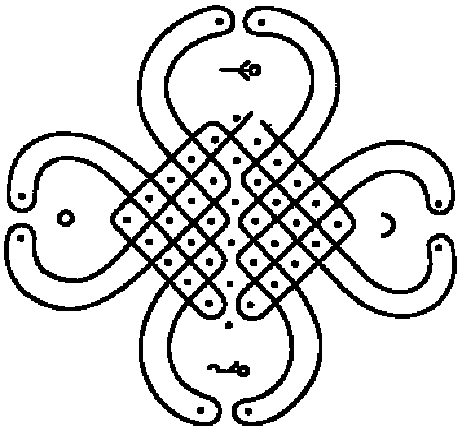
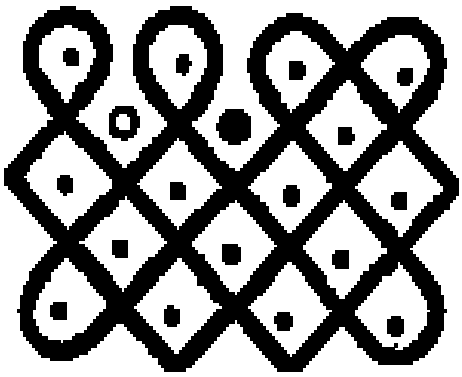
¶ 1. The Chokwe people live in southwestern Africa, in a region located in the northeast of Angola, northeast of Zambia, south of Republic of the Congo.

They are well-know for their decorative arts: woven mats and baskets, pottery, wood sculpture; all featuring impressive geometric designs.

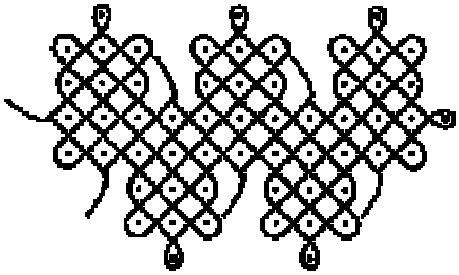
¶ 2. The Chokwe people are also well-known for their story telling. They use art in their story telling, making beautiful sand drawings, known as *sona* (plural), *lusona* (singular), to illustrate their stories. Here are some samples:



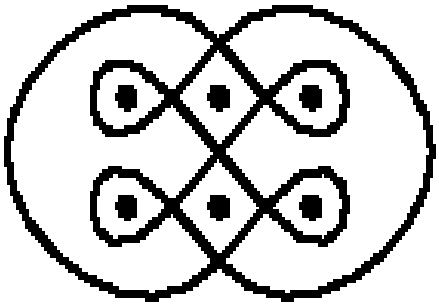
(Beginning of the World)



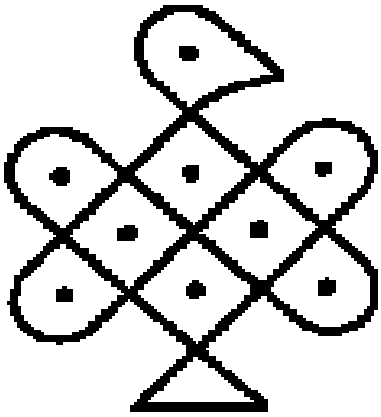
(Hunter and dog)



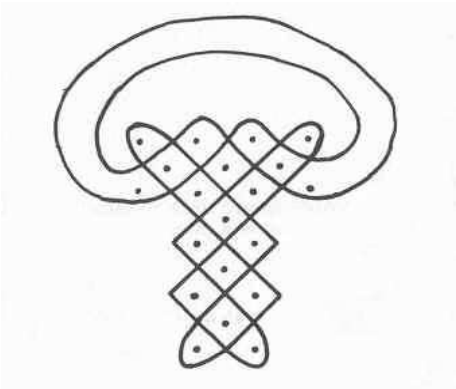
(Leopard with five cubs)



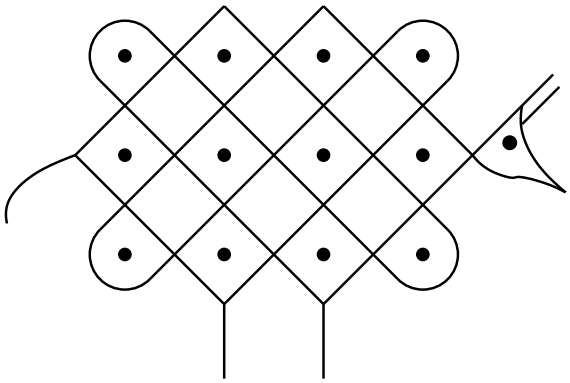
(Antelope's paw)



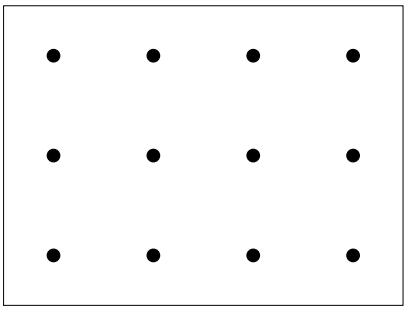
(Bird)

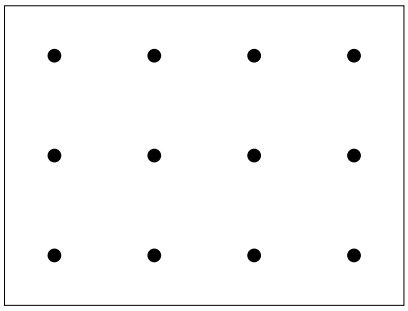


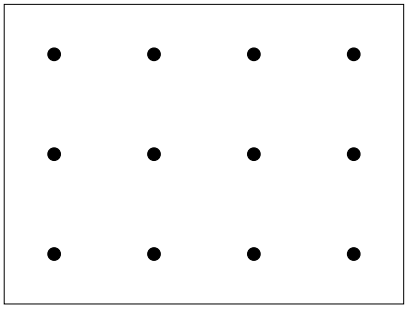
¶ 3. This lusona drawing depicts an antelope. It is drawn on a 3×4 array of dots.

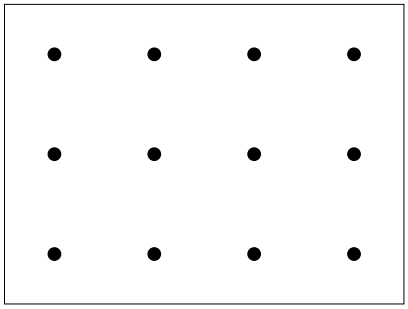


On the arrays below, try to redraw the body of the antelope. The border is there to help you complete your drawing. Your line starts anywhere in between two dots and proceeds at 45 degrees. As your line hits that border, reflect by 45 degrees and continue until you reach the place where your line started. If there are points in the array that have not been completely surrounded by your line, choose a new starting place for your next line and proceed as before.
The question is to drawn the body of the antelope with the least number of closed curves possible. How many do you need?

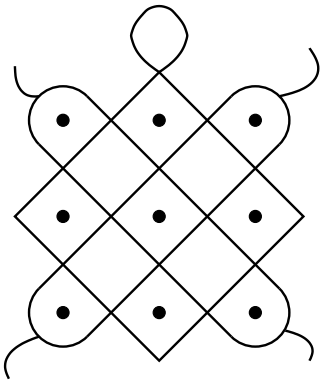


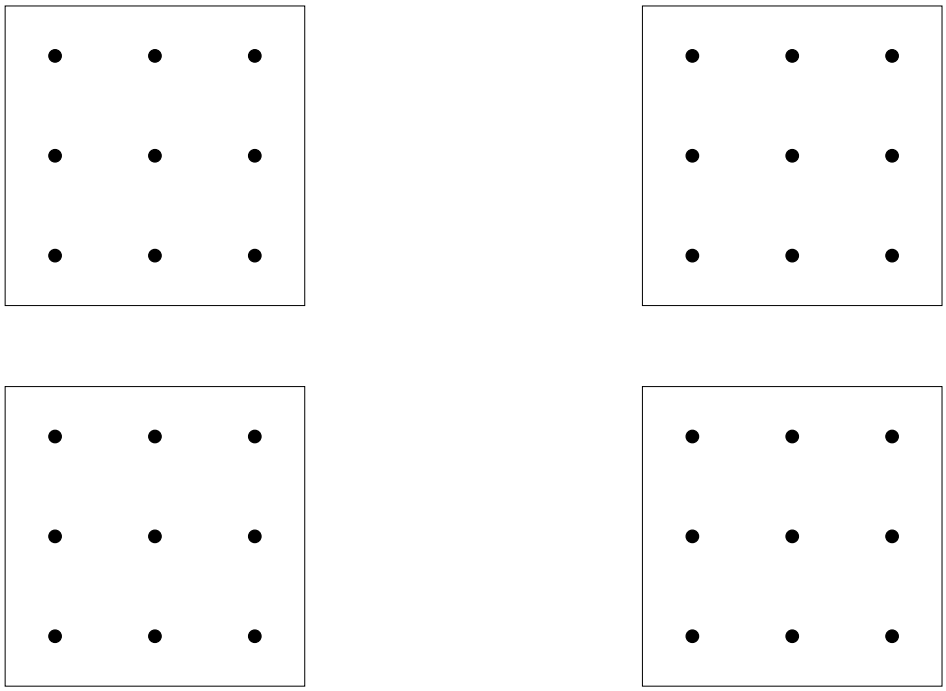




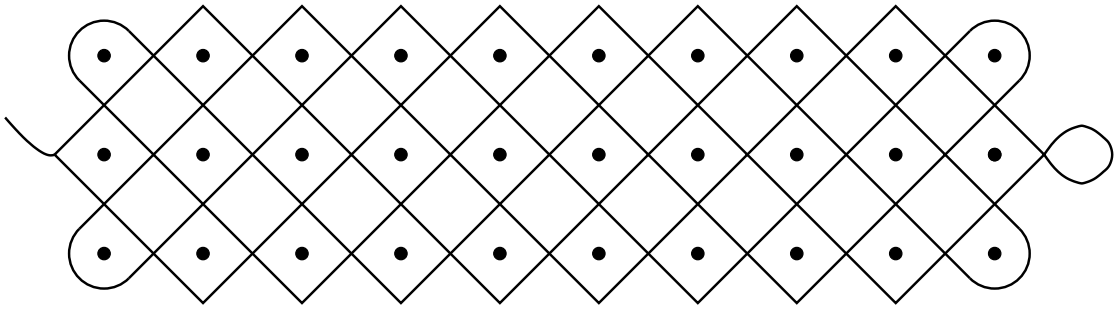


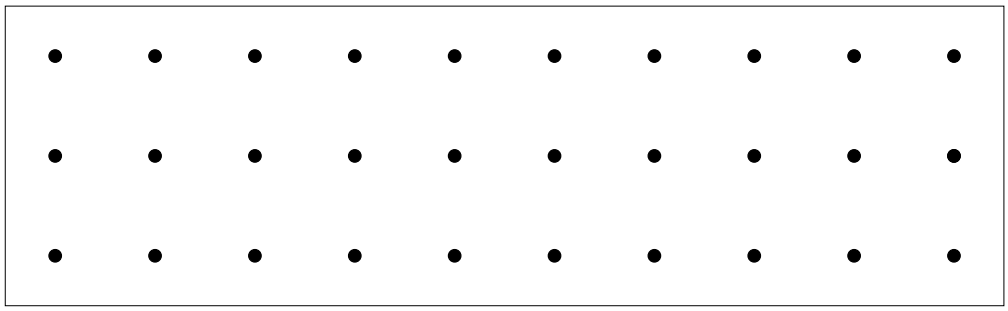
¶ **4.** The lusona drawing that depicts a turtle is drawn on a 3×3 array of dots.

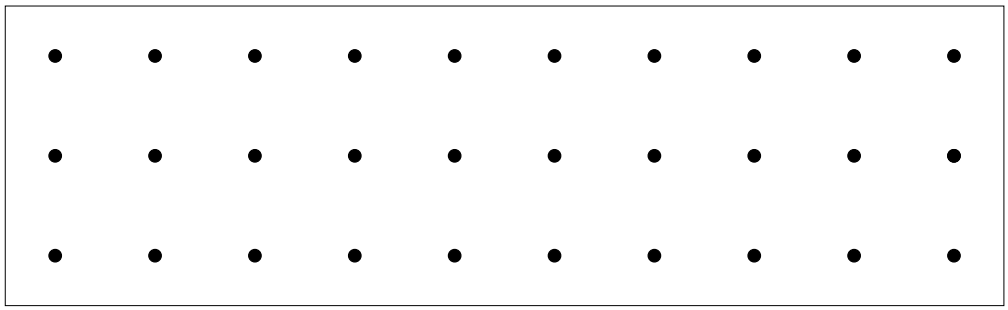




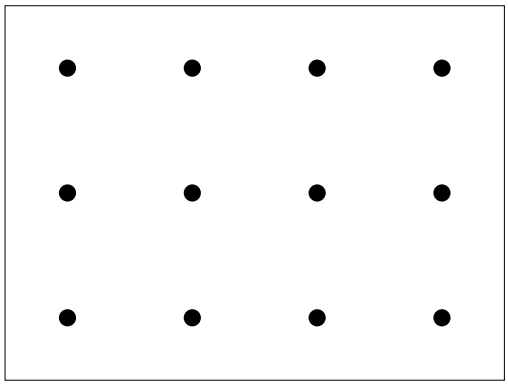
¶ **5.** The lusona drawing that depicts a lioness is drawn on a 3×10 array of dots.

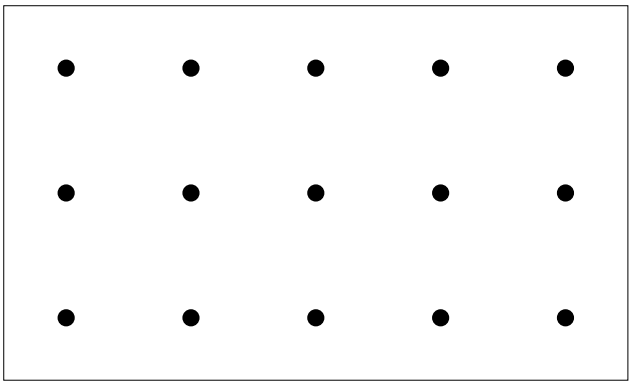


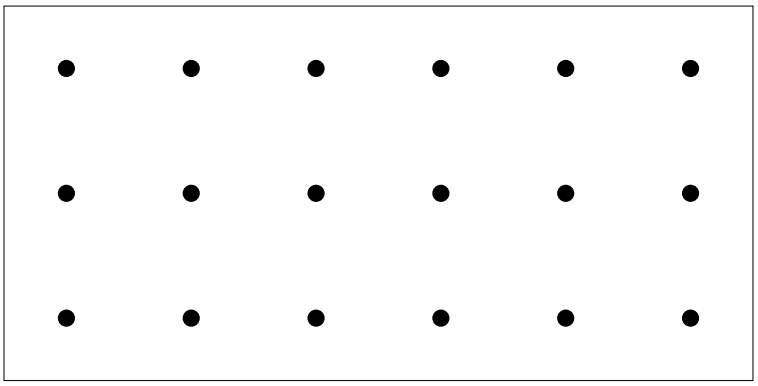


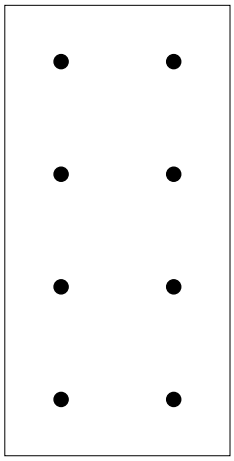


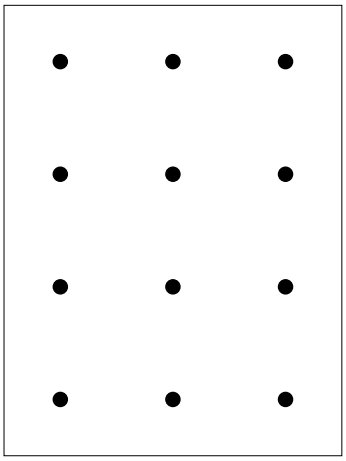
¶ 6. The sona represented by the antelope and the lioness could be completed with just one closed line. The lusona depicting the turtle required three closed lines at least. It turns out that the minimum number of closed lines required to complete any sona drawing based on a rectangular array depends on the number of columns and on the number of rows in a precisely mathematical relation. Many Chokwe people know how many closed lines are required as soon as the story teller has finalized laying out the array of dots on the sand. For example, they know that a 5×4 array requires just 1 closed line, but that a 6×4 array requires 2 closed lines.
The purpose of this problem is to discover this relationship between the number of closed lines and the dimensions of the array.

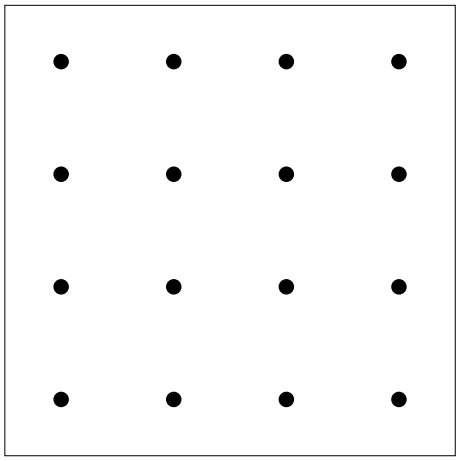


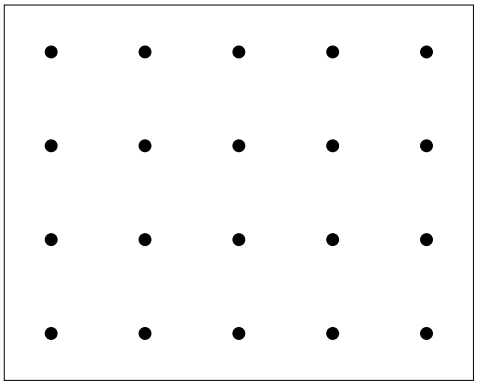


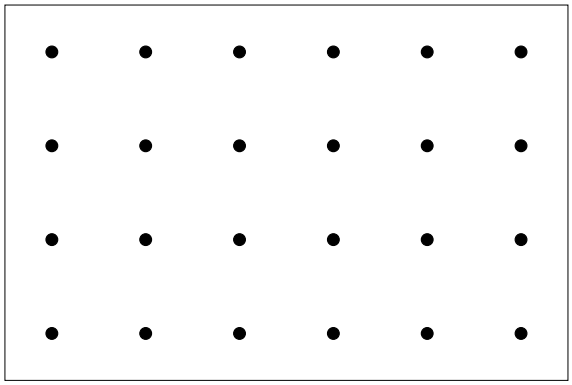












¶ **7.** Can you figure out the minimum number of closed lines required to complete an array of `p` rows by `q` columns? Fill in the table below as many values as you can, and then try to deduce a relationship between number of rows, number of columns, and number of closed lines.

Number of Rows	Number of Columns	Number of closed lines

¶ 8 (The Euclidean Algorithm)**.** It turns out that the minimum number of closed lines to complete a lusona based on an array of size $q \times q$ is the largest number that divides both p and q.
Given two integers p and q, their greatest common divisor, denoted $\gcd(p, q)$ or simply (p, q), is the largest integer which divides both a and b.
If we know the prime factorizations of p and q then (p, q) is easy to find. For example, if $p = 3 \cdot 5^2 \cdot 11 = 825$ and $q = 3^2 \cdot 5 \cdot 13 = 585$, then $(p, q) = 3 \cdot 5 = 15$.
But in general, factoring numbers takes a long time, so using prime factorization for finding the g.c.d. of two integers p and p; this is the **Euclidean algorithm**:

$$\begin{aligned}
p &= a_1 q + r_1, \qquad 0 < r_1 < q \\
q &= a_2 r_1 + r_2, \qquad 0 < r_2 < r_1 \\
r_1 &= a_3 r_2 + r_3, \qquad 0 < r_3 < r_2 \\
\cdots &= \cdots, \qquad \cdots \\
r_{n-3} &= a_{n-1} r_{n-2} + r_{n-1}, \qquad 0 < r_{n-1} < r_{n-2} \\
r_{n-2} &= a_n r_{n-1} + 0, \qquad 0 = r_n
\end{aligned}$$

That is, we use long division at each step. We find the remainder r_1 of dividing p by q, then the remainder r_2 of dividing q by r_1, and so on, until the remainder $r_n = 0$. The last nonzero remainder r_{n-1} is the g.c.d. of p and q.

¶ 9. Use the Euclidean algorithm to find the g.c.d. of the following pairs of numbers:

(a) $p = 6381$ and $q = 5163$.

(b) $p = 13876$ and $q = 15794$.

(c) $p = 3528$ and $q = 17455$.

¶ 10. We summarize the Euclidean algorithm in the following sequence of instructions. It is like a computer program, and will help us later to write a program that works on the TI-84. (The technical name is "pseudo-code.")

1 Input integers a and q.

2 If p divides q, then $(p, q) = p$. Done

3 If not, replace p by q and q by the remainder of dividing p by q.

4 Go to Step 2

¶ 11. We write a TI-84 program which implements the Euclidean algorithm for finding the greatest common divisor of two numbers.

```
PROGRAM:GCD
:Prompt P, Q
:abs(P)►P
:abs(Q)►Q
:While Q≠0
:P-iPart(P/Q)*Q►R
:Q►P
:R►Q
:End
:Disp ‘‘GCD IS’’, P
```

Literature

[1] Paulus Gerdes, *Geometry from Africa*, The Mathematical Association of America.

[2] Paulus Gerdes, *Sona: Sand Drawing from Africa*. Available at `http://www.exploratorium.edu/store_images/publications/masc_sona.pdf`