Math 550. Homework 7. Due 10/29/03

Problem 1. A vector X in $\mathbf{R}^n$ is called a probability vector if its coordinates are all nonnegative and add up to 1. An $n \times n$ matrix is an stochastic matrix if its columns are probability vectors. Use the Brouwer fixed point theorem to prove that if A is a 3×3 stochastic matrix then there is a probability vector X such that $AX = X$.

Problem 2. Let f, g be two continuous mappings from X into the unit circle S^1. Prove that if $|f(x) - g(x)| < 2$ for all x in X, then f and g are homotopic.

Problem 3. Prove that if the sphere is cover by three closed subsets, then one of them must contain a pair of antipodal points.

Note. This is Proposition 4.33 in the textbook. The proof there is not complete; you have to provide a complete proof.

Problem 4. Prove that the sphere can be covered with four closed subsets, neither of them containing a pair of antipodal points.

Problem 5. Suppose that A is a connected closed subset of $\mathbf{R}^2$, and P is a point in A. Prove that $[\omega_{P,\theta}] = 0$ in $H^1(\mathbf{R}^2 \setminus A)$ if and only if A is unbounded.