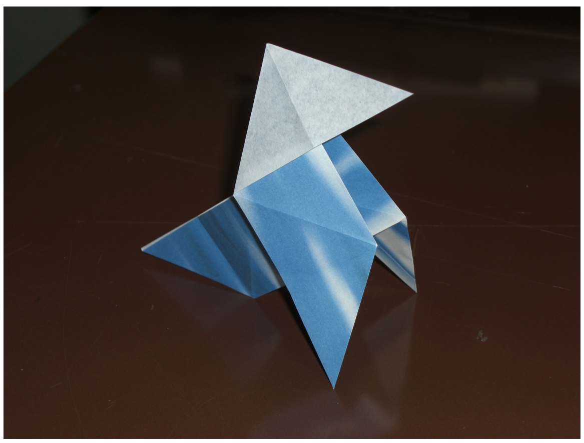
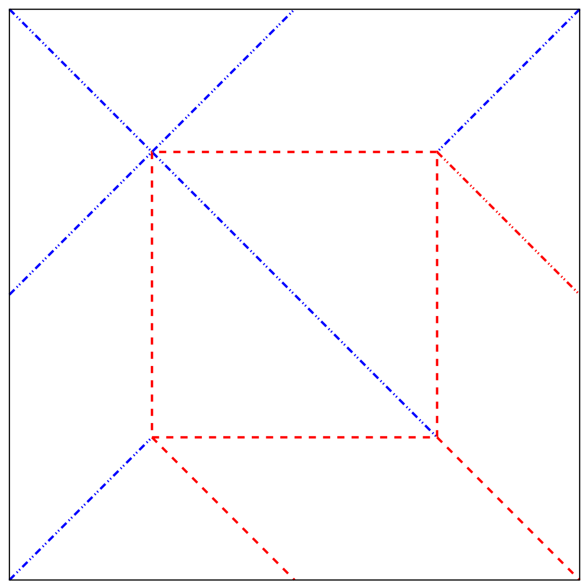
¶ **1.** Computational Origami is a rapidly growing area of computational geometry with many applications in computer science and engineering. One of the big open problems is to devise a computer program and to design a machine that could fold a sheet of material into any origami object. The computational complexity of such program is tremendous. But applications are equally tremendously important: airbag design, machine folding, protein folding, and a variety of others.
In this activity we well examine some basic aspects of this problem and try to determine if a crease pattern drawn in a sheet of paper can be folded flat.

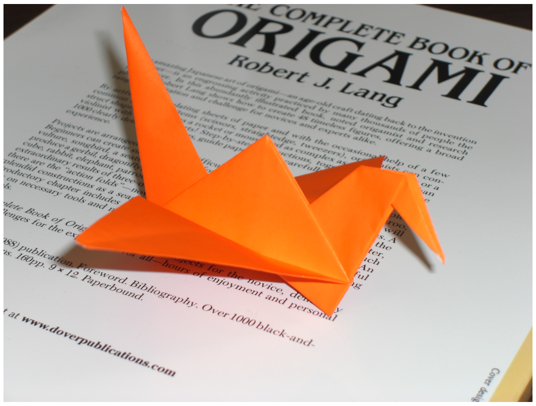
¶ **2.** If we fold an origami figure, the creases along which folds have take place form a pattern of segments in the square sheet called the **crease pattern** of the figure. The crease pattern of an origami figure is like the blueprint of a house. However, it may be quite difficult to realize the origami figure from the crease pattern: the order in which the folds take place is important; often intermediate creases are formed which are not part of the crease pattern. A basic example that illustrates this is the **pajarita**, a traditional origami model from Spain.

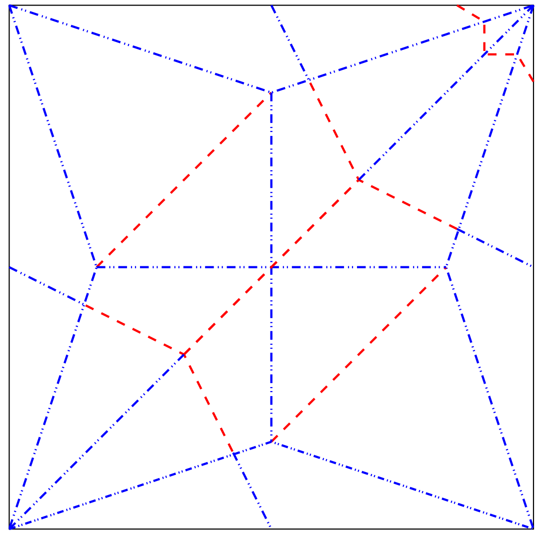




The diagram on the right is the crease pattern for the pajarita. You will notice that your paper has more creases than are drawn here. Those are the intermediate creases, used to construct landmarks for subsequent creases, but not part of the creases in the final model. Creases in the pattern come in two flavors: mountain creases (drawn as $- \cdots - \cdots -$) and valley creases (drawn as $- - - - - -$). For the purpose of this activity, we need not distinguish them.
The pajarita is one example of a flat origami model or 2D origami model: it can be pressed flat between the pages of a book without crushing it (ignoring the thickness of the paper).
Not all origami model are flat. For example, the body of your car is not a flat origami model; it can be pressed flat between two metal plates, as they do in scrap metal plant, but that alters considerably its appearance.

¶ **3.** Another traditional model is the flapping bird shown below.





Our first problem is to understand some basic facts about the computational geometry of origami. For example, you draw a pattern of segments and ask the question: is this the crease pattern of a flat origami model?

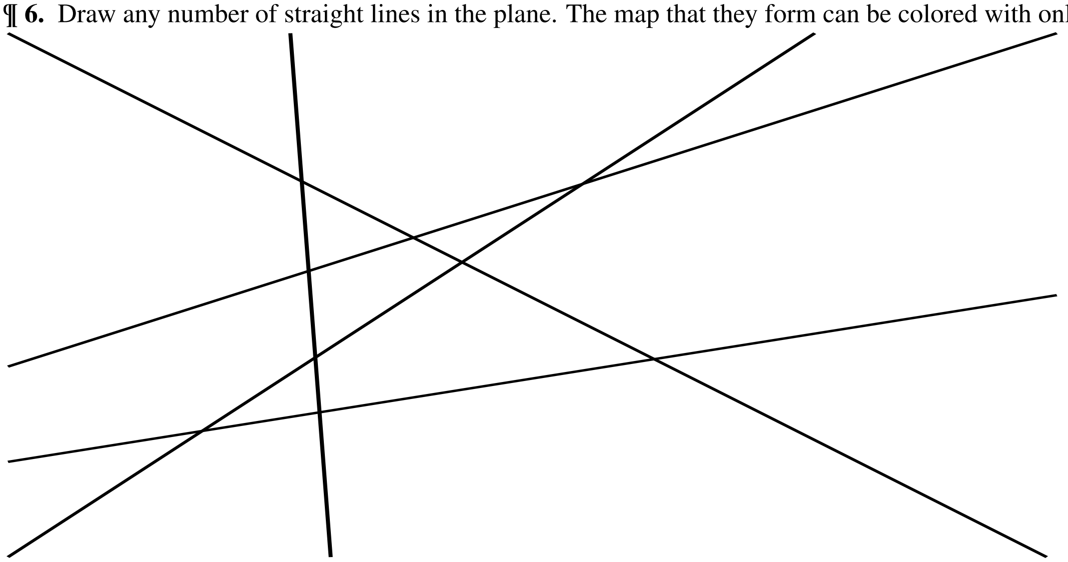
¶ **4.** (a) Draw the crease pattern for your flapping bird model. For that, carefully unfold your model and draw with a pen the creases that are actually part of the finished design, ignoring all other creases that played an intermediate role.

(b) Then color the faces of your crease pattern with as few colors as possible in such a way that no two adjacent faces (that is, no two faces that share a whole edge) have the same color. What is the fewest number of colors that you can use?

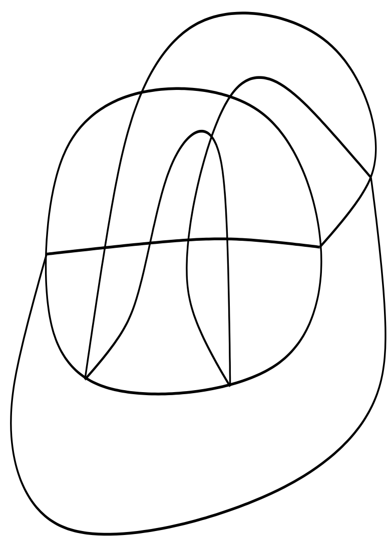
(c) What will the coloring look like when you refold your model?

¶ **5.** The coloring activity should already give you a hint as to a necessary condition for a pattern to be the crease pattern of a flat origami model. State your conjecture.

¶ **6.** Draw any number of straight lines in the plane. The map that they form can be colored with only two colors.



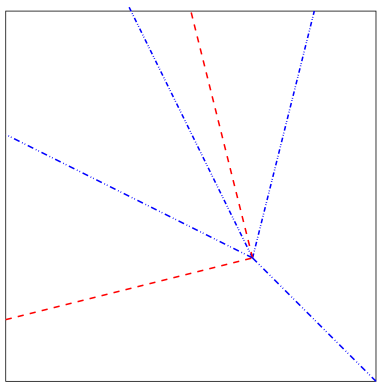
¶ **7.** You are given a drawing of a map. Each country is represented by a region, as in the diagram below. Now you travel through all the countries in the map as if you were driving by car: if you are in country A you can go to country B if and only if countries A and B share a boundary.

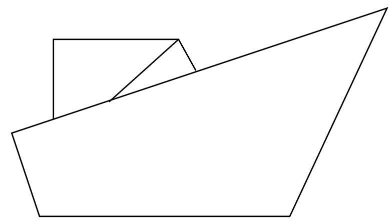


For this map, find a travelling plan so that you visit all the countries exactly once. Number the countries in the order in which you visited them, and the color those countries that have an even number.

¶ **8.** If you can color the regions of a map using only two colors, then at any boundary point is shared by an even number of regions.

¶ **9** (Realizing crease patterns)**.** Next we examine in more detail single vertex folds of flat models. Here we take into account whether the creases are mountains or valleys. The general problem is to determine which patterns are possible at a vertex of a flat model. Here is one example:



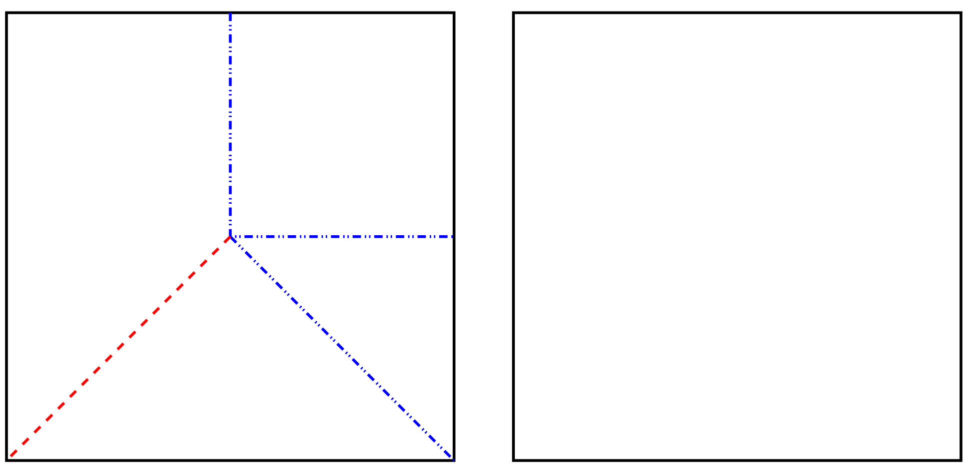


Boat

¶ **10.** For the single vertex crease pattern on the left:

(a) Can you fold flat the single vertex crease pattern below with the mountain-valley creases as assigned?

(b) Can you rearrange the mountain-valley crease assignment so that the pattern can be folded flat?



¶ **11.** Take a square piece of paper and make a single vertex crease pattern that folds flat. Place the vertex near the center of the square, draw some crease lines coming out of it, and then add more as needed to make the whole thing fold flat. The purpose of this problem is to formulate as many conjectures as you can about how such folds work. Use the blank squares below to record your vertex crease patterns before formulating you conjecture or conjectures.

Conjectures:

(a)

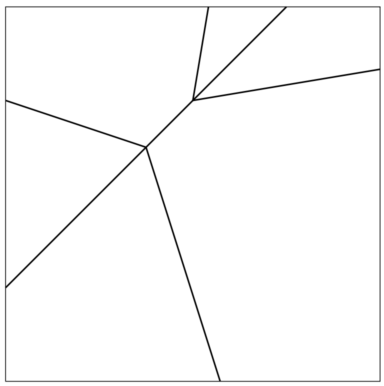
(b)

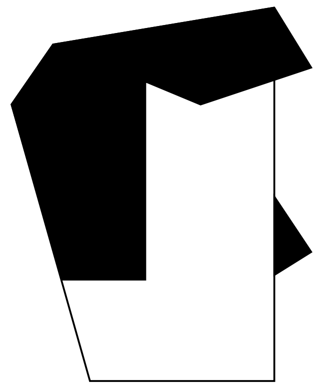
Theorems:

(a)

(b)

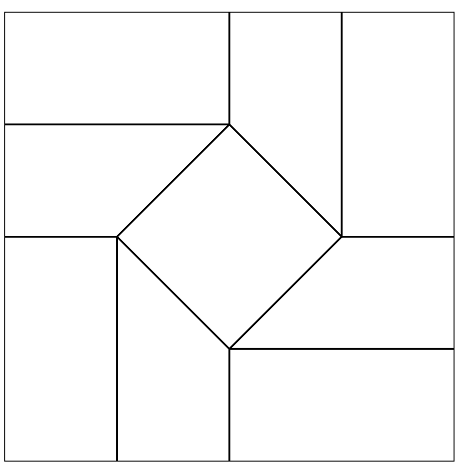
¶ **12** (Folding flat crease patterns)**.** The previous problems concerned with what happen at a single vertex of a crease pattern. The crease pattern of a flat origami figure will have many vertices, and it is expected that there would be some interaction between them.



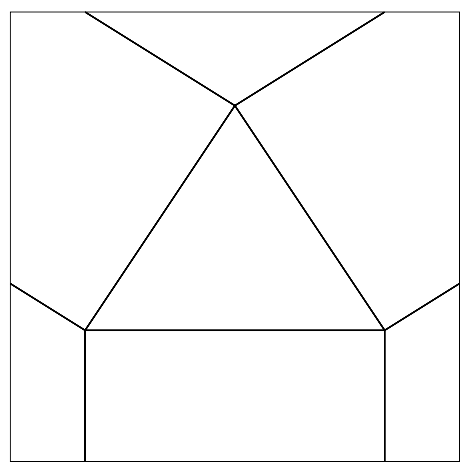


Elvis' Hairpiece (after Peter Budai)

¶ **13.** Given a pre-crease pattern on a square sheet of paper, we try to answer the question: is it possible to make a mountain-valley assignment to the segments so that the resulting pattern is in fact the crease pattern of a flat origami model? While it may be possible to realize each vertex individually, it may not be possible to realize the whole pattern.
Here are two examples. Your task is to find out what they can fold into. You may not add more crease lines, but you have the choice of declaring which creases are valleys and which are mountains.

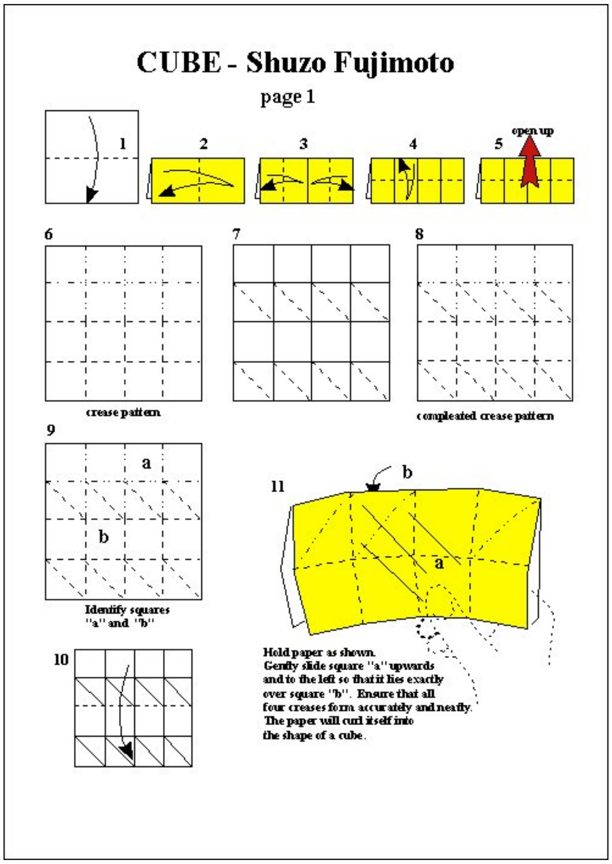


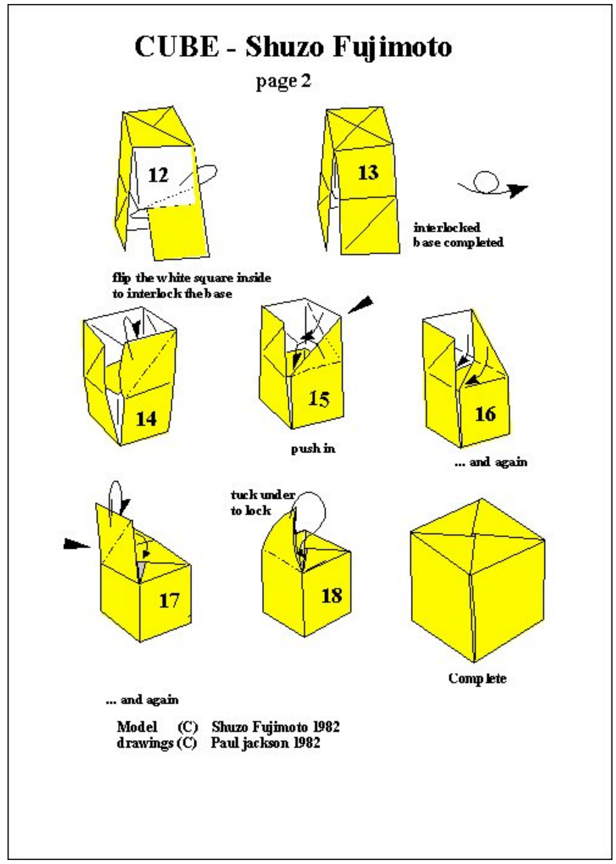
(Twist Pattern)



(T. Hull Pattern)

¶ **14** (Fujimoto Cube). Here are the crease pattern and folding isntructions for the Fujimoto Cube.

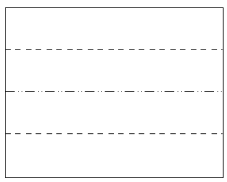




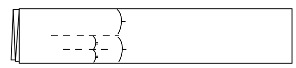
¶ **15.** Here are the instructions for the Miura Map Fold (by Thomas Hull)

The Miura Map Fold

This fold was invented by the Japanese astrophysicist Koryo Miura as a method for deploying large solar panel arrays on space satellites. It's also been used to easily fold and unfold maps.

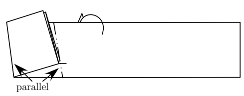


(1) Take a rectangle of paper and mountain-valley-mountain fold it into 1/4ths lengthwise.

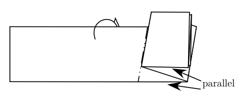


(2) Make 1/2 and 1/4 pinch marks on the side (one layer only) as shown.

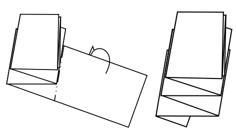
(3) Folding **all layers**, bring the lower left corner to the 1/4 line, as in the picture.



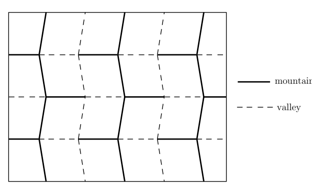
(4) Fold the remainder of the strip behind, making the crease parallel to the previous crease.



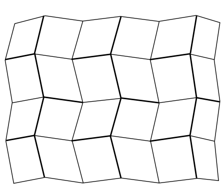
(5) Repeat, but this time use the fold from step (3) as a guide.



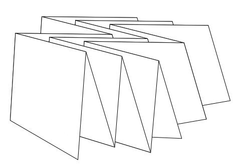
(6) Repeat this process until the strip is all used up. Then **unfold everything**.



(7) Now re-collapse the model, but change some of the mountains and valleys. Note how the zig-zag creases alternate from all-mountain to all-valley. Use these as a guide as you collapse it...



In the end the paper should fold up neatly as shown to the left. You can then pull apart two opposite corners to easily open and close the model.



Literature

[1] Thomas Hull, *Project Origami*, A.K.Peters Ltd., 2006.

[2] Erik Demaine and Joseph O'Rouke, *Geometric Folding Algorithms*, Cambridge University Press, 2007.