

## Math 623. Homework 5. Due 04/21/04

**Problem 1.** Show that the hyperbolic distance  $d(z, w)$  between points  $z$  and  $w$  in the unit disk satisfies the following identity

$$\operatorname{Tanh} \frac{1}{2} d(z, w) = \left| \frac{z - w}{1 - \bar{z}w} \right|.$$

**Problem 2.** A circle in  $\mathbf{H}^2$  centered at  $p$  with radius  $r$  is the set of points at hyperbolic distance  $r$  from  $p$ . Show that (hyperbolic) circles are Euclidean circles (but perhaps with different center and radius).

**Problem 3.** What is the circumference of a circle of (hyperbolic) radius  $r$ ?

**Problem 4.** (a) What is the area of the (hyperbolic) disk of center 0 and (hyperbolic) radius  $r$ ? (Unit disk model).

(b) Let  $A(r)$  denote the area of a ball of (hyperbolic) radius  $r$  and  $L(r)$  the length of its circumference. Examine the ratio  $L(r)/A(r)$  as the hyperbolic radius  $r \rightarrow \infty$  and compare with the corresponding ratio in Euclidean geometry.

**Problem 5.** Show that the sum,  $S$ , of the interior angles of a hyperbolic polygon is always less than the sum of the angles of a Euclidean polygon with the same number  $n$  of sides, and the deficiency  $(n - 2)\pi - S$  is the area of the polygon.