Okay. In this section of inventory control, situation in which we have also price discount. In the previous model we said there is no price discount. Let me clarify what I mean. It is obvious that throughout the year we need D units. Correct? And we should pay. We should buy these D units no matter how we order. By the end of the year we should have this many units. This is our demand. We have assumed that demand is fixed and constant. However, as we discussed in the previous section, we usually don’t order all the D units at the beginning of the year. Instead, each time we order Q units. Now, in the case of not having price discount, it simply says no matter if you order 1 unit, 2 unit, 10 unit, Q unit, or D units at a time, the price is fixed, and your total purchasing price cost is equal to D times P. That was the original model. And because that purchasing price does not depend on our Q, we just took it out of our total cost system and we just focused on ordering cost, which decreases as order quantity increases, and ordering cost which increases as order quantity increases. And we said the optimal ordering policy is at intersection of these two lines.

And we found the formula of EOQ is equal to square root of 2DS divided by H. Now the question is this: should I order EOQ and minimize my total ordering and carrying cost? And all order more than EOQ and take advantage of price discount while my inventory system cost will go up. That is what we want to discuss in this section. So our total cost curve is like this. This is inventory carrying cost plus inventory ordering cost, and it will reach to a minimum. This is our total cost curve without taking into account the purchasing cost. But I should also order D units throughout the year with price of P. Therefore, my purchasing cost is this much. It is P times D. Because there is no price discount, it is a straight line. No matter if I order this much, this much, or this much, price is P and multiplied by D. Now, if I want to include this purchasing cost into carrying cost and ordering cost, I should add these two curves together. When I add these curves, what does it mean? That curve will shift up this much. Therefore, that is total cost with purchasing cost. However, there would be no change in EOQ, because this curve just goes up this much. The whole curve will go up, therefore, the minimum of it will stay in the same place. Look here. This is our ordering cost for our prototype example. This is our carrying cost. When I add these two together, that would be my total cost. Now, if, for example, this product was $1 per unit, the total carrying cost and ordering cost was here, and reached its minimum at 300. Because I need 9,600 units a year and because I assume the price is $1, therefore, 9,600 will be added to all of these numbers. That much will be added to all of these numbers, and therefore, the new curve is this curve, which is my total cost curve. The minimum is still at the same 300 point, but the curve entirely shifted up.

By quantity discount we mean the price per unit decreases as order quantity increases. If I order more, my unit cost per unit will come down in some structure, which we go through it later. When quantity discounts are offered, we have more than one U-shaped curve for our total cost, and our total cost, don’t forget, now in the case of price discount is carrying cost, ordering cost, and also purchasing cost, because purchasing cost depends on our strategy. Therefore, for any given purchasing cost, I will have a different total cost curve, which is parallel to the other cost curves, but is higher than some of them and lower than others.

For lower prices, the curves are lower. For higher prices the curves are higher, but they are parallel to each other. At each range, when each price is valid, a different curve is there. I will explain in a minute. Then, therefore, the questions of should I make EOQ or more than EOQ because I may get price discount. Look at this problem. This problem clarifies what I mean. Demand for a product is 816 units per year; therefore, D is equal to 816. Ordering cost $12 per order. Carrying cost is $4 per unit per year. We have the following price schedule. The price is $20 each time I order something between 1-49. It’s $18 if I order something between 50 and 79. It’s $17 if I order something between 80 and 99, and it’s $16 if I order 100 or more. Don’t forget in all cases throughout the year I should order 816 units, but should I order 100 or more, then my total cost of purchasing is $16 multiplied by this. But if I order less than 49 or 49, my total purchasing cost is 816 multiplied by 20. This is my purchasing cost. Carrying cost we discuss it in the last session, how do we – carrying and ordering cost we discussed in last session, how do we calculate ordering cost and carrying cost, and now we learn how to calculate purchasing cost. And the best strategy is the strategy for which? The total cost is minimal. Suppose I solve this problem, and I get EOQ is greater than – EOQ is equal to 120. What will happen? You use EOQ formula, which is equal to 120. What do you do? You do nothing, nothing more. EOQ is more than 100, and at more than 100 you get the maximum price discount. It is the best for EOQ and the cheapest price. If you use 40, then you know 40 is the best for carrying cost and ordering cost, but maybe 50 is better than 40, because if EOQ is 40, if I order 50, first one thing decreases, as order quantity goes up, ordering cost goes down, but because order quantity goes up, carrying cost goes up. However, because I move from range of 1 to 49 to range of 50 or more, my price drops from 17 to 18; therefore, maybe while EOQ is 40, maybe it is better to order 50.

Also, if I go and order 80, what will happen? My ordering cost goes down, my carrying cost goes up, but maybe my purchasing discount plus reducing ordering cost is – leads to a cost which is better than this cost and better than this cost. So our total cost for price of $20 is like this, but if price drops to 18, the total cost would be like this. Parallel to that but lower. If the price drops to 17, it will be this curve. If the price drops to 16, it will be this curve, but this curve is valid up to some point. After that this one is valid up to some point, and finally this is valid. But EOQ in all of them is at the same point.

This is a slide you need to read. Smaller unit prices will raise total cost curve less than larger unit prices. For each price there is a separate U-shaped total cost curve. Note that no single curve is applied to the entire range of quantities. Each curve is applied to a portion of the range. If I order larger quantities, I will have price discount, therefore, purchasing cost comes down. I have fewer orders; therefore, ordering cost comes down, but I have more inventory, therefore carrying cost goes up. Our objective is to minimize the total annual cost of ordering cost, carrying cost, and purchasing cost. In our previous model we assume price was fixed, therefore, we did not include purchasing cost. P1, EOQ is valid up to here. So in this case, in this specific example, if P1 is valid up to this point, EOQ is after P1, therefore, I will never consider P1 because I could go and order more because EOQ is telling me order more, and then I not only minimize my carrying and ordering cost, but also because my total cost curve will drop. I will also benefit from lower purchasing cost. But this P2 is valid up to this point. However, after that, after this point, for example, if I order quantities more than this one, I still have a lower price like this. Now the question is should I order EOQ, purchase at this cost and my total cost is this one? Or I order more than EOQ and purchase at this cost with this total cost? But this cost is also valid up to a point and after that I have further reduction in cost, therefore, if EOQ is here, is it my benefit to order EOQ or this much or this much? It depends on the parameters of the problem. It is possible that I have a situation like this. This is price 1, after that price 2, after that price 3, after that price 4. This is the shape of the curve. In this specific figure I have drawn the curve in a way that this one is better. If this is true, this is better. But it is not always the case. Sometimes another one is better.

Here is a procedure you don’t need to memorize. I will explain it in detail. Right now I will just read it. Solution procedure. 1, compute EOQ without price considerations. This EOQ is the same for all prices. So we just compute EOX. But if this EOQ is feasible for only one price, identify the corresponding price and quantity. If EOQ is feasible for the lower price, it is the solution. If it is not, then calculate: total cost for EOQ and corresponding feasible prices. Note that total cost is equal to this. Calculate total cost for all Qs of price breaks and compare them. The one with the lowest price, total cost is the best. You really don’t need this page, some students just want to memorize. This is something to memorize. You don’t need it. You just need to understand.

Total cost curve could be like this. In this situation the best strategy is this, but total cost curve could come out like this. I compare this point with this point and this point and in this situation this is the best. Or I may have a situation like this, again, EOQ is here. The first, second, third, and fourth price break, here EOQ is the best. Let’s go back to the example, and it will clarify everything. The first step is to ignore prices, and compute EOQ. I have this EOQ which is equal to 70. 70 – this is my price structure. It says I am here. Correct? Now what should I do? Obviously I never go and consider 49. Why? I will tell you in a minute. Because EOQ is here. And total cost reaches minimum at EOQ, after that my cost goes up. Before EOQ my cost goes up. So if EOQ is here, if I go back to 49, what will happen? I am moving up on the curve. I am going up. This point it higher than this point. This point is higher than this one. If I go from 70 toward 49 and 50 or towards 75 and 80, my total inventory cost, carrying cost, and ordering cost goes up. If I go in this direction it goes up, but it is also possible my price goes up. So both things in this direction are negative. My inventory cost goes up, my price also goes up. So I never go in this direction if EOQ is here. But what about this direction? If I go in this direction my inventory cost, summation of carrying cost and ordering cost goes up, but I hope that here at this point the reduction in purchasing cost is so high that brings this point in total lower than that point. Because I go up in this direction, but at this point I drop. My total cost drops. Furthermore, here at this point I will not go to the right, because if I go this much to the right, my total cost goes up, however, maybe it is best to go to the right because here I have another drop, and maybe this point is greater than this point, is small than this one.

So here EOQ is 70, which is between 50 and 79 range, therefore, the corresponding price is $18. Obviously we do not consider P equal to 20. If I go from 70 toward 49, my ordering and carrying cost goes up. At the same time I may go from purchasing cost of 18 to 20. What about ordering 80 or 100? Maybe these result in lower total cost. Look, I never consider 81, because cost of 81 is greater than 80. Because of cost of 79 is greater than cost – cost of 89, 99, is greater than 80. This curve goes up. However at 100 I have a drop. So maybe cost of the 100 is less than cost of 80, but 81, 82, 83, 84, 85, 89, 95, 96, 98, 99 all are greater than 80, so I never consider that. Also here cost of 100 maybe less than 80, but cost of 101, 102, 103 are all greater than cost of the 100, so I never consider them.

I hope you have got this discussion. If you have got this discussion, if it is clear, then you are done. You really are done. Is Q = 70 at P = 18 better? Or Q = 85 and P = 17 or Q = 100 and P = 16? I write the total cost. Here red numbers, characters means those things which are not known. For example Q is not known. It is different for different strategies. As soon as Q is identified, P which was not known is identified. I should put Q either 70 and P is 18. If I put Q is 80, then P is 17. If I put Q is 100, then P is 16. All other things, H, S, D are known. They are parameters. They don’t depend on policy. So Q equals 70, P equal to 18. 4 is carrying cost. 70 is maximum inventory. 70 divided by 2 is average inventory. It is cycle inventory. 12 is ordering cost. 816 is total demand divided by 70, which is quantity that we ordered each time, multiplied by ordering cost. This is ordering cost, total cost, total purchasing cost. This is total cost of EOQ, $14,968. I’m sorry, Dr. Evil, it is not $100 million.

Okay. But the second strategy is 80 and 17. I put 80 and 17 here. The total cost is this, $14,154. Finally I put 100 and 16 in this equation and the total cost is $13,354, which is lower than the two others. So optimal strategy is 100 units. Note that under 100 units, this cost is greater than this cost. And this cost is greater than this cost, but this cost is quite low, and therefore the total cost of this strategy is better than the other two. Now we are in a good position to solve this problem. Read it. Stop the video. Pick up a piece of paper. If you need to go back to the lecture, but at this point I have transformed enough knowledge to be able to solve this problem. Once again, I recommend you not to go forward. Go sit down, solve the problem yourself. The material in the book is also the same as I discussed. If you need to go to the book, you can read the power points, but don’t go further until you solve the problem yourself. Don’t look at the solution.

Okay. I hope you have spent enough time to solve the problem, and your brain now is really ready to make sure you know everything or you have some minor difficulties that when I explain it, if you do have that, I believe after explaining this second example, you will not have any problem. So in this problem, D is known, S is known, H is known, and price structure is known. I hope that I go ahead and find EOQ, and I hope EOQ is 600 or more. Then in that case I don’t need to do any other computation. I say EOQ which minimizes my total carrying cost and ordering cost is at the range when I get the maximum price discount. Therefore, I don’t need to consider anything else. I don’t consider if EOQ is 610, I don’t consider 611 because 611 has the same price but total cost greater than 610 because summation of carrying cost and ordering cost goes up if I do that. EOQ is at the bottom of the valley of the curve. Any movement to the right or to the left it increase my cost. The only way to decrease is if I reach to a more price discount, but on this section after 600 there is no price discount, therefore, I stop at that EOQ. But if EOQ is 300 I should consider 400 and 600 and compare which is better. If EOQ is 500 I don’t consider 399 because 500 is EOQ. If I move from EOQ to the left or to the right my ordering cost and carrying cost and summation of ordering cost and carrying cost goes up. But in this direction I am also able to go to a higher price range, therefore I never move in this direction toward 399, but I may move in the direction of 600 and see if ordering 600 is better than ordering something between 400 and 599.

Let’s go ahead and solve the problem. EOQ is here. EOQ is 490. This is my price structure. I never consider 399. I never move from 490 to 399 because as I said my curve is like this. 490 is at the bottom moving to the left and right will increase my ordering cost. But I may also go to a price of 10. I never go from 490 to less than 490. But what about 600? When I go from 490 to 600, my total carrying cost and ordering cost goes up, but maybe my purchasing cost drops in a way that total cost here is less than 490. So I have two alternatives. Should I order 490 at price of 9 and 600 at price of 8? As soon as I am done with that, I make my best decision. 490 at price of 9 is better or 600 at price of 8? We should compare total cost. This is total cost. This is total cost for 490 and 9. If I do the computation, EOQ, these two costs are equal. They are not exactly because of the rounding, but they are close. In your computation if they are very close, that means you have made a mistake in your rounding, but they should be very very close to each other. That is examination for EOQ is correct. This is purchasing cost of $45,979.8. Here then I put 600 at price of 8. Same parameters over there. The price is $41,000. This is not the case in all examples we always use the maximum price discount. This is the structure and parameters of this problem. So, my friends, we have enough knowledge about EOQ models when we apply price discount. Now you are in a position to be able to solve this problem completely, totally, and correctly. Sit down, do it yourself. I beg you to do it yourself then find a friend and discuss with him or her to see if you have found the same thing. If not, discuss it with each other and see what is the problem. Then finally come to class and see how do I solve the problem. Thank you very much. Have a nice time.