Math 103. Practice 1st Midterm. Solutions

Problem 1 Consider the line through $(-2, 7)$ and $(3, -5)$.

1. Find an equation for that line.

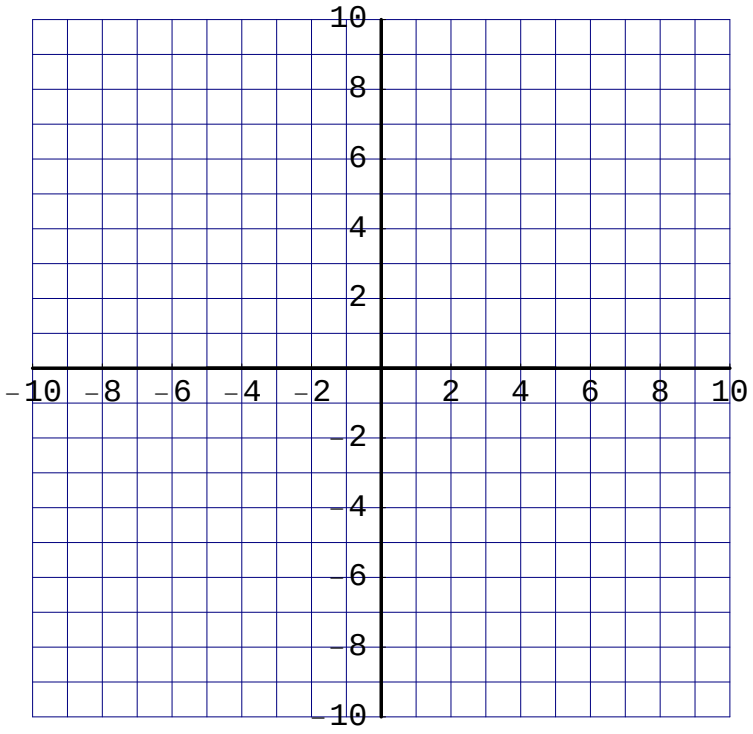
   $\boxed{12x + 5y - 11 = 0}$ or $\boxed{y = -\frac{12}{5}x + \frac{11}{5}}$

2. Find the x and y intercepts.

   The x-intercept is $\boxed{\left(\frac{11}{12}, 0\right)}$

   The y-intercept is $\boxed{\left(0, \frac{11}{5}\right)}$

3. Graph that line on the axis below. Make sure to label 2 points on the line.



Problem 2 The cost of incurred by the Math Department in preparing the Calculus Lecture Notes for printing was \$5700. Printing charges are \$2 per copy. The Math Department determined that the price-demand equation for selling the Lecture Notes is $p = 27 - 2x$, where p is measured in hundreds of dollars and x is measured in hundreds of copies sold.

(a) Determine a cost function $C(x)$, where $C(x)$ is in hundreds of dollars and x is in hundreds of copies sold, for the Calculus Lecture Notes.

$\boxed{C(x) = 57 + 2x}$

(b) Write an expression for the revenue function $R(x)$ (in hundreds of dollars) as a function of x.

$\boxed{R(x) = 27x - x^2}$

(c) For what values of x will the Math Department break even?

$\boxed{x = \frac{19}{2} \text{ and } x = 3}$

(d) For what values of x will a profit occur?

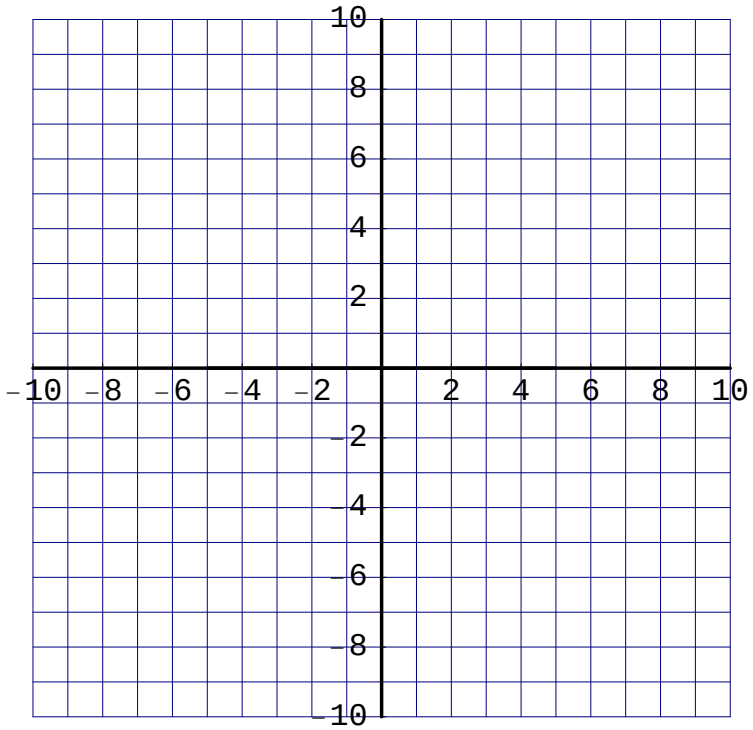
$\boxed{(3, 19/2)}$

(e) For what values of x will a loss occur?

$\boxed{[0, 3) \cup (19/2, \infty)}$

Problem 3 Consider the function $f(x) = -\frac{1}{2}|x - 4| + 8$

(a) On the axes below, sketch the graph of f. Be sure to label at least 2 points on the graph.



(b) Determine the range of f (interval notation)

(c) What are the x-intercepts?

Problem 4 The marketing research department for a company that manufactures and sells notebook computers has determined that the cost is modeled by the cost function $C(x) = x^2 + 2x$, where x is measured in number of notebooks and $C(x)$ is measured in thousands of dollars.

(a) What is the cost of producing 100 notebooks?

$\boxed{C(100) = 10200}$

(b) Evaluate and simplify $C(x + 1) - C(x)$.

$\boxed{2x + 3}$

(c) Explain what $C(1001) - C(1000)$ means.

$\boxed{\text{This is the cost of producing the 1001th notebook.}}$

Problem 5 For the function $f(x) = -2x^2 - 12x + 5$, determine:

(a) The vertex.

$\boxed{(-3, 23)}$

(b) The y-intercept. $\boxed{(0, 5)}$

(c) The x-intercept(s).

$\boxed{(-3 - \sqrt{14}, 0) \quad \text{and} \quad (-3 - \sqrt{14}, 0)}$

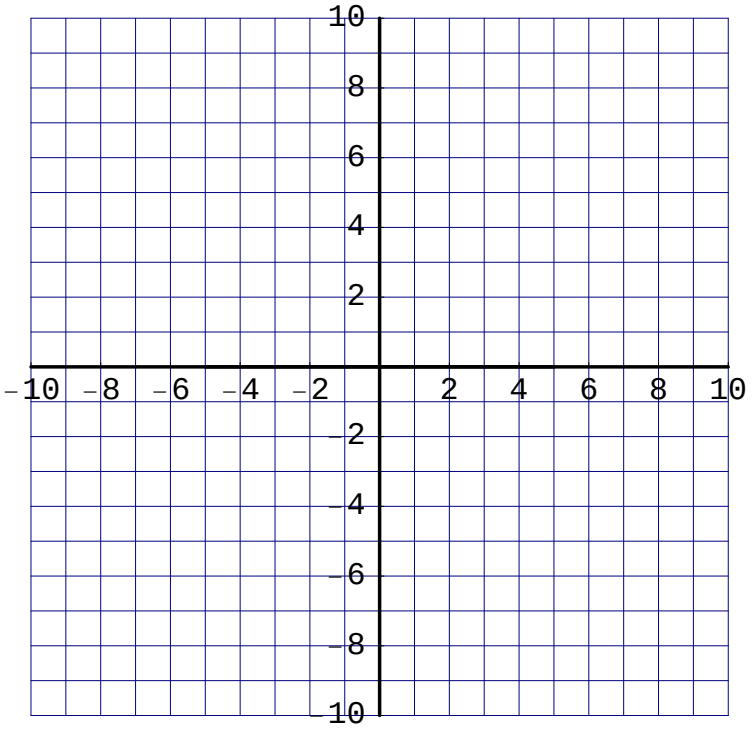
(d) The range.

$\boxed{(-\infty, 23]}$

(e) The maximum or the minimum value.

$\boxed{\text{Maximum value: } 23}$

(f) Sketch the graph of f in the axis below.



Problem 6 The water rates charged by the Turbulent Water Co per month are as follows: a Base charge of \$12; the first 500 gallons or less at a rate of \$0.5 per gallon, and all gallons over 500 at a rate of \$1 per gallon.

(a) What is the monthly cost of using 500 gallons of water?

\$262

(b) What is the monthly cost of using 501 gallons of water?

\$263

(c) Write a piecewise definition for the cost, $C(x)$, of using x gallons of water (fill in the boxes)

$$C(x) = \begin{cases} x/2 + 12 & \text{if } 0 \leq x \leq 500 \\ x - 238 & \text{if } 500 < x \end{cases}$$

Problem 7 The cost, $C(t)$, of renting a boat at Lake Balboa, is given by the function below

$$C(t) = \begin{cases} 10, & \text{if } 0 < t \leq 4; \\ 30, & \text{if } 4 < t \leq 6; \\ 3t + 12, & \text{if } 6 < t; \end{cases}$$

where t is the duration of the rental measured in hours, and $C(t)$ is the cost of the rental measured in dollars.

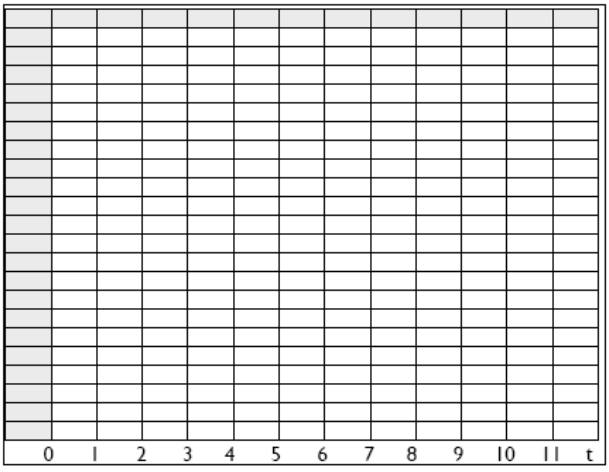
(a) What is the cost of renting a boat for 4 hours? 5 hours? 10 hours?

4 hours is \$10

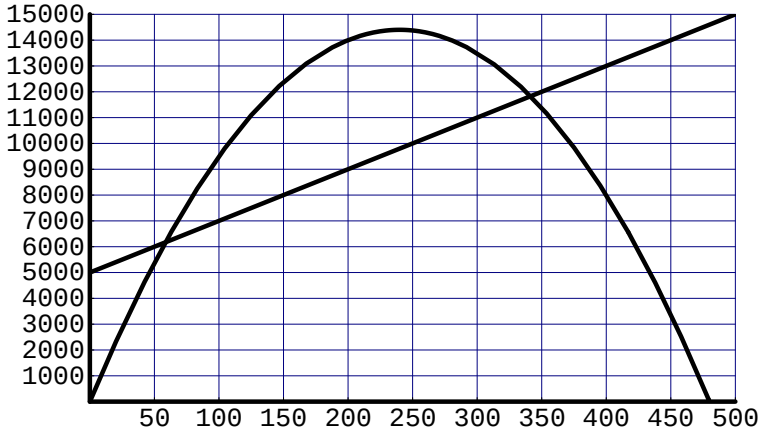
5 hours is \$30

10 hours is \$42

(b) On the grid below mark the vertical scale and graph the cost $C(t)$.



Problem 8 The graphs of the revenue, R, and cost, C, for the production and sale of x units of certain product are shown below. The straight line is the cost function and the other curve is the revenue function.



(a) Use the graph to estimate the production level x that maximizes the profit.

$\boxed{x = 200 \text{ units}}$

(b) Mark the points $(x, C(x))$ and $(x, R(x))$ on the graphs of the cost and the revenue functions corresponding to the level of production x that maximizes profit.

(c) What is the maximum profit?

$\boxed{\text{Max profit is } P = 5000}$

Problem 9 For the function $f(x) = \dfrac{3x-7}{x-2}$:

(a) Find the vertical asymptote and the horizontal asymptote.

Vertical asymptote is $x = 2$

Horizontal asymptote is $y = 3$

(b) Find the x intercept and the y intercept.

x-intercept: $(7/3, 0)$

y-intercept: $(0, 7/2)$

(c) Find the domain

$(-\infty, 2) \cup (2, \infty)$

(d) Find the range

$(-\infty, 3) \cup (3, \infty)$

(e) Sketch the graph in the axis below. Make sure to graph also the asymptotes and to label the intercepts