Benford's Law[1]

¶ **1.** Examine the list of numbers that you have been provided. You will be collecting some statistical information about those numbers

(a) How many numbers are there in your list?

(b) The first significant digit of a number is the first non-zero digit of that number, ignoring decimal points, if any. For example, the first significant digit of 01212 is 1, the first significant digit of 2.4579 is 2, and the first significant digit of 0.07843 is 7.

For each of the nine digits $d = 1, 2, \cdots, 9$, count how many of the numbers in your table have first significant digit equal to d, and record that value in the table below.

(c) Do the same for the last significant digit of the numbers in your list.

First Significant Digit d	Occurrences
1	
2	
3	
4	
5	
6	
7	
8	
9	

Last Significant Digit d	Occurrences
1	
2	
3	
4	
5	
6	
7	
8	
9	

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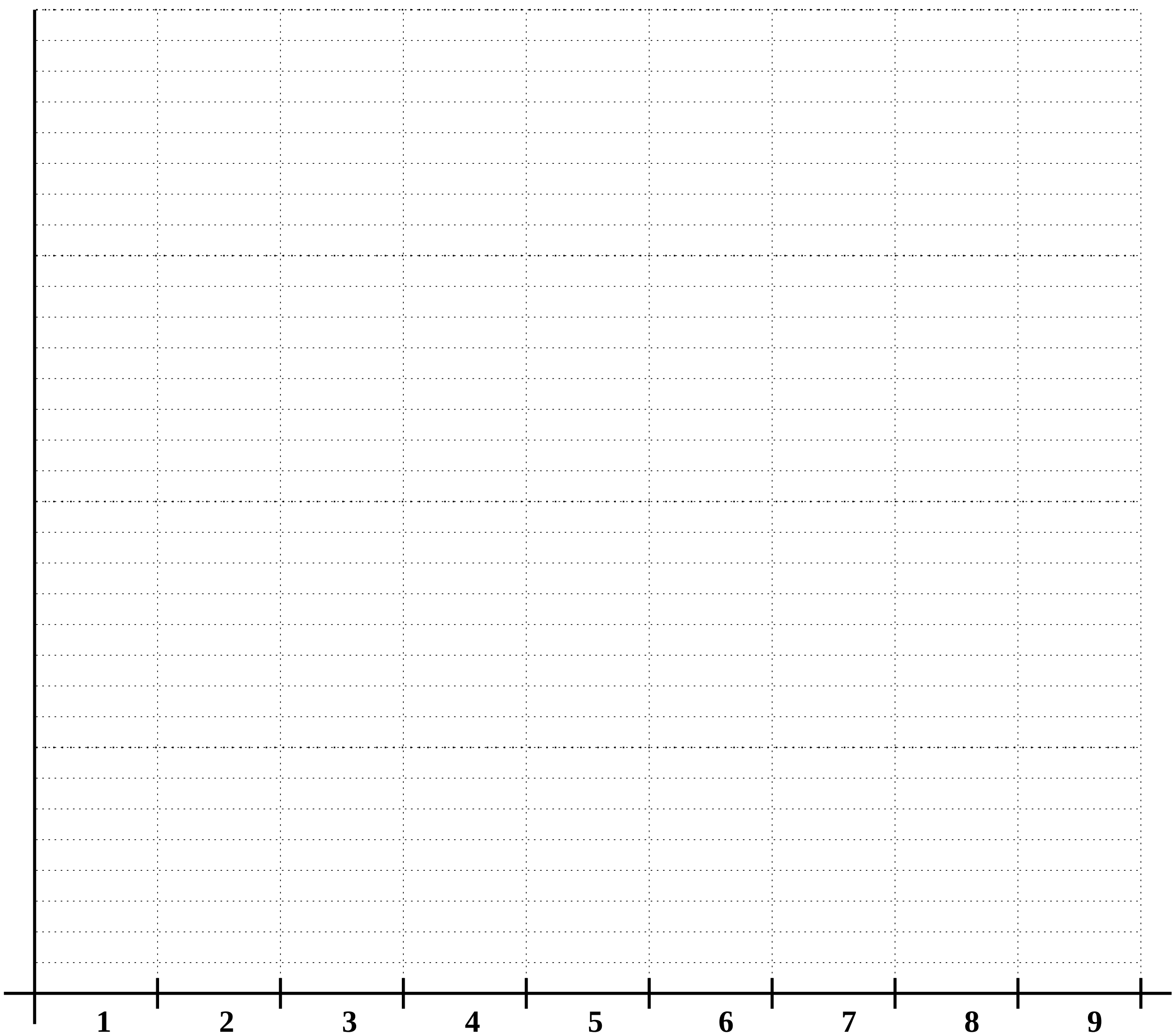
¶ **2.** The nature of the data that you have collected will be more apparent if you display it in a bar chart.

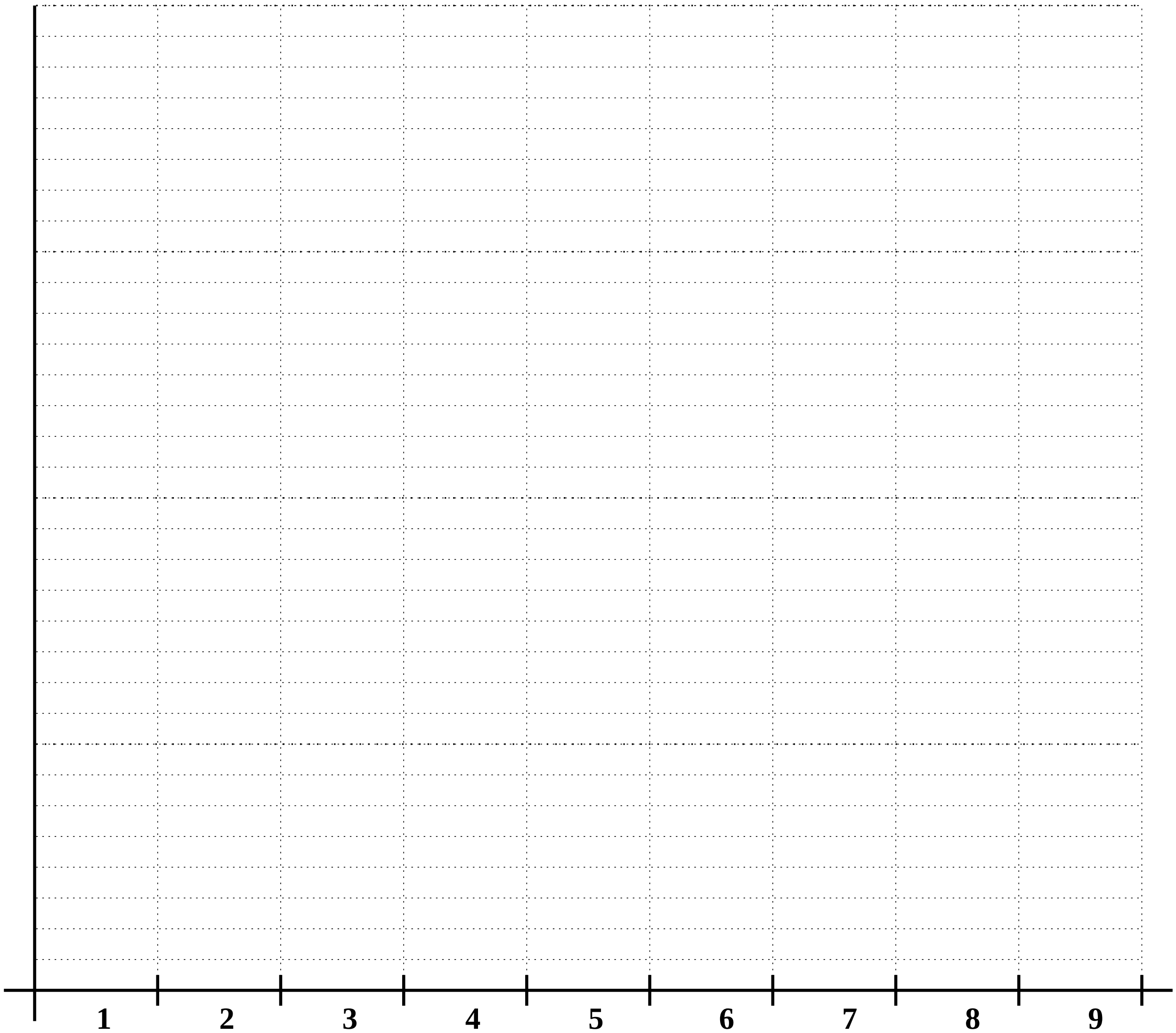
(a) Use the graph below to construct a bar chart showing the frequencies of the first significant digits of your numbers.

(b) Use the graph on the next page to construct a bar chart showing the frequencies of the last significant digits of your numbers.

(c) Compare your chart to the charts constructed by other students.



Bar Chart for the First Significant Digit



Bar Chart for the Last Significant Digit

¶ **3.** Benford's describes the fact that in many naturally occurring data sets, the first significant digit is more likely to be 1 or 2 that any other number. More precisely, it states that the probability that the leading digit be d is $p(d) = \log\left(1 + \frac{1}{d}\right)$.

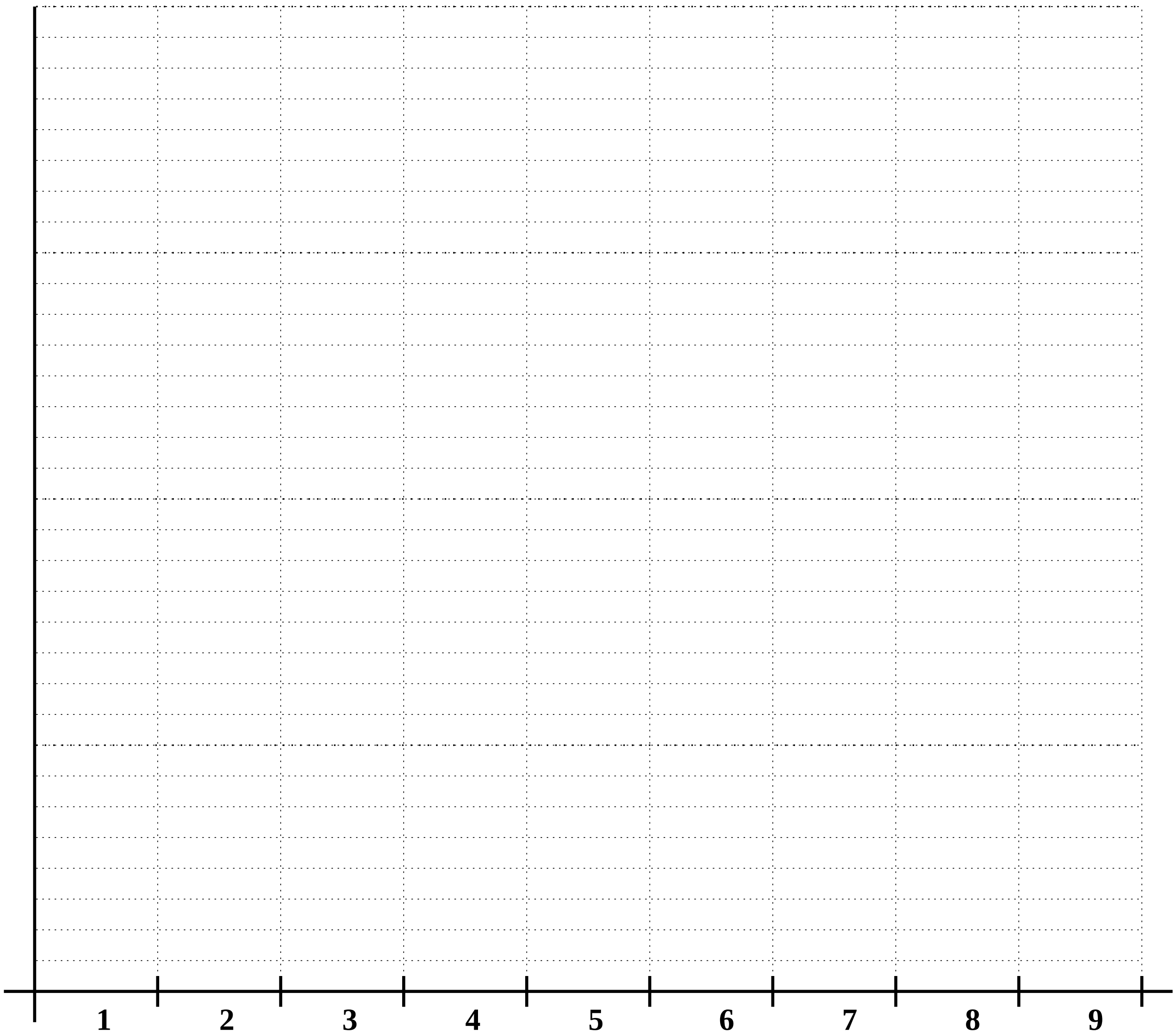
(a) Verify that $p(d)$ makes sense as a probability. That is, verify that the sum

$$p(1) + p(2) + p(3) + \cdots + p(9) = 1.$$

(b) Use this formula and your calculator to complete the following table of probabilities, and the :

d	$p(d)$
1	
2	
3	
4	
5	
6	
7	
8	
9	

(c) Now complete the bar chart below for that table of probabilities:



¶ **4.** You are the manager of a research team overseeing 3 employees. You have been using data collected by them and suspect that one of them is fabricating his or her numbers. After counting the occurrence of the first significant digits in each employee data set, you construct the table below. Which employee is most likely to have fabricated his or her data?

1st significant digit	Number of occurrences		
	Peter	Paul	Mary
1	2917	2876	4010
2	1825	1872	3412
3	3216	2145	2249
4	1245	3122	1981
5	1943	2678	2318
6	673	1988	1997
7	784	931	2023
8	921	2331	1491
9	256	2412	1530

¶ **5.** Assume, in a bull period, a market average indicator starts with an average of $1,000$ that grows 0.2% a day.

(a) Make a table with the market averages for the following 2 weeks.

(b) How long will it take for the market average to reach $2,000$?

(c) For how long will the market average indicator stay between $2,000$ and $2,999$?

(d) Determine the number of days of the first year during which the first digit of the indicator be a 1?

¶ **6.** Would you take this bet? Go to the library and pick a book at random. If the number of pages has leading digit 1, 2, or 3, then you pay me \$5; otherwise, if the number of pages has leading digit 4, 5, 6, 7, 8 or 9, then I pay you \$5. What about if you pay me \$2 if I win and I pay you \$3 if you win?

¶ **7.** The Fibonacci numbers are constructed recursively as follows: $F_0 = 1$, $F_1 = 1$ and $F_n = F_{n-1} + F_{n-2}$ for all $n = 2, 3, \cdots$. The first numbers are $1, 1, 2, 3, 5, 8, 13, 21, 44, \cdots$.

(a) Find the frequencies of the first significant digits of the first 100 Fibonacci numbers.

(b) How do they compare to the frequencies predicted by Benford's law?

(c) Try with different seeds and with different recurrence formulas. For example, $a_0 = 1$, $a_1 = 1$ and $a_n = na_{n-1} + n^2 a_{n-2}$ for $n \geq 2$.

(d) The Fibonacci numbers appeared in a problem originally posed by Leonardo de Pisa in one of his books. Find out more about Leonardo de Pisa, and about this problem and the book where it appeared.