Math 550. Homework 6. Due 10/22/03

Definition 1. Two continuous mappings $f, g : X \to Y$ are homotopic if there is a continuous mapping $H : X \times [0,1] \to Y$ such that $H(x,0) = f(x)$ and $H(x,1) = g(x)$ for all x in X.

Problem 1. Let C and C' be circles.

(i) Prove that if a continuous mapping $F : C \to C'$ is not surjective, then $\deg F = 0$.

(ii) Find an example of a continuous mapping $F : C \to C'$ that is surjective but has $\deg F = 0$.

Problem 2. (i) Prove that two continuous mappings $F, G : C \to C'$ are homotopic if and only if they have the same degree.

(ii) Conclude that a continuous mapping $F : S^1 \to S^1$ has degree n if and only if F is homotopic to the map $z \mapsto z^n$ of S^1 onto itself.

Problem 3. (i) Let $F, F' : X \to Y$ and $G, G' : Y \to Z$ be continuous mappings. Prove that if F is homotopic to F', and G is homotopic to G', then the composite $G \circ F$ is homotopic to $G' \circ F'$.

(ii) Let F, G be continuous mappings from the unit circle S^1 into itself. Prove that

$$\deg(G \circ F) = \big(\deg F\big) \cdot \big(\deg G\big).$$

Problem 4. Suppose that F is a continuous mapping from the positive octant $\{(x,y,z) \mid x \geq 0, y \geq 0, z \geq 0\}$ to itself. Show that there is a unit vector P in this octant, and a nonnegative number λ, such that $F(P) = \lambda P$.

Problem 5. Let $f : C \to C'$ be a continuous mapping between circles.

(i) Prove that if $f(P^*) = f(P)$ for all P, then the degree of f is even.

(ii) Prove that if $f(P^*) \neq f(P)$ for all P, then f is surjective.