

Math 350. Quiz 4. Date: 2/25/09

Name: _____

Problem 1. A sequence x_1, x_2, x_3, \dots of real numbers is called a Cauchy sequence if for any $\epsilon > 0$ there is a natural number N such that, if $n, m > N$, then $|x_n - x_m| < \epsilon$.

Prove that a sequence of real numbers is convergent if and only if it is a Cauchy sequence.

Problem 2. Let S be a subset of $E = \mathbf{R}^k$ and let p be a point of E (p may or may not be in S). Prove that the following are equivalent:

1. Any ball $B(p, r)$ contains a point q of S such that $q \neq p$. That is, for any $r > 0$, the intersection $(B(p, r) \setminus \{p\}) \cap S \neq \emptyset$.
2. Any ball $B(p, r)$ contains infinitely many points of S .
3. There is a sequence q_1, q_2, q_3, \dots of points of S such that $q_n \neq p$ and $\lim_{n \rightarrow \infty} q_n = p$.