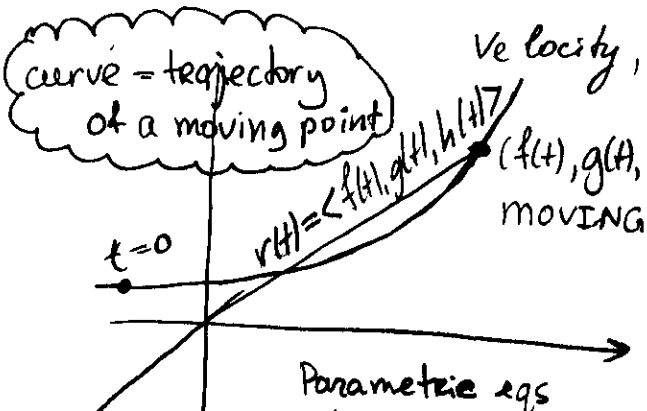
Velocity, Acceleration, and Curvature

curve = trajectory of a moving point



position at t : $\bar{r}(t) = \langle f(t), g(t), h(t) \rangle$

position at $t+\Delta t$: $\bar{r}(t+\Delta t) = \langle f(t+\Delta t), g(t+\Delta t), h(t+\Delta t) \rangle$

displacement for the period Δt:

$$\Delta\bar{r}(t;\Delta t) = \bar{r}(t+\Delta t) - \bar{r}(t) =$$

$$= \langle f(t+\Delta t) - f(t),\ g(t+\Delta t) - g(t),\ h(t+\Delta t) - h(t) \rangle$$

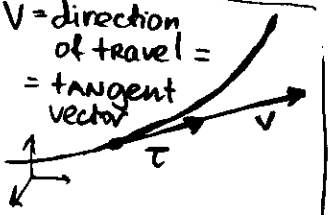
Parametric eqs of a 3-D curve

$$\begin{cases} x = f(t) \\ y = g(t) \\ z = h(t) \end{cases}$$

average velocity for the period Δt:

$$\bar{V}_{av}(t;\Delta t) = \frac{\Delta\bar{r}(t;\Delta t)}{\Delta t} = \left\langle \frac{f(t+\Delta t)-f(t)}{\Delta t},\ \frac{g(t+\Delta t)-g(t)}{\Delta t},\ \frac{h(t+\Delta t)-h(t)}{\Delta t} \right\rangle$$

$\bar{V}$ = direction of travel = tangent vector



Instantaneous velocity, or velocity:

$$\bar{V}(t) = \lim_{\Delta t \to 0} \frac{\Delta\bar{r}(t;\Delta t)}{\Delta t} = \left\langle \lim_{\Delta t\to 0} \frac{f(t+\Delta t)-f(t)}{\Delta t},\ \lim_{\Delta t\to 0} \frac{g(t+\Delta t)-g(t)}{\Delta t},\ \lim_{\Delta t\to 0} \frac{h(t+\Delta t)-h(t)}{\Delta t} \right\rangle$$

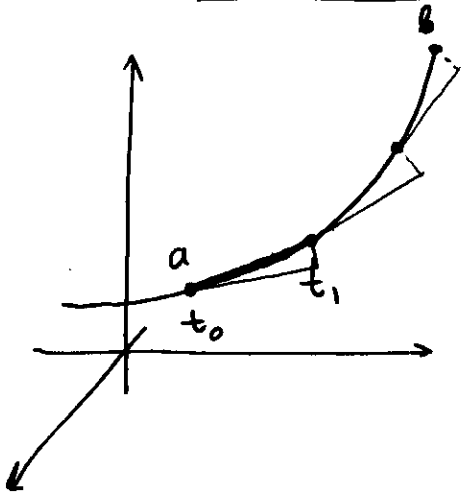
$$= \langle f'(t), g'(t), h'(t) \rangle$$ ⇦ TANGENT vector

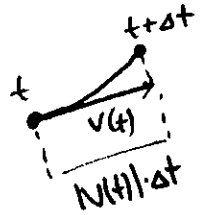
SPEED = $\|V(t)\|$

Acceleration:

$$\bar{a}(t) = \frac{d\bar{V}(t)}{dt} = \langle f''(t), g''(t), h''(t) \rangle$$

Lenght of the curve





Lenght of the piece ≅ speed $\times \Delta t$

$$\text{lenght of the curve} = \lim_{\Delta t\to 0} \sum_{n=1}^{N} \|V(t_n)\| \Delta t_n$$

$$= \int_a^b \|V(t)\|\,dt =$$

$$= \int_a^b \sqrt{(f'(t))^2 + (g'(t))^2 + (h'(t))^2}\ dt$$