Cartesian Coordinate System

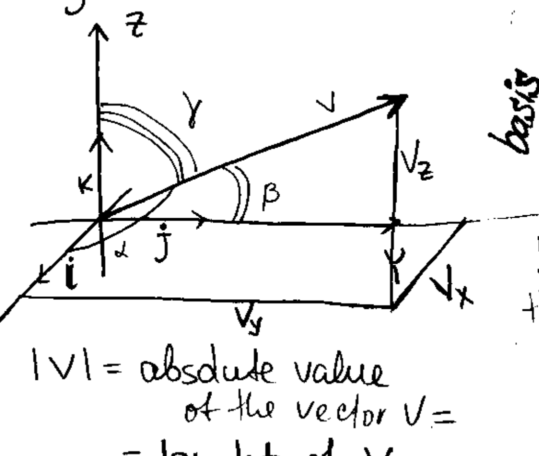
1) Orthogonal axes
2) Same unit lenght for all axes
3) Right-handed System

distance $d(P_1,P_2)=\sqrt{(x_2-x_1)^2+(y_2-y_1)^2+(z_2-z_1)^2}$

eq. of a sphere with center (h,k,ℓ) & radius r

$$(x-h)^2+(y-k)^2+(z-\ell)^2=r^2$$

Vector "=" lenght + direction



$V=(V_x,V_y,V_z)$

or $V=$ lenght + direction angles

$|V|=$ absolute value of the vector $V=$ = lenght of V

basis vectors
- i – unit lenght vector parallel OX
- j – " OY
- k – " OZ

Direction angles: angles between the vector and the basis vectors

i	j	k
α	β	γ

$$\cos\alpha=\frac{v\cdot i}{|V|}\quad \cos\beta=\frac{V\cdot j}{|V|}\quad \cos\gamma=\frac{V\cdot k}{|V|}$$

Dot Product:

Def #1 $\quad U\cdot V=|U||V|\cos\theta$

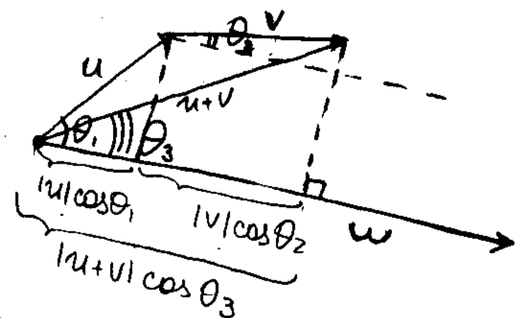
(θ is the angle between u and v)

$\Updownarrow$

Def #2 (in Cartesian Coordinates)

$$U\cdot V=u_xV_x+u_yV_y+u_zV_z$$

1) Prove that $(u+v)\cdot w=u\cdot w+v\cdot w$



So,

$$|u+v|\cos\theta_3=|u|\cos\theta_1+|V|\cos\theta_2$$

$\Rightarrow$

$$(u+v)\cdot w=|u+v||w|\cos\theta_3=$$

$$=|u||w|\cos\theta_1+|V||w|\cos\theta_2$$

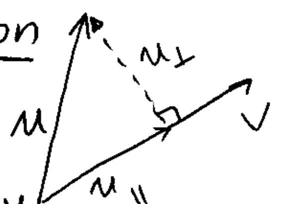
$$=u\cdot w+v\cdot w \blacksquare$$

2) Use the fact that

$$u=u_xi+u_yj+u_zk$$

$$V=V_xi+V_yj+V_zk$$

Scalar Projection



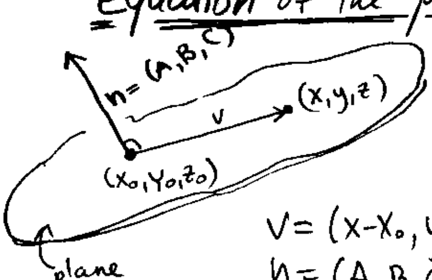
$$u=u_{\parallel}+u_{\perp}$$

$$U\cdot V=(u_{\parallel}+u_{\perp})\cdot V=u_{\parallel}\cdot V$$

$$u_{\parallel}\cdot V=|u_{\parallel}||V|\underbrace{\cos\theta}_{=1}=|u_{\parallel}||V|$$

$\Rightarrow \quad |u_{\parallel}|=\dfrac{U\cdot V}{|V|}, \quad u=\dfrac{U\cdot V}{|V|^2}V$

Equation of the plane



Plane = three points =

= two intersecting lines

= normal vector + point

$$\left.\begin{array}{l} V=(x-x_0,y-y_0,z-z_0)\\ n=(A,B,C)\\ V\perp n \Leftrightarrow v\cdot n=0 \end{array}\right\}\Rightarrow$$

Eq. of the plane

$$A(x-x_0)+B(y-y_0)+C(z-z_0)=0$$

or

$$Ax+By+Cz=D$$