

Math 350. Quiz 3. Date: 2/18/09

Name: _____

Problem 1. Prove that the set S of real numbers given by $S = \{x \in \mathbf{R} \mid 1 < |x|\}$ is open.

Solution. There is a variety of ways of solving this problem. One of them (worked out in class 2/18) is to verify that S satisfies the definition of open set: for every x in S there is a ball with center x contained in S .

Note that S is the union of two intervals $S = (-\infty, -1) \cup (1, \infty)$. If x is in S , then either $x > 1$ or $x < -1$. If $x > 1$, then $x - 1 > 0$ and the ball with center x and radius $x - 1$, which is just the interval $(1, 2x - 1)$, is contained in S . Similarly, if $x < -1$, then the ball with center x and radius $-1 - x > 0$ is contained in S .

You could also show that S is open by showing that the complement of S is closed. The complement of S is the interval $[-1, 1]$. This is a closed set because it is the closed ball with center 0 and radius 1. Or because if x_n is a sequence in $[-1, 1]$ and $\lim_{n \rightarrow \infty} x_n = x$, then x is in $[-1, 1]$, by the “squeeze lemma.” \square