The Cross product

Def. Let $u = \langle u_1, u_2, u_3 \rangle$ and $v = \langle v_1, v_2, v_3 \rangle$ be two vectors in Cartesian coordinates. The cross product of vectors u and v is the vector

$$u \times v = \langle u_2 v_3 - u_3 v_2,\ u_3 v_1 - u_1 v_3,\ u_1 v_2 - u_2 v_1 \rangle$$

The cross product CAN also be calculated using the determinant of 3×3 matrix:

$$u \times v = \begin{vmatrix} i & j & k \\ u_1 & u_2 & u_3 \\ v_1 & v_2 & v_3 \end{vmatrix}$$

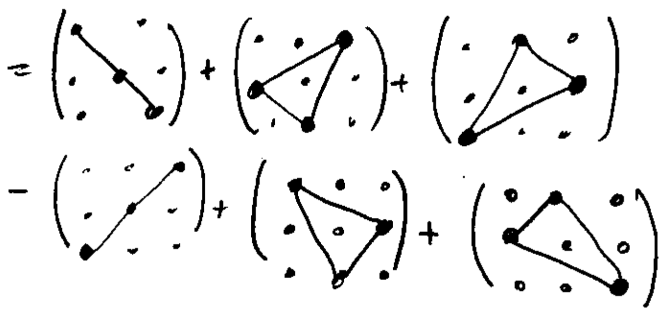
or

$$u \times v = i \begin{vmatrix} u_2 & u_3 \\ v_2 & v_3 \end{vmatrix} - j \begin{vmatrix} u_1 & u_3 \\ v_1 & v_3 \end{vmatrix} + k \begin{vmatrix} u_1 & u_2 \\ v_1 & v_2 \end{vmatrix}$$

Determinant

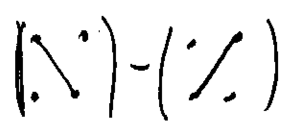
For a 3×3 matrix

$$\begin{vmatrix} a_1 & a_2 & a_3 \\ b_1 & b_2 & b_3 \\ c_1 & c_2 & c_3 \end{vmatrix} =$$



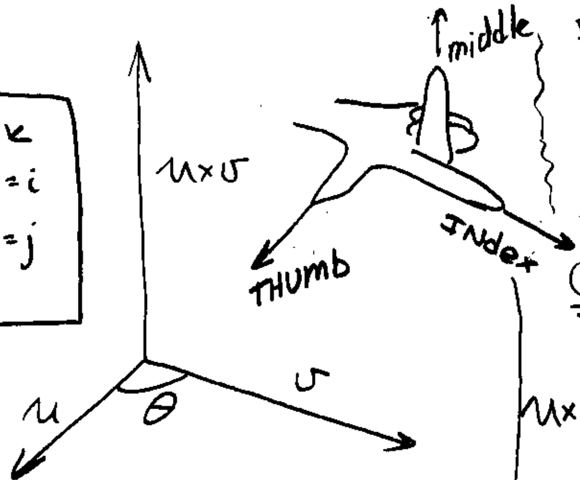
For a 2×2 MATRIX

$$\begin{vmatrix} a_1 & a_2 \\ b_1 & b_2 \end{vmatrix} =$$



$$= a_1 b_2 - a_2 b_1$$

$i \times j = k$
$j \times k = i$
$k \times i = j$



Geometric interpretation (Thm A)

u, v – vectors, θ is the angle between u & v

$u \times v =$
1) $u \cdot (u \times v) = 0$, $v \cdot (u \times v) = 0$ $\Leftrightarrow$ $u \times v$ is perpendicular to both u and v
2) u, v, $u \times v$ form a right triple
3) $|u \times v| = |u||v| \sin\theta$

Corollary: (Thm B)

u parallel $v \Leftrightarrow u \times v = 0$.

Thm C Algebraic Properties

1) $u \times v = -(v \times u)$ ANTIcommutative
2) $u \times (v + w) = u \times v + u \times w$, $(v + w) \times u = v \times u + w \times u$ left and right distribution
3) $k(u \times v) = (ku) \times v = u \times (kv)$, k – scalar

4) $u \times 0 = 0 \times u = 0$, $u \times u = 0$

5) $(u \times v) \cdot w = u \cdot (v \times w)$

6) $u \times (v \times w) = (u \cdot w) v - (u \cdot v) w$