How big is the Earth?

¶ 1. What is the diameter of the Earth? What is the radius of the Earth? Of course, you can Google this numbers in no time. But you can arrive at a rather precise estimate using readily available information and some elementary mathematics.
For example:

1. How far is Los Angeles from New York City? (Hint: how long does it take to travel by plane from LAX to JFK? How fast do modern passenger planes fly?)

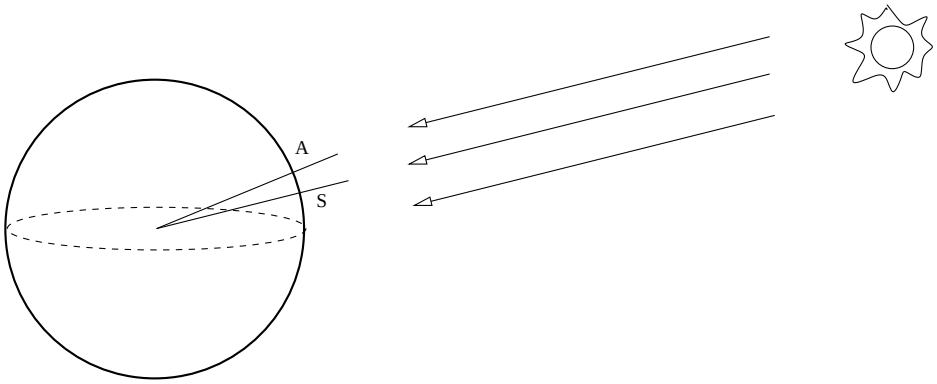
2. What proportion is the distance from Los Angeles to New York to the perimeter of the Earth? (Hint: How many time zones are there?)

3. Your estimate for the perimeter of the earth is:

4. Your estimate for the radius of the Earth is?

5. The actual value for the radius of the Earth is: 6,378 km or 3963 miles.

¶ 2. The first estimate of the diameter of the Earth of which we have records today was done by Eratosthenes of Cyrene (276-194 B.C.). He knew that at certain time of the year (midday of June 21 by our current calenders) in the ancient town of Syene the light rays of the sun felt straight down on the Earth, the sun made no shadow. However, at that say time, in Alexandria, which is straight north from Syene, the shadow at noon made an angle of about 1/50 of a full circle.



Eratosthenes Of Cyrene

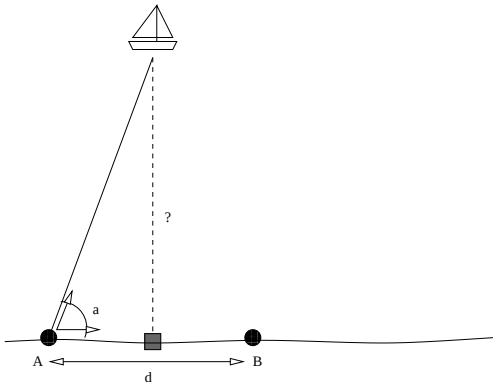
1. The distance from Syene to Alexandria was recorded as being $5,000$ stades, and each stadium is believed to be about $1/10$ of a mile long (or about 528 feet long). Using these units, what is the diameter of the Earth?

2. The ratio of the perimeter to the diameter of a circle is π. At the time of Eratosthenes, π was estimated to be $3\frac{1}{7}$. Using this value for π, What was Eratosthenes' estimate in miles for the radius of the Earth?

How big is the Moon? How far is it?

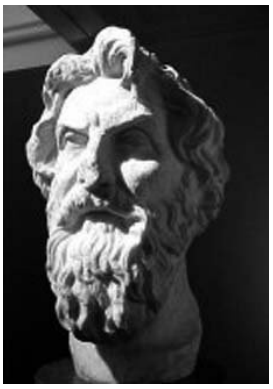
¶ 3. The next measurement that we want to perform is the distance to the Moon. The ancient Greeks had methods for estimating distances: about 500 years before Eratosthenes, Thales of Miletos devised a method for estimating the distance from the shore to a ship at see. The method was based on estimating the angle between the shoreline and the line from the ship to a location in the shoreline.

For this, two observation towers A and B were set at each side of the city so as to lie in straight line. An observer in tower A would then use an instrument similar to a compass to estimate the angle made between the line from that tower to the ship and the shoreline.



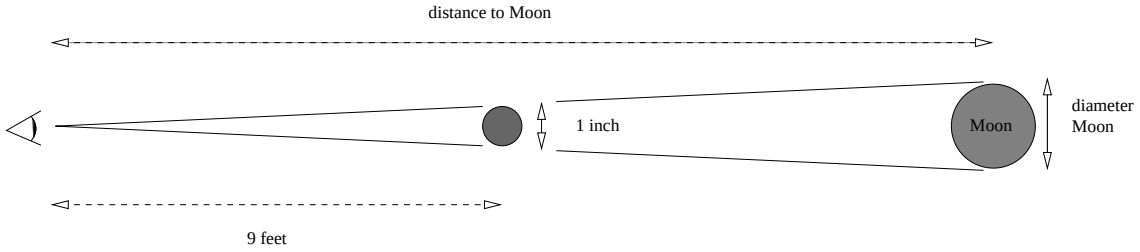
In the figure, the angle α is $1/5$ of a full circle. The distance d between the two observation points A and B is two stades. Using the approximate value of $3\frac{1}{7}$ for π, what is the distance ? from the ship to the city in the shoreline?

¶ 4. It may now seen obvious that to estimate the distance from the earth to the Moon we should estimate the angle difference between to points on the surface of the Earth, like two cities a few miles apart. Unfortunately, the angles are so small that the resulting errors in measurement are too big to have any value. However, Aristarchus of Samos (310-230 B.C.) came up with a novel idea for estimating the distance from the Earth to the Moon. By observing lunar eclipses and carefully measuring how long it took for the Moon to travel under the shadow of the Earth, he was able to determine that the diameter of the Earth was about three and a half times the diameter of the Moon.



Aristarchus of Samos

You may produce a "lunar eclipse" by holding a coin in front of you so as to cover the Moon exactly. A little experimentation will show you that You can exactly cover the whole Moon with a quarter held about 9 feet in front of you.



1. The diameter of 1 US quarter is almost 1 in. Explain why is the following relationship true:

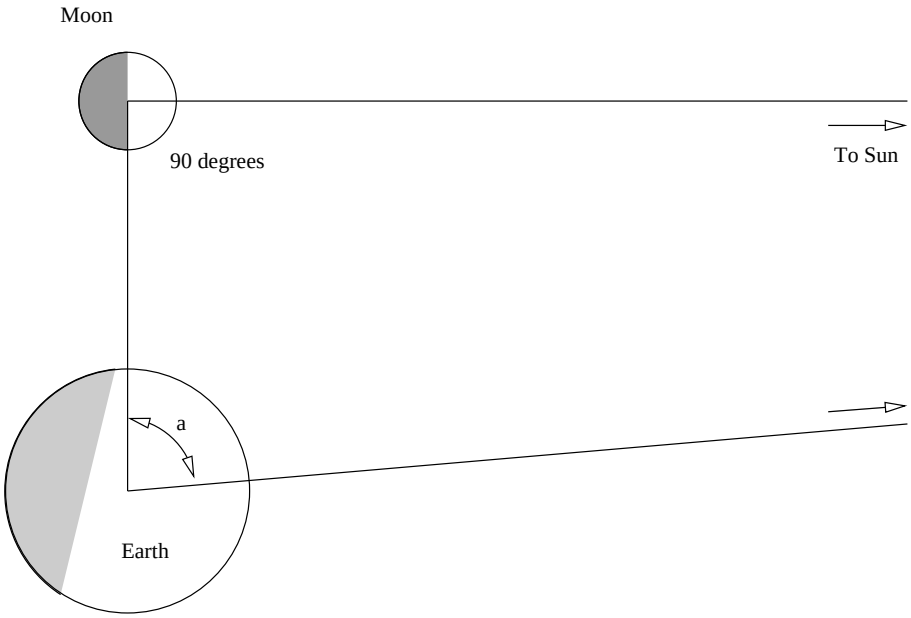
$$\frac{1\text{in}}{9\text{ft}} = \frac{\text{diameter of Moon}}{\text{distance to Moon}}$$

2. Taking into account the estimate that the diameter of the Earth is $3\frac{1}{2}$ times bigger than that of the Moon, what is the distance from the Earth to the Moon?

3. The actual mean distance to the Moon is $238,857$ miles.

1 How big is the Sun?

¶ 5. Estimating the distance from the Earth to the Sun was a more difficult problem. The ancient Greeks did attempt to obtain such an estimate, but they incurred in large errors. They method was based on the following diagram:



They assumed that, when the Moon appears to be exactly half full, the line from the Moon to the Sun was perpendicular to the line from the Earth to the Moon. Since the distance from Earth to Moon had already been estimated, the distance to the Sun could be found once the angle α was determined. However, the angle α turns out to be about $1/6$ of a degree smaller than a right angle, too small to measure accurately.

1. Aristarchus estimated the angle α to be 1 degree short of right angle. Using this value for the angle α, and the value of $3\frac{1}{7}$ for π, what was Aristarchus estimate for the distance to the Sun?

2. From this, Aristarchus concluded that the Sun was much bigger than the earth. How much bigger?

3. It turns out that the distance is about 93 million miles. In fact, as it was established later by Kepler and other astronomers, the distance is not constant, as the Earth revolves around the sun in an elliptical orbit, with the Sun located in one of its foci. The maximun distance id about 94.5 million miles and the minimun disatnce is about 91.5 million miles.

   Using this information, and assuming a circular orbit, how fast (in miles/hour) does the Earth travel around the Sun?