

Math 512A. Homework 8. Due 10/7/07

Problem 1. Find the maximum value of $f(x) = x^3 - 9x$ in the interval $[-3, 3]$. **Note:** No derivatives!

Problem 2. Find an integer n such that the polynomial equation $x^3 - x + 3 = 0$ has a solution between n and $n + 1$.

Problem 3. Prove that there is some number x such that $\sin x = x - 1$.

Problem 4. (i) Suppose that f is continuous on the interval $[0, 1]$ and that $0 \leq f(x) \leq 1$ for all x in $[0, 1]$. Prove that $f(x) = x$ for some number x in $[0, 1]$.

(ii) Let f be continuous and bounded above and below on \mathbf{R} . Prove that there is some number x such that $f(x) = x$.

Problem 5. A function f defined on an interval I has the Intermediate Value Property on I if for any two numbers $a < b$ in I and every x strictly between $f(a)$ and $f(b)$, there is c in (a, b) such that $f(c) = x$.

(i) Prove that the function f given by $f(x) = \sin 1/x$ if $x \neq 0$ and $f(0) = 0$ has the Intermediate Value Property on the interval $[0, b]$, for any $b > 0$.

(ii) Prove that if f is nondecreasing on the interval I and has the Intermediate Value Property on I , then f is continuous on I . (Recall that f is said to be nondecreasing on I if $f(x) \leq f(y)$ whenever $x < y$ in I .)