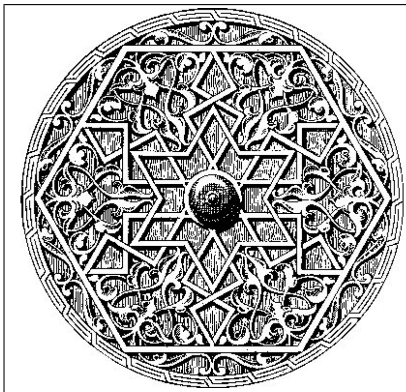
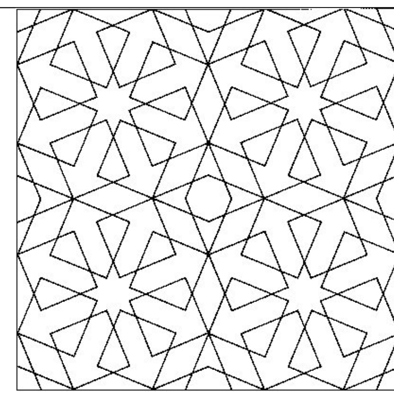
¶ 1. Symmetries of patterns

Given a pattern, we analyze it by determining which rigid motions preserve the pattern. These are referred as the symmetries of the pattern, and form a group. Pattern in the plane are usually divided into three groups:

- those that repeat in no direction (rosette patterns).
- those that repeat in exactly one direction (frieze patterns)
- those that repeat in more than one direction (wallpaper patterns)

		
(Rosette)	(Frieze)	(Wallpaper)

¶ **2. Mirror Symmetry** If a figure can be divided by a line so that the part on one side of that line is the mirror image of the part of the figure on the other side then we say that the figure has mirror symmetry, and the line marking the division is called the mirror line (or reflection line, or line of symmetry).

Many letters in the alphabet have one or more mirror lines. (Sometimes that depends on the font.) Many numerals (both Arabic or roman) have mirror lines. Identify the mirror lines for each of the letters or words below.

A	★	WAVYTUMMUTYVAW
B	✻	XIXC
H	bdbdbd	❈
pId	✼	MOON
bId	✾	ABRACADABRA
➽	dbpqdb	WOW
✖		31

¶ **3.** Identify the mirror lines (if any) of the following commercial logos:

¶ **4. Rotational Symmetry.** Some figures remain unchanged when rotated about their center by certain angle; they have gyrational or rotational symmetry. We say that a figure has rotational symmetry of order n if it coincides with itself exactly n times in one complete turn around its center. Thus, the figure coincides with itself after a rotation by an angle of $360°/n$.

Identify the order of rotational symmetry of the following figures:

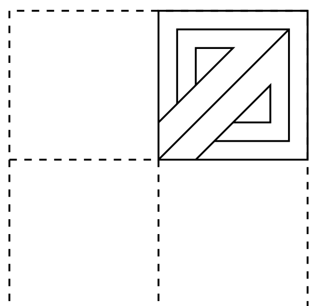
⬠	✱	❀
⬡	✷	❈
⯃	✾	❂

¶ **5. Rosette Patterns** A rosette pattern describes the possible symmetries of a flower, or a pinwheel. The repetition aspects of a symmetry describes the petals around stem. Translations or glide reflections do not come into play. The pattern is preserved under a rotation by certain angles corresponding to the number of petals on it. There may or may not be mirror lines, that is, reflections that preserve it.

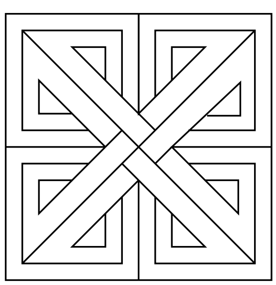
Rosette patterns were classified by Leonardo da Vinci. He realized that there were two classes of rosettes: the ones with mirror lines (reflection symmetry), like a sunflower, and the ones without mirror lines, like a pinwheel. The notation is D_n (for dihedral) and C_n (for cyclic), where n indicates the number of times that the rosette pattern coincides with itself in one complete turn around the center (thus n is largest number such that a rotation by $360°/n$ about its center brings the pattern into coincidence with itself).

When constructing rosette patterns with a given rosette group, it is important to take into account that combination of two reflections in non-parallel lines is a rotation about the point of intersection by twice the angle between those lines.

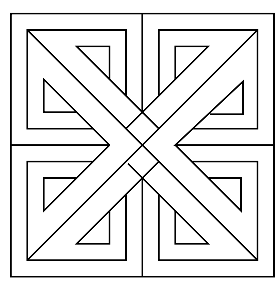
¶ **6.** Step patterns are found in Celtic manuscripts, metal work and stone crosses. Square one were constructed by first designing one quarter of the pattern (like the top right corner) using horizontal, vertical and diagonal lines to produce a unit like the following (Mark Lynch, Glasgow Calendonian Univ., Scotland):



Then three copies were added to construct the full square, either by (a) rotating the original successively by 90°; or (b) reflecting it across its left and bottom edges, giving rise to the two designs pictured below:



(a)



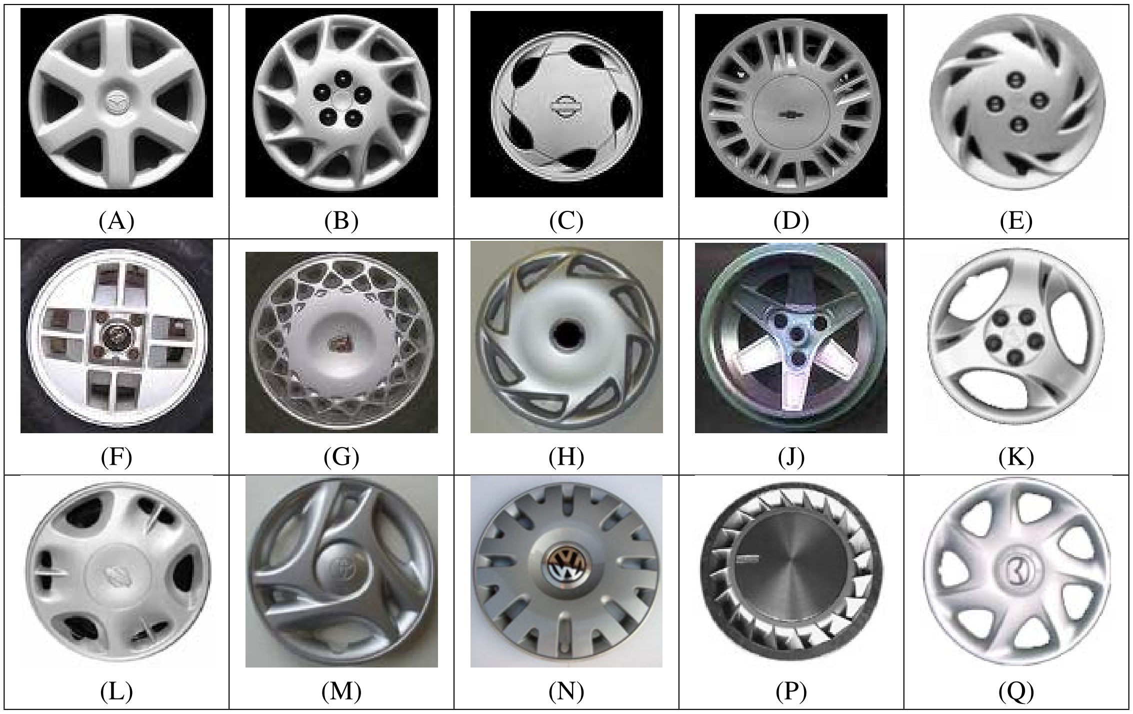
(b)

Identify the rosette group of each pattern.

(a) ____________________

(b) ____________________

¶ **7.** Identify the symmetry groups (e.g, give the notation such as C_4 or D_7) for the symmetry groups of the rosettes in the car wheel hubcaps pictured below (disregard the logos and bolts, if any, in the centers).



(A) ______________ (B) ______________ (C) ______________ (D) ______________ (E) ______________

(F) ______________ (G) ______________ (H) ______________ (J) ______________ (K) ______________

(L) ______________ (M) ______________ (N) ______________ (P) ______________ (Q) ______________

¶ **8.** Use the pentagons picture below to draw rosette patterns that have symmetry groups D_5 and C_5.