Foliations II

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To our wives Juana and Jackie

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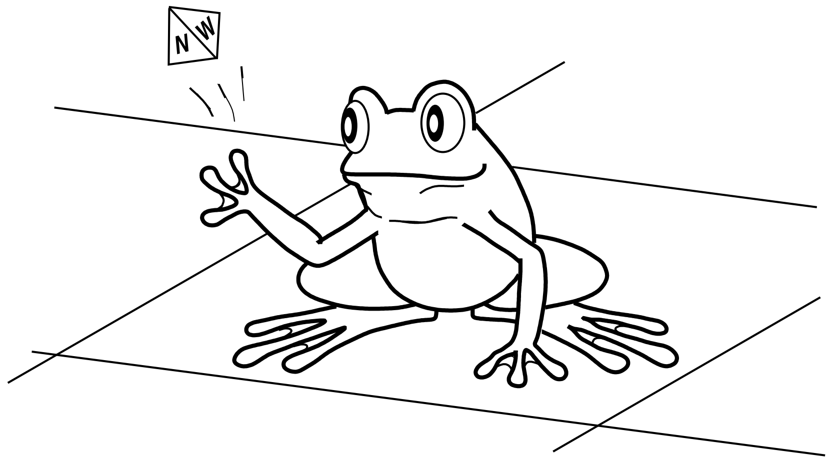

Figure C.7.1. Random frog

If the behavior of the frog is accepted as a discrete version of the movement of a Brownian particle, then it is reasonable to expect that the solution to the Dirichlet problem on a bounded domain D of the manifold X with boundary data φ will be given by

$$f(x) = E_x \left[\varphi(\omega(T_D(\omega)))\right]$$

where T_D is the first exit time from D.

The random frog will now be put to work toward a solution to the Poisson problem, submitting her to the following process. Positioned at time 0 at the point (mq, nq), let her jump at will (at discrete times $t = 0, 1, 2 \dots$) to one of the neighbouring lily pads with the same probability as before. If at time T she hits a boundary pad, then assign the first exit time $T = T(\omega)$ to the sample Brownian path. While it may or may not be possible to explicitely compute the expectation $E_{(m,n)}[T]$, it turns out that it satisfies an important identity.

As before, if the frog is at (mq, nq) at time t, then at time $t + 1$ she is going to be at one of the neighboring lilies $(m'q, n'q) = ((m + s)q, (n + s)q)$ with probability 1/4. It follows that

$$E_{(m,n)}[T] = \left(\sum_{-1 \leq r,s \leq 1} p_{rs} E_{(m+r,m+s)}[T] \right) + 1.$$

Equivalently, the function $f(mq, nq) = E_{(m,n)}\,[T]$ satisfies the equation

$$\triangle f(mq, nq) = -1$$