Math 550. Homework 1. Due 9/3/2003

Problem 1 Let U be an open subset of the plane. Prove that U is connected if and only if every locally constant function on U is constant on U.

Problem 2 Let $\omega = ydx - xdy$ and let γ be the line segment from $(0,0)$ to $(1,1)$. Compute the path integral

$$\int_\gamma \omega.$$

Problem 3 Let f, g be two smooth functions on U. Prove that $df = dg$ on U if and only if $f - g$ is locally constant on U.

Problem 4 Let ω_θ be the 1-form on $\mathbf{R}^2 \setminus \{(0,0)\}$ given by

$$\omega_\theta = \frac{1}{x^2+y^2}\Big(-ydx + xdy\Big).$$

On which of the following open sets U is there a smooth function g with $dg = \omega_\theta$ on U? (Prove your answers.) (i) The upper half plane $\{(x,y) \mid y > 0\}$. (ii) The union of the upper half plane and the right half plane. (iii) The annulus $\{(x,y) \mid 1 < x^2 + y^2 < 2\}$.

Problem 5 Is

$$\omega = \frac{1}{(x^2+y^2)^2}\Big(xdx + ydy\Big)$$

the differential of a function on $\mathbf{R}^2 \setminus \{(0,0)\}$?