Math 350. Quiz 6 Prep

Quiz 6 will feature one of more of the following questions, or a variation thereoff.

Problem 1. Prove that if f is continuous and $K \subset \mathrm{dom}(f)$ is compact, then f is uniformly continuous on K,

Problem 2. (i) Suppose that f is real valued and continuous on the interval $[0, 1]$, and that $0 \leq f(x) \leq 1$ for all x in $[0, 1]$. Prove that $f(x) = x$ for some number x in $[0, 1]$.

(ii) Let $f : \mathbf{R} \to \mathbf{R}$ be continuous and bounded above and below. Prove that there is some number x such that $f(x) = x$.

Problem 3. Prove that if $f : \mathbf{R} \to \mathbf{R}$ is a continuous function and $f(x)$ is a rational number for any x in $\mathbf{R}$, then f is constant.