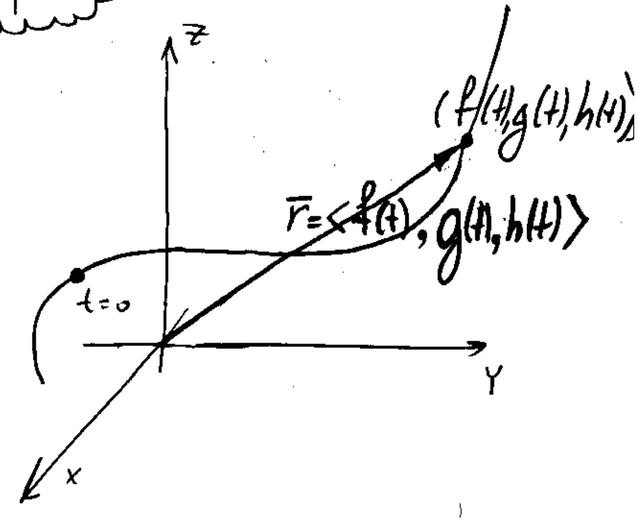
Lines and curves in three-space

Def The set of points determined by a triple of parametric equations

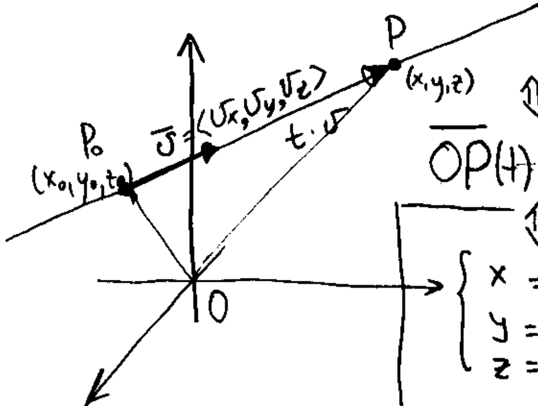
$$\begin{cases} x = f(t) \\ y = g(t) \\ z = h(t) \end{cases}, \quad t \in I \subset \mathbb{R}$$

is called a space (3-D) curve



Alternatively, line can be determined by the position vector $\bar{r}(t) = \langle f(t), g(t), h(t) \rangle$
$= f(t)i + g(t)j + h(t)k$

Equation of the Line



$\overline{P_0P} = t \cdot v$

$\Updownarrow$

$\overline{OP}(t) = \overline{OP_0} + t \cdot v$

$\Updownarrow$

$$\begin{cases} x = x_0 + t v_x \\ y = y_0 + t v_y \\ z = z_0 + t v_z \end{cases}$$ parametric eq. of the line

$\Rightarrow$

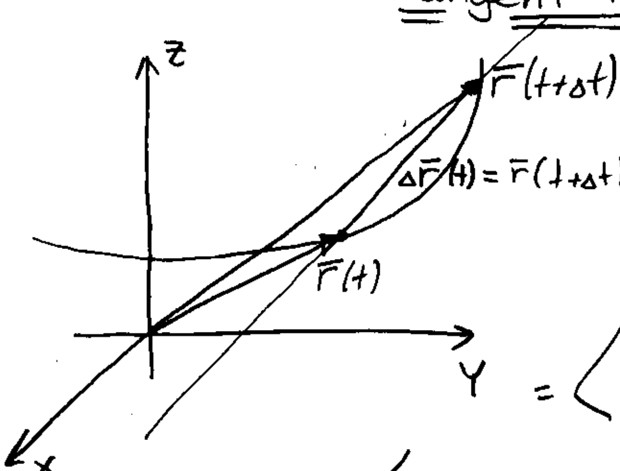
$$\begin{cases} t = \dfrac{x - x_0}{v_x} \\ t = \dfrac{y - y_0}{v_y} \\ t = \dfrac{z - z_0}{v_z} \end{cases}$$

$\Uparrow$

$$\frac{x - x_0}{v_x} = \frac{y - y_0}{v_y} = \frac{z - z_0}{v_z}$$

symmetric eq. of the line

Tangent line to a curve



$\Delta \bar{r}(t) = \langle f(t+\Delta t) - f(t), g(t+\Delta t) - g(t), h(t+\Delta t) - h(t) \rangle$

$\boxed{\text{Def}}$

$$r'(t) = \lim_{\Delta t \to 0} \frac{\Delta \bar{r}(t)}{\Delta t} =$$

$$= \left\langle \lim_{\Delta t \to 0} \frac{f(t+\Delta t) - f(t)}{\Delta t}, \lim_{\Delta t \to 0} \frac{g(t+\Delta t) - g(t)}{\Delta t}, \lim_{\Delta t \to 0} \frac{h(t+\Delta t) - h(t)}{\Delta t} \right\rangle$$

$$= \langle f'(t), g'(t), h'(t) \rangle$$

How fast the curve is changing in each direction

$\parallel$

TANGENT VECTOR