Math 311. Polyhedra A Candel
Name: CSUN Math

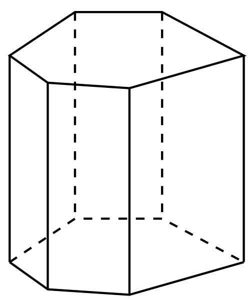
¶ **1.** A *polygon* may be described as a finite region of the plane enclosed by a finite number of segments, arranged in such a way that (a) exactly two segments meets at every vertex, and (b) it is possible to go from any segment to any other segment by crossing vertices of the polygon.

A *polyhedron* (plural: polyhedra) is a three dimensional figure enclosed by a finite number of polygons arranged in such a way that

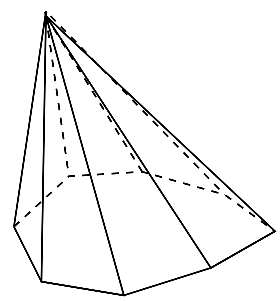
(a) exactly two polygons meet (at any angle) at every edge.

(b) it is possible to get from every polygon to every other polygon by crossing edges of the polyhedron.

¶ **2.** The simplest class of polyhedra are the simple polyhedra: those that can be continuously deformed into spheres. Within this class we find prisms and pyramids.



Prism



Pyramid

If you round out the corners and smooth out the edges, you can think of a polyhedra as the surface of a spherical body. An so you may also think of the polyhedra, with its faces, edges, and vertices, as determining a decomposition of the surface of the sphere.

¶ **3.** A polyhedron is called convex if the segment that joins any two of its points lies entirely in the polyhedron. Put in another way, a convex polyhedron is a finite region of space enclosed by a finite number of planes.

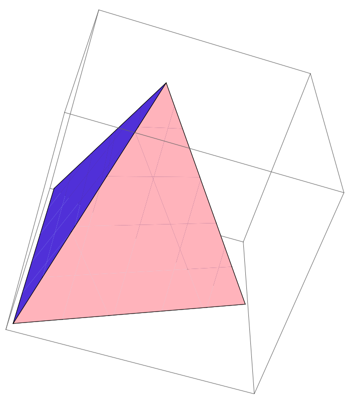
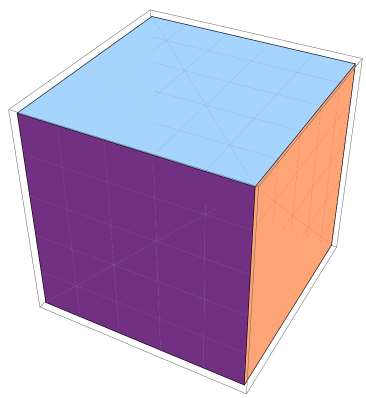
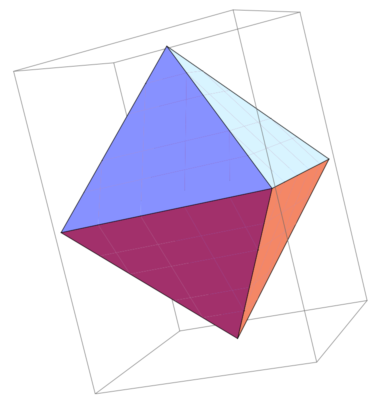
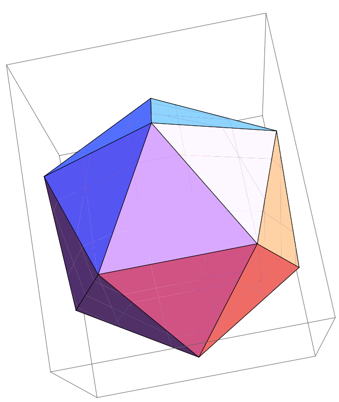
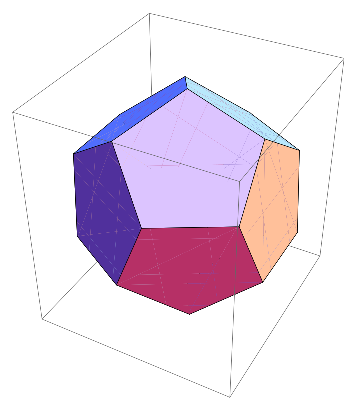
Convexity is a geometric property, it can be destroyed by deformations. It is important because a convex polyhedron is always a simple polyhedron.

(a) Can you draw a simple polyhedron that is not a convex polyhedron?

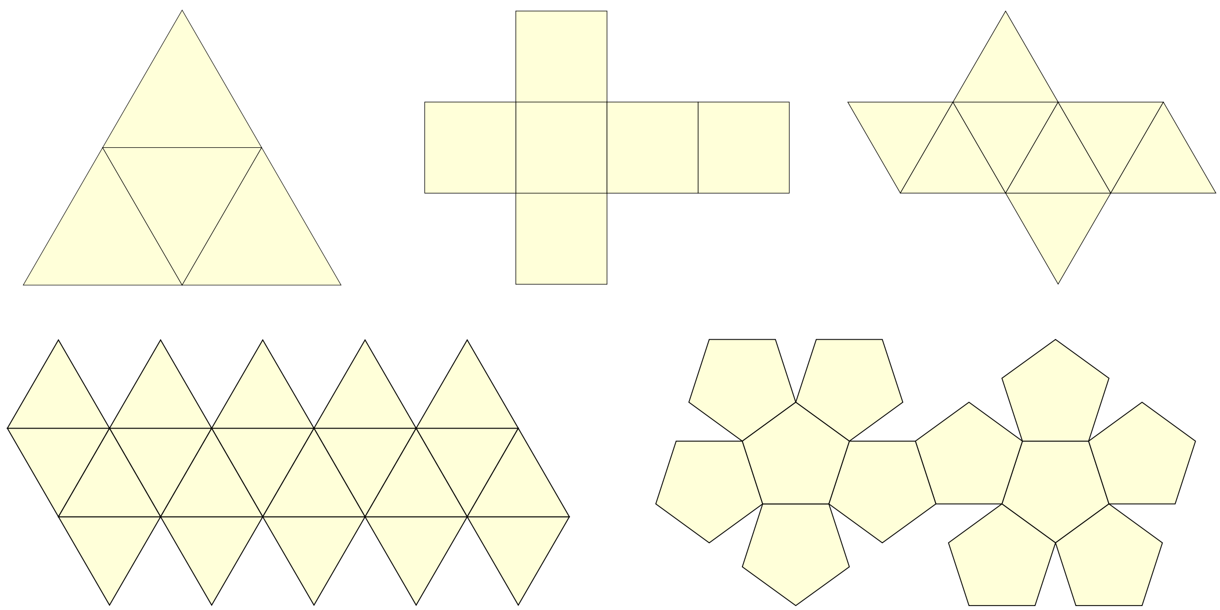
¶ 4. A (convex) polyhedron is called a *regular convex polyhedron* if all its faces are congruent to a regular polygon, and all its vertices are surrounded alike. Plain experimentation with sticks will allow you to easily construct 5 regular polyhedra: tetrahedron, cube, octahedron, icosahedron and dodecahedron.

These were know to the Greeks; they were regarded by Plato as symbolizing the four basic elements: fire, earth, air, and water. The fifth polyhedron, the dodecahedron, he assigned to the whole universe. Today, these polyhedra are also called *Platonic Solids*.

¶ 5. Here are pictures of the five Platonic solids.

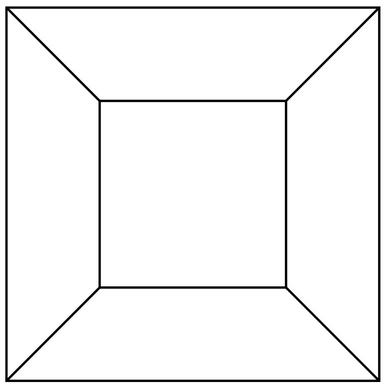






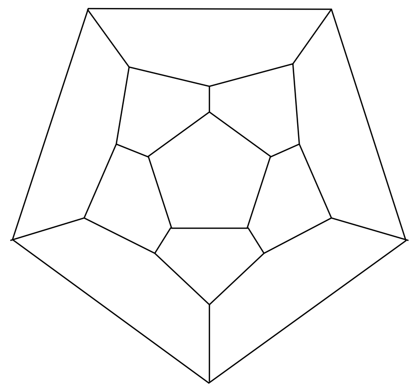
¶ **6.** When you cut apart a polyhedron along its edges and laid it flat on the plane, you obtain the net of the polyhedron. Here are the nets of the Platonic solids:



(a) Draw a prism and a pyramid (with any polygon as base), and then sketch a net for it.

¶ **7.** Another way of obtaining a planar diagram for a polyhedron is by means of the o called Schlegel diagram. Imagine that the polyhedron is a wire frame model with clear faces, and that a light is suspended directly over one the center of one of its faces. The wires (that is, the edges) will cast a shadow on the plane where the polyhedron rests. For example, here are the Schlegel diagrams for a cube and for a dodecahedron





(a) Draw a Schlegel diagram for a tetrahedron.

(b) Draw a Schlegel diagram for a octahedron.

(c) Draw a Schlegel diagram for a icosahedron.

¶ 8. We can classify the regular polyhedra by a method similar to the method used to classify the regular and semiregular tilings of the plane.

For a regular polyhedron:

(a) what is the least number of (regular) polygons meeting at any vertex?

(b) what is the greatest number of (regular) polygons meeting at any vertex?

¶ 9. The Schläfli symbol of a regular polyhedron whose faces are regular polygons with `p` sides, and `q` of meeting at any vertex is `p.....p` (a `p` repeated q times).

For example, the Schläfli symbol of a cube is `4.4.4`. Fill out the following table for all the Platonic solids.

Polyhedron	Face type	Symbol	F (# faces)	E (# edges)	V (# vertices)
Tetrahedron					
Cube					
Octahedron					
Dodecahedron					
Icosahedron					