Math 350. Quiz 1. Date: Name: _____

Problem 1. Find, giving reasons, the least upper bound and the greatest lower bound of the set A of real numbers given by

$$A = \left\{0, \frac{1}{2}, \frac{2}{3}, \frac{3}{4}, \frac{4}{5}, \cdots\right\} .$$

Solution. The set A consist of all numbers of the form $\dfrac{n-1}{n}$ where $n = 1, 2, 3, \cdots$ is a positive integer. Since 0 is a lower bound for A that also belongs to A, it must be that $0 = \text{g.l.b.}\, A$.

The set A is bounded above: for any n in N, we have $n - 1 < n$ and so $\dfrac{n-1}{n} < 1$. Hence 1 is an upper bound for A. Moreover, if $x < 1$, then $1 - x > 0$ and there is, by the Archimedian property, a positive integer n such that $\dfrac{1}{1-x} < n$. Writing this inequality in the form $x < 1 - \dfrac{1}{n} = \dfrac{n-1}{n}$ we see that no $x < 1$ can be an upper bound for A. That is, if x is an upper bound for A, then $x \geq 1$ and so $1 = \text{l.u.b.}\, A$. □