Math 550. Homework 8. Due 11/10/03

Problem 1. Let U, V be open subsets of $\mathbf{R}^2$. Prove that the coboundary map $\delta : H^0(U \cap V) \to H^1(U \cup V)$ is a homomorphism of vector spaces.

Problem 2. Let $F : U \to V$ be a smooth map from the open set $U \subset \mathbf{R}^2$ into the open set $V \subset \mathbf{R}^2$. In a previous homework set we showed that F induces a "pull-back" operator F^* that sends n-forms on V into n-forms on U. Prove that F^* induces a linear map of vector spaces $F^* : H^n V \to H^n U$, for $n = 0, 1$.

Moreover, prove that if F is a diffeomorphism, then $F^* : H^n V \to H^n U$ is an isomorphism of vectors spaces.

Problem 3. Prove that if U and V are connected, and $H^1(U \cup V) = 0$, then $U \cap V$ is connected.

Problem 4. Prove that if the open set $U \subset \mathbf{R}^2$ can be written as an union $U = U_1 \cup \cdots \cup U_n$, where each U_j is a convex open set, then $H^1 U$ is a finite dimensional vector space.

Problem 5. Let $U \subset \mathbf{R}^2$ be the complement of n points. Prove that $H^1 U$ is a vectors space of dimension n, and find a basis for it.