Okay. Now we will go to a new concept. In the previous talks we discussed how much to order. In the case that there is no price discount, we always order EOQ. When there is a price discount we may order EOQ, or we may order more than EOQ. So in those sessions, we addressed how much to order. Now we want to know when to order. For how much to order we started from EOQ, and here for when to order, when there is a periodic inventory system we always order at the start of the period, but in a perpetual inventory system, reorder point is defined as quantity in terms of quantity, and therefore we want to find out at which level of inventory should we reorder. Therefore, this topic is mainly on reorder point or what we call ROP.

As we said in periodic inventory system, ROP is the start of the period, but in perpetual inventory system or in continuous reviewing inventory system, reorder point is when inventory on hand drops to a predetermined quantity. If there is no variation in demand, because in EOQ model we assume that there is no variation in demand. Demand is known, constant, and it will remain constant. If there were no variation in demand and demand was really constant, then ROP is when inventory on hand is equal to average demand during lead time. And don’t forget if demand is constant, then demand is always a specific number, and therefore, average demand during lead time is equal to demand during lead time. While in a case where demand is variable, average demand during lead time differs from actual demand during the lead time. But here if the demand is constant, no variation in demand, demand during lead time is average demand during the lead time. And reorder point is a point when inventory on hand is equal to demand during lead time.

Lead time is the time interval from placing an order until receiving the order. Again, I want to emphasis this. If demand is fixed, we know what is the demand during lead time. We know it. But if demand is variable, we only know average demand during lead time, which really high probabilities is different than demand during lead time. The greater the variability of demand during lead time, the greater the need for additional inventory. These additional inventories to reduce the risk of shortage, so that additional inventory that I need is called safety stock. Therefore, the more variability in the demand during lead time, the more safety stock I need. When variability exists in the demand or lead time, actual demand during lead time will be different than average demand during lead time. It may exceed average demand during lead time. It may be less than average demand during lead time, but if there is no variation in demand during the lead time, then demand during the lead time is equal to average demand during the lead time, and we know it in advance. In the constant case in advance I know how much is the demand during the lead time, but in variability case I only know how much is the average demand during the lead time.

If I order at a point when inventory on hand is equal to average demand during the lead time, because it is always varied, to assume demand during lead time has normal distribution and because normal distribution is a symmetric bell-shaped curve and probability of being greater than average is equal to 50 percent and probability of being less than average is 50 percent, therefore, if my reorder point is at the point when inventory on hand is equal to average demand during the lead time, inventory on hand is equal to average demand during the lead time. Then there is 50 percent probability that demand during lead time exceeds the average demand during the lead time. What that means in practical sense, that means there is 50 percent probability that you come to my store and ask for the product, and I say I don’t have it. Sorry. I don’t have it. 50 percent probability to come here and I don’t have the product to satisfy your demand. So what will happen? The first thing will happen I lose the profit that I could make on that product if I would have sold it to you. But the more serious impact is when you come to my store for one or two times with 50 percent probability and I tell you I don’t have the product, the next time you go to another place and buy the product over there. And I lose you forever. So usually retailers, manufacturers, distributors don’t want to have a 50 percent probability of shortage. They want 5 percent, 1 percent, 10 percent. Therefore, we would like to have a probability of 99 percent, 95 percent, or 90 percent to be able to fulfill your request. Therefore, I should order when inventory on hand is more than average demand during lead time. I should order at the point where inventory on hand is more than average demand during lead time. How much more? That much which make it possible for me to say I don’t have the product only to 1 percent or 5 percent or at most 10 percent of my customers. Then I am able to satisfy 99, 95, or 90 percent of my customers. Safety stock is what I have to average during the lead time, then I order at the point when inventory on the hand is equal to average demand during lead time plus safety stock. Then when I order at this level of inventory on hand, it will reduce the probability of stockout during the lead time. Reorder point in perpetual inventory control is the inventory level equal to the average demand during lead time plus a safety stock.

Look at this example. Average demand for an inventory item is 200 units per day. Lead time is 3 days. There is no variation in demand, and no variation in lead time. Therefore, demand is constant and is completely known in advance, and therefore, I really don’t need any safety stock. Safety stock is equal to 0. I order at the point when inventory on hand is equal to the demand during the lead time. And demand during the lead time is known. There is no variation in it. Therefore, reorder point is 3 times 200, and whenever inventory drops to 600, I place an order. The demand during those 3 days is exactly 600 units. And I will receive my order exactly in 3 days, therefore, immediately after inventory reached to 0, I will get the next order.

Okay. Now let’s go through this second simple example. Average demand for an inventory item is 200 units per day. Lead time is 3 days, and safety stock is 100 units. What is the reorder point? In a perpetual inventory system the reorder point is the point at which inventory on hand is equal to the average demand during lead time plus safety stock. So lead time is 3 days, average demand during lead time is 200. 3 times 200 is 600 plus 100 is 700. When inventory level drops to 700 units, we place an order. Since there is variation in demand during the next three days, our demand is not 600. It may be 500, 400, 429, 678, whatever, 750 and so on so forth. Okay. Now, after this introduction, after these two simple examples, I think we have set the stage to go through a little bit more complex concepts.

This is inventory on hand. We have already received this order. Okay. This is time. We start consuming that inventory. Gradually it goes down. Because demand is constant, it goes down in a linear fashion, like a straight line, not like a curve. So it goes like this, straight line. It goes down, down, down, down, down. If lead time is this much, suppose this is three days, if lead time is this much, this is when we expect that inventory reaches 0. Okay. This is 3 days. So from that point because we know this curve in advance, because we assume there is no variation in demand and because there is no variation in demand, therefore, we know this line in advance. Therefore, because I assume it reads 0 at this point and because lead time is three days or whatever days, I go back that much from this point, that many days from this point, and then I draw a perpendicular line, and therefore, I find out whenever inventory reached this level, when inventory on hand reached to this level, I should place an order. Okay. When I place an order at this level of inventory, after this many days I will receive the next order. No variation in demand, no variation in lead time. So this is lead time. This is the point that I expected the demand to become equal to 0. At this point when inventory on hand is this much, I place an order. I consume it exactly the way that I was supposed to consume it, and I get the next order. Immediately at the point when inventory on hand reaches 0. Again the same thing. This is inventory on hand. This is time. I expect to consume the inventory like this. It reaches 0. If this is my lead time, I go perpendicular, and whenever the demand reached to that inventory level I will order. But you know, it is possible that I consume the inventory with a faster rate, for example, like this. I consume it faster. And I wish to reach that reorder point earlier than the previous one. Here reorder point was at this period of time, but here the reorder point is at this point of time. Alternatively, I may consume it with a slower rate. In that case the reorder point is sometime here. But in all three cases, inventory whenever inventory reaches this level I will put an order. In this case I will put my order at this point of time. In the previous case, I will put my order at this point of time. In the first case I will put my order in this point of time. Reorder point is determined based on inventory on hand. Inventory on hand is determined based on consumption rate. If it is faster, inventory on hand reached that specific level sooner, if it is slower, inventory on hand reaches to that specific level later. Sooner, later. Sooner, later.

So I have ordered this much, and I have started – and I have consumed it. And I have reached the reorder point. At this point I reorder, and I expect to receive the new order at this point. And I expect to consume this inventory at the same linear rate, therefore, exactly at the point that inventory on hand reaches 0, I will get the next order. But who can guarantee that customers come always to my shop based on the schedule I have assumed? No one can guarantee that. Indeed it will never happen like that. Therefore, it is possible that my product is consumed with a faster rate. In that case inventory on hand reached 0 at this point while for all of these days I have no inventory. Any customer who comes to my shop or to my distribution center or to my retail store asking for product will not get the product because I am out of product and I should wait until this point to get the product. In general, customers do not come to my place to ask for product in a linear fashion. One day there will be more customers, the other day there will be less customers. Therefore, in contrast to the basic assumptions of the inventory model that demand is known and is constant, no, demand is not known. Only average demand is known, and I need my computation for EOQ based on average demand. However, some days I may have more orders. Some days I may have less orders. Here inventory drops quickly. Here inventory drops very slowly. But as soon as inventory on hand reaches to the level in which I think inventory on hand is enough for average inventory during lead time, which is this much, I order. And I expect to receive the order at this point, and I expect to consume this inventory at this rate. But it is possible that I consume it in a faster rate. In that case, I will be out of stock for this much time. So I reach to the reorder point, but the demand after that was quite fast, was quite high, therefore, before I get the next order, all my inventory on hand was consumed. Alternatively, the demand may be less than what average demand that I have assumed. In that case I wish to reorder point not at this point in time, but at this point of time. So I put my order at this point when inventory on hand is equal to the average demand during lead time. And I expect to consume that inventory in a linear fashion, and I expect that at the point that I get the next order, inventory on hand is equal to 0. But maybe I consume that inventory at the lower rate and therefore when I get the next order I still have some inventory on hand. Why we need safety stock? We need safety stock because I want to reduce this probability. I want to reduce the probability of being short of stock. This axis shows quantity, and by quantity, I mean it shows inventory on hand. This axis shows time. This is the order that I have received, it is either EOQ or something greater than EOQ. I have received it. That is how to order. It doesn’t have to when to order, doesn’t have anything when to order. It is either EOQ or in case if I have price discount it is possible that it is greater than EOQ to take advantage of price discount. How much to order? When to order is something different, and now we want to talk more about when to reorder, when is the ROP.

Look. I consume that inventory in a non-linear fashion, a curve, which sometimes is steep, sometimes slow. But I assume in my model that it is consumed in a linear fashion. And to find the slope of that line, I will use average inventory. But in reality sometimes demand is less than average inventory, is less than average demand. Look at this pictorial representation. Vertical axis shows quantity. Horizontal axis shows time. In this pictorial representation we want to show how inventory is consumed over time. This is how much do I order. It is either EOQ or if there is a price discount it may be something more than EOQ to take advantage of price discount. The caution that we are opening the stage for now is when to order that is ROP, reorder point. This inventory is consumed in a non-linear fashion. For example, it may be consumed like that, which is slow. It may be consumed like this, which is fast. Suppose it will be consumed like this in which here it is consumed in a slow rate. Here it is consumed in a fast rate in a relatively slow rate, in a relatively fast rate. But in our model we assume that it is consumed in a linear rate, in a linear fashion. And the slope of that line is equal to average demand. So we assume that it is consumed in a linear way with this slope, but in reality it may be consumed in a rate lower than that rate or sometimes in a rate higher than that rate, in a rate lower than that rate, in a rate higher than that rate. So while actual demand is non-linear, we assume it is linear because we want to model it and get some conclusions, some guidelines from that model. Don’t forget whenever we want to model a portion, we need to simplify it. Then based on that simplifications, we would be able to develop some mathematical formulation and collect some information about the simplified version of the real world. However, later we need to incorporate at least some parts of complexities of real world into our simple model.

Okay. Now suppose lead time is this much. At this point we assume it is consumed in a linear fashion like that. We assume it has reached 0 at this point. And, therefore, we go this many days back from this point, and we reach to that point, when inventory reaches that level, we reorder. And we expect to consume that inventory at this rate. And as soon as inventory on hand reaches 0, I expect to get the next order. That is average demand during lead time. Whenever inventory on hand reaches average demand during lead time, I place an order. I expect it is consumed in this way, or even if it is not consumed in this way, I expect for some period it is consumed say more than or less than that rate and for some other periods it is consumed less than or more than that rate, and I expect on the average I will go with that rate during this period, and when it has reached 0, I expect to get the next order. So my reorder point was a point in which inventory on hand is equal to average demand during the lead time. But it is possible that during lead time the demand is quite higher than what I expected. And by higher we mean this line will not go in this fashion because that is average slope. That is slower slope. But when we say demand is large, we have a lot of demand. That means this line will become steeper. It will go like this. Correct? It is possible that it is steeper it goes like this. Therefore, all of these inventory is consumed not during this period but quickly during this period. If I wanted to have the whole demand which I may need during this period, then I should have ordered when inventory on hand was equal to this much, not when it was equal to this much. So because demand during lead time was quite higher than the average demand, at this point inventory reaches 0. Therefore, for this much of time, which is almost half of the lead time, I don’t have any product to give it to my customers. Therefore, in order to reduce this probability, I need safety stock. Safety stock reduces risk of stockout during the lead time. And it is safety stock if I could have ordered at the point when inventory on hand was equal to this much and that much is, for example, something here. When inventory on hand is almost equal to this much, then I would have been – probably could have been sure that I will not be in a position of having no stock. That’s what we add to average inventory during lead time, and that is what we call safety stock.

Let me quickly go through this discussion once again. This is inventory on hand when I received the order. This is reorder point when I reorder. And reorder point up to now we know is equal to average demand during lead time. Actual demand is like this. Non-Linear I assume it is linear, and it is equal to average demand. This point, this point when inventory on hand is equal to average demand during lead time I reorder. I expect to consume that inventory at the average rate, and therefore I go this many days back. That is my reorder point. Demand during the lead time is not constant. Demand during lead time has normal distribution. That amount is only average of the demand during lead time. Actual demand during lead time could be any of these numbers. If it is more than average, probability of stockout is 50 percent. Therefore, if I want probability of stockout not to be 50 percent but to be smaller amount, then I will add safety stock. And I order not at the point when inventory on hand is equal to this much but at the point when inventory on hand is equal to this much and that is this point. Inventory on hand is equal to this much.

When to reorder. Demand during lead time has normal distribution. This is what we assume. So it’s like this. Look, this is not different than the normal distribution you have seen. I have just rotated it 90 degrees to show my concept. This is the average. So this is normal distribution. This is average. There is 50 percent probability that demand is less than this average, and there is 50 percent probability that demand is greater than this average. Therefore, if I order at the point when inventory on hand is equal to this much, and it is equal to average demand during the lead time, there is 50 percent probability that the demand during lead time is less than this average. Demand could be this much. It could be this much. It could be this much. It could be this much, this much, this much, or this much, or this much. In all of these demand levels, I can satisfy the demand. But it is also possible that demand during lead time is greater than average demand during lead time. There is 50 percent possibility that demand is greater than the average demand. Demand could be this much, which is greater than the average demand. It could be this much from here to here, which is greater than average demand, greater than average demand. Here demand during lead time is this much, average demand is this much. In all of these cases, I want to be able to satisfy the demand. I don’t want probability of facing shortage to be 50 percent. I want it to be less than 50 percent. But still I cannot make it shorter. With 100 percent probability I can satisfy all the demand. Therefore, I am ready to accept some risk to be out of stock, but that risk – probability of that risk I don’t want it to be 50 percent. I want it to be less than 50 percent. I want it to be 1 percent, 5 percent, 10 percent, but not 50 percent. Therefore, I am ready to reorder when inventory on hand with this much probability, this much probability already under the normal curve up to this point with this much probability which is very close to 1. It may be .95, .9, .99, with this probability to be able to satisfy demand, but I am ready to accept some risk that with some small probability, with 1 percent 5 percent, 10 percent probability I will not be able to satisfy the demand, the whole demand during the lead time.

So this is normal distribution. This is average demand during lead time. I am ready to accept this much risk to be out of stock, but not this much risk. This risk could be 1 percent, 5 percent, 10 percent, depends on managerial policies. Probability of no stockout is quite high. Risk or probability of stockout is quite low compared to this probability. This is what we call a service level. If probability of risk is 5 percent, service level is 95 percent. If probability of risk is 1 percent, service level is 99 percent. If probability of risk is 10 percent, service level is 90 percent. So management says with 90 percent probability I want to be able to satisfy the demand during lead time, but I am ready to accept the risk of 10 percent. This is the safety stock which is added to average inventory to ensure that this probability is not 50 percent but less than 50 percent. And this probability is not 50 percent but quite more than 50 percent; therefore my order point is not here anymore. It is here when inventory on hand is equal to this much. And this is what we call reorder point.

I’m sorry, Dr. Evil, but I should go through statistics a little bit. If you really have forgotten whatever you have learned in statistics Dr. Evil, then you can go to Mini Me and ask for a mini book on the statistics. But I have already contacted Mini Me, and he has given me some very straight forward direction that in a minimum effort we can learn some basics of statistics, a normal distribution which is related to our discussion. So if you are patient, and if you carefully listen to me, you will be okay. Each normal variable X, what is a normal variable X? Like, for example, demand during lead time. Demand during lead time is a normal variable X. What do you know about this variable? It’s normal, so its distribution is bell-shaped. And when we say it’s a normal variable X, we should also know its average, which we could say is a hundred units during lead time, and we should have its standard deviation. So when we talk about a normal variable, we know it’s normal and has a specific mean, which could be anything, and has a specific standard deviation, which could be anything. So this normal variable, this random normal variable has two attributes, mean, and a standard deviation, which could be any number. But the good news is that for a normal variable X is associated with a standard normal variable Z. What is the attributes of a normal variable X? It has a mean and a standard deviation. A mean, an average, and a standard deviation. But those two attributes in standard normal variable Z are 0 and 1, therefore, standard normal variable Z has mean of 0, and a standard deviation of 1. And therefore, some people have developed a table for standard normal distribution. Given any probability of not exceeding a specific Z, that table will give us the value of Z. But this Z is a standard normal variable. It is not the variable X that I am looking for and it has a specific average and a specific standard deviation. In a minute we will know the relationship between Z and X. And also given any value for Z, the table will give me the probability of not exceeding Z.

So, if I know the service level or risk, which is one service level, then I can compute a Z, a specific Z, which is related to that risk and that service level. If I know a specific Z, then I can find out what is the service level for that specific Z and what is the risk for that specific Z. So I have a table, and that table tells me give me service level or give me risk. I will give you Z. Give me Z. I will give you service level, and I will give you risk. And there is a relationship which tells me give me Z and I will give you X. Give me X, I will give you Z. And that’s all we need. So while we are talking about a general normal variable with a specific average and a specific standard deviation, there is a relationship which connects that general normal variable X with a standard normal variable Z. So while we are talking in an X world, we can go to Z world, use those tables, make some conclusions, and take those Z and come back to X world. Not from X values to Z values, not back from Z values to X values. Why we do need this mapping? Because we don’t have a table of probabilities for all types of normal variables. And by all types of normal variables we mean with different average and different standard of deviation, but we have that table for standard normal variable which has mean of 0 and a standard deviation of 1, and we have a relationship which transforms any normal variable with any mean and any standard deviation into standard normal variable.

So if risk is 10 percent, service level is 1 minus 10 percent, which is 90 percent, and if we go do normal table, it will tell us Z value is 1.28. So for normal standard Z variable, for standard normal variable, the Z value for which probability of service level is 90 percent and probability of risk is 10 percent, that specific Z is 1.28. So if the average demand during lead time was 0, and standard deviation of lead time was 1, then if inventory on hand is 1.28, then there is 90 percent probability that I could satisfy all the demand. And there is 10 percent probability that I cannot satisfy all the demand. But that is for Z distribution. I usually don’t have Z distribution for my demand during the time. I have normal distribution, which I can easily transform into Z distribution or standard normal distribution. That standard normal distribution table tells me if risk is 5 percent, then therefore service level is 95 percent, then your Z value is 1.65, and if risk is 1 percent and service level, therefore, is 99 percent, then the Z value is 2.33. So it simply tells me that the Z value for which the probability of being greater than Z is 1 percent, and therefore, the probability of being less than or equal to Z is 99 percent. That Z is equal to 2.33. I can now use this Z or this Z or this Z and find out what should be my inventory on hand at the time that I reorder such that the probability of being out of stock could be 10 percent, 5 percent, or 1 percent.

If I take any normal variable X, and if I subtract it from its average, and if I divide it by its standard deviation, it becomes Z. So if demand during lead time I have it, if I subtract it from average demand during lead time, and if I divide it by the standard deviation of demand during lead time it becomes Z. Then I know that what is Z for which risk is 10 percent. What is Z for which risk is 5 percent. What is Z for which risk is 1 percent. I know that. And therefore, I can find Z for which this probability is equal to 90 percent, or 95 percent, or 99 percent. I put it over there as Z, and therefore, I know average demand during lead time, I know standard deviation of demand during lead time. I can find the reorder point. Thank you, Mini Me, for your mini book. X is equal to average X + Z times the standard deviation of X. Mu is average of X. Sigma is the standard deviation of X. X is equal to Mu + Z times sigma. So if I have Z, I can transform it into X. If I have X, I can transform it into Z. And I know given any service level or any risk I can easily compute Z value. LTD, lead time demand. Reorder point is average lead time demand + Z times the standard deviation of lead time demand, and Z I can find it over there. Reorder point is equal to average lead time demand + Z times the standard deviation of lead time demand and that is what we call safety stock. We say it if we only reorder at a point that inventory on hand is equal to average lead time demand, then probability of stockout is 50 percent. But if I add safety stock to it and reorder at the point where the inventory on hand is equal to average lead time demand + safety stock, then the probability of stockout would be quite low. It could be 10 percent, 5 percent, or 1 percent. Therefore, if you want probability of stockout to be 10 percent, you will find 1.28, which is Z. Correct? Then you take that Z. You take that Z here and you multiply it by the standard deviation of lead time demand and that would be safety stock. Then you add it to average lead time demand, and that would be your reorder point. If you order at that reorder point, probability of stockout would be 10 percent. Alternatively, if you want probability of stockout to be one percent and service level to be 99 percent, then you use normal table. You find Z value is equal to 2.33. Now, you have Z, and you have average lead time demand, and you have standard deviation of lead time demand. You multiply it by Z, and you get reorder point. This is safety stock, which is equal to Z times standard of deviation of lead time demand. This is average lead time demand. When you add them up together, that is your reorder point corresponding to that specific service level of 99 percent, 95 percent, or 90 percent.

In the test, I will give you this table. Therefore, for any service level, and risk, you will have the corresponding Z value. Now we know everything to be able to solve this problem. Therefore, I would like to ask you to take a piece of paper, spend 5, 10, 15 minutes to solve this problem. Try to solve it completely and thoroughly. If you find out that you cannot solve the problem, don’t push forward, push backward. Go to the lecture. Go to that part which I borrowed from Mini Me. Listen to Mini Me talk and then come back and try to solve this problem. Believe me, if you try to solve the problem yourself, your grade and midterm would be B and more. The most important thing you need is to stop whenever I say that now it is your time to contribute.

Okay. So stop this talk. Solve the problem, and then come back.

What is the service level? Here probability of risk is 5 percent therefore service level is 1 minus 5 percent, which is 95 percent. I want to find the reorder point for a specific X. I know service level and risk for that X, but I don’t know the value of the X. I don’t have a table to use to find that value of X, but I have a table to find Z for that risk and that service level. I have a relationship to transform that Z to an X. So what is the Z value corresponding to this service level of 95 percent? Z value, if you go to the table, to those tables which I showed you before is 1.64. And I know there is a relationship between Z and safety stock. Safety stock is equal to Z times standard deviation of lead time demand. Standard deviation of lead time demand is 5 tons. Z is 1.64. If I multiply 1.64 by 5, that would be my safety stock, which is 8.2. I mean all of these problems when we solve these things don’t change real numbers into integer numbers. Safety stock is 8.2.

Okay. What is the reorder point? Reorder point is equal to average demand during lead time plus safety stock. And we have all those information. This is average demand during lead time. This is the Z value. This is standard deviation demand during lead time. Therefore reorder point is as the point when inventory on hand is equal to 58.2 tons. Let’s go through the second example, and that will clarify everything. Average demand during lead time is 75 tons. Standard deviation of demand during lead time is 10 tons. Assume that the management is willing to accept a risk of no more than 10 percent. So risk is 10 percent. Service level is 90 percent. Then I go to the table. Risk, 10 percent; service level, 90 percent, and Z value is equal to 1.28. So that is a Z value for which a standard normal variable has a probability of 90 percent to be less than this value and probability of 10 percent to be greater than this value. That’s what I want. Z is equal to 1.28. What is safety stock? Safety stock is Z times standard deviation of demand during lead time. Z is known. Standard deviation of demand during lead time is known. It is 10. Standard deviation of demand during lead time 10, Z 1.28, and therefore safety stock is 12.8. Reorder point is a point in which inventory on hand is equal to average demand during lead time. And average demand during lead time is 75 tons plus safety stock, and safety stock is Z times standard deviation of demand during lead time, and therefore, it is 75 + 12.8, which is 87.8. Whenever inventory on hand reaches 87.8, we reorder. Under this reorder strategy, the probability of stockout is 10 percent. The relationship between service level, risk, and safety stock, when service level goes up, risk goes down. When service level goes up, safety stock goes up. We should have more cushioning against variation in demand. When service level increases, risk decreases. When risk decreases, safety stock increases, because safety stock is Z times the standard deviation of demand during lead time. The standard deviation of demand during lead time is fixed, but as service level goes up, Z goes up, therefore, Z times the standard of deviation of demand during lead time, which is safety stock, goes up; therefore, our service level goes up. Safety stock goes up.

Okay. Now we want to add one more point to the complexity of discussion. In the previous discussion demand during lead time and the standard deviation of demand during lead time were not known. Demand during lead time and the standard deviation of demand during lead time were known. Here in this new example, demand during lead time is not known. Demand per day or per week is known. And the standard deviation of demand per day or per week is known, and lead time is known, therefore, we should transform this problem into the previous problem. As soon as I can transform this problem into the previous problem, then I am done. Let’s look at it. Let’s see what is the situation. If demand is variable and lead time is fixed, so demand is variable but our lead time is fixed, lead time is LT, so lead time is 2 days, 3 days, 4 days or it is 2 weeks, 3 weeks, 4 weeks. D is demand. D bar is average demand per day or per week not for the lead time. Per day or per week. D is demand per day or per week. D bar is average demand per day or per week, lead time in terms of days or weeks. Average demand during lead time is equal to lead time times average demand per day. So if D bar is average demand per day and lead time is LT days, therefore, average demand during lead time is equal to lead time times average demand per that period. If lead time is 3 days and this is per day, I multiply 3 days by this. If lead time is 2 weeks, and this is in terms of week, I multiply 2 by this. And that will give me average demand during lead time. Sigma is the standard deviation of demand per day or per week. But I don’t need a standard deviation of demand per day or per week. I need standard deviation of demand during the lead time. Standard deviation of demand during lead time is shown by sigma of demand during lead time. And that standard deviation of demand during lead time is square root of lead time times standard deviation of demand. The example will clarify everything.

The only formula we need to know to remember is this one, which is a little bit – something to remember, I will give it to you on the test. You will have it as a formula. The other formula you should know is this one. It is very intuitive. We can – if demand per day is D bar, average demand per day is D bar, lead time is say 5 days, 5 times that would be demand during lead time.

Let’s go to the example. Average demand of sand is 50 tons per week. A standard deviation of weekly demand is 3 tons. Lead time is 2 weeks. Risk is 10 percent. Compute the reorder point. Compute safety stock. Look. The difference between this problem and the previous problem is in what? In the previous problem we knew the demand during lead time was something. But here the demand per week is something, and the lead time is say 2 weeks, 3 weeks, 5 weeks, whatever weeks it is. Then we have standard deviation of demand per week, which is 3. So in the previous example, we had average demand during the lead time and the standard deviation of demand during lead time, but here we have average demand per week or we could have it per day, and we have a standard deviation of demand per week, or we could have it per day. And the lead time is 2 weeks. Or we could have it in terms of days. So here we have average demand per week, standard deviation of demand per week, and lead times number of weeks. We should transform this information into the previous information in which we had demand during lead time and the standard deviation of demand during lead time. Therefore the key question is how could I transform average demand per week into average demand during lead time, and how could I transform standard deviation of demand per week into standard deviation of demand during lead time? And the answer is quite simple. Average demand during lead time is lead time times average demand per week. The standard deviation of demand during lead time is standard deviation of the demand per week multiplied by square root of the lead time. Quite straight forward.

Because service level is 90 percent or risk is 10 percent, Z value is 1.28. That’s exactly the same as the previous one. Lead time is 2 weeks. Average demand is 50 tons, therefore, average demand per week is 50 tons. Lead time is 2 weeks. Therefore average demand during lead time because lead time is 2 weeks and average demand is 50 tons, therefore average demand during lead time is equal to D, which is 50 tons, multiplied by lead time, which is 2. Therefore, average demand during lead time is 100. Standard deviation of demand during lead time is equal to square root of lead time multiplied by standard deviation of demand. Z is 1.28. I put it over there. Lead time is 2 weeks. Square root of 2 weeks. This 2 is under the square root. Outside the square root I have standard deviation of demand per week. Therefore, this is 100. 1.28. Square root of 2 multiplied by 3, and it is 100 + 5.43. This is standard deviation of demand during lead time. This is Z. Average demand during lead time. Average demand during lead time was lead time multiplied by average demand per week and standard deviation of demand during lead time was equal to square root of lead time multiplied by standard deviation of demand per week. Very straight forward.

Okay. Now here we will talk about the situation when lead time is variable and demand is fixed. Look whenever we say something is variable. The way to find out which one is variable, if demand is variable or lead time is variable, the way to find it out is to see we are talking about standard deviation of which one? In the previous example we were talking about standard deviation of demand. We didn’t talk about standard deviation of lead time. Look at here. Standard deviation of demand is 3 tons. So that means that demand is variable. Lead time is fixed. But if lead time is variable, then we will talk about standard deviation of lead time. Look at this problem. If lead time is variable and demand is fixed, demand is equal to D, and that is equal to average demand and the standard deviation of demand is 0. Lead time, we show it by LT. Average lead time, LT bar, and average demand during lead time if average lead time is LT bar and demand is D, then average demand during lead time is LT bar times D.

Standard deviation of lead time if it is sigma LT, then standard deviation of demand during lead time is D, not square root of D times standard deviation of lead time. Standard deviation of lead time demand is equal to D times standard deviation of lead time. Look. In the previous formula we had standard deviation of lead time demand is square root of lead time multiplied by standard deviation of demand. But in this new formula, standard deviation of lead time demand is demand times standard deviation of lead time, no square root here. These formulas will be given to you during the test.

Look at this example. Demand of sand is fixed, and it is 50 tons per week. Average lead time is 2 weeks. Standard deviation of lead time is .5 weeks. Service level is 10 percent. Compute reorder point and safety stock. So here we are talking about standard deviation of lead time. So lead time is variable. Demand is fixed. And what do you need? We want to compute average demand during lead time, which is simply multiplication of these two, and standard deviation of demand during lead time. Standard deviation of demand during lead time we say it is equal to demand multiplied by standard deviation of lead time. Then we multiply these two by each other. That would be standard deviation of demand during lead time. Acceptable risk is 10 percent. Z is equal to 1.28. That part is done. D is 50 tons, no variation. Average lead time is 2 weeks. Standard deviation of lead time is .5 weeks, therefore, average demand during lead time is average lead time multiplied by demand, which is 100, 2 times 50. Standard deviation of demand during lead time is equal to demand multiplied by standard deviation of lead time, which is .5 times 50. 50 times .5, that would be standard deviation of lead time demand, average lead time demand, and Z. 50 times 2 + 1.28 times 50 times .5. Reorder point is equal to 100 + 32. Safety stock, average demand during lead time. If I reorder during this point, it is 50 percent that I am out of stock during lead time but if I order when inventory is 1.32, then there is only 10 percent that I don’t have stock during lead time. This concludes our discussion. I have asked you to solve this problem. This is a complete review on whatever I have said in this lecture. The best thing to do is to sit down with a friend, solve each part, compare your results together, discuss it, and then if you cannot come to a unified conclusion, go and look at the solution.

Okay. This concludes our discussion on inventory systems, and I wish you luck and happiness.