Today we will talk about inventory models.  Inventory models are perfect examples of applying mathematical models to real world problems.  Inventory is important because about 20 percent of the budgets of hospitals are spent on medical, surgical, and pharmaceutical supplies. For all hospitals in the United States, it adds up to $150 billion annually.  The average inventory in the United States economy is about $1.13 trillion, and that is for $9.66 trillion of sales per year.

What happens if a company with a large work in process and finished goods inventory finds that the market is shifting from one product to another product? Well, I do have a lot of work in process and a lot of finished goods inventory of a product that customers have already shifted to another product.  I have two choices.  One is to fire-sell what I have. I fire-sell all inventories and finished goods, and that is a significant loss because I should sell them at 20 percent, 10 percent, 30 percent of their actual value. The second way that I have is to sell my finished goods and also at the same time, turn my work in process into finished goods and sell that one too. That means there will be a lot of delay in entering the product into the market, and I could lose a substantial portion of market share. Therefore, in both of these alternatives, they lead to loss. What is the message? I need to reduce my inventory as much as possible, minimal inventory.

There are three types of inventory.  Input inventory is composed of raw materials, parts, components, sub-assemblies that we buy from outside. In-process inventory are parts and products, sub-assemblies and components that are being processed; part, products, sub-assemblies, and components that are there to decouple operations.  For example, if operation B follows operation A, in order to not hundred percent have operations B dependent on operations A, we may put a little bit of inventory between those two.  And the third type of in-process inventory is when we realize that if we buy at large volumes, we get lower expense due to economies of scale. Then I have output inventory.  I need to have some inventory over there, because when customers come, I can’t tell them ‘wait, I will make your product.’  The best strategy is if I can have such a low flow time in which I can deliver manufactured product and give it to the customer, but we are not there.  Therefore, I should have inventory on shelf when customers come to satisfy demand.  Also, sometimes, demand in one season is high and in another season is low; therefore, I should produce in low season and put it in inventory to satisfy demand in high season.

Another type of output inventory is my products that are in a pipeline from manufacturing plants to warehouses or to distribution or to retailers.  That is also inventory. If you look at our highways, huge volumes of inventory on our highways, those are pipeline or in transit inventories.

Poor inventory management hampers operations, diminishes customer satisfaction, and increases operating costs.  A typical firm probably has about 30 percent of its current assets in inventories or about 90 percent of its working capital, and working capital as you may remember from accounting and from finance is the difference between current asset and current liabilities.  90 percent of working capital, they may switch to another company forever or another vendor.  Understocking is not good because customer comes there and when we don’t have the product, we will have dissatisfied customers.  We will have loss of sales, and the customer may go to another vendor forever.  Overstocking is not good too because it has three types of costs. Financial costs, instead of having our money in a city or in a profitable business, we put it in inventory.  Physical cost, we should put that in safe keeping somewhere.  We should lease a warehouse or allocate a portion of our shop to a physical location of these products.  And finally we may have obsolescence cost, because we buy a part and component to manufacturing product. If we bought a lot, it takes a long time to consume them into a final product, to assemble them into our final product, then customers may shift from this product to another, and therefore, those products may be obsolete.  You may never be able to sell them.

We have two types of inventory systems—inventory counting systems.  One of them is called periodical.  The other is called perpetual.  In periodical inventory system, at the beginning of each period, the existing inventory level is identified and the additional required volume to satisfy the demand during the period is ordered.  So the quantity of order is variable, but the timing of order is fixed.  It is the beginning of each period.  Re-order point (ROP) – when we reorder, is defined in terms of time.  It is beginning of the period.  How do we order?  At the beginning of the order – at the beginning of the period, I go and look at my inventory.  You may imagine it like a one-bin system, that is, one bin and my products are there.  I look and see that it is filled up to this point and the remainder is empty.  So I order that much.  Next period I go and see it is filled up to here and the remainder is empty so I order this much.  Each time I order enough to refill the single bin.  The quantity that I order each time depends on how much I need to fill the bin, but the timing is exact.  It is at the beginning of the period.  Reorder point is defined in terms of time.

The disadvantage of this system is that during this period, I don’t have information about my inventory.  I don’t look at that virtual bin during the period.  I only look at it at the end of each period, which is the beginning of the next period.  The advantage is this: because my inventory counting system for a lot of items at one day, then at one day I could order for a lot of items, so I make orders for many items at the same time.  My ordering cost perhaps will go down.

Perpetual inventory system is entirely different.  When inventory reaches reorder point, I order a specific quantity, and usually I order economic order quantity (EOQ), which I will discuss later.  So the quantity of order unlike periodic inventory system is fixed, but the timing of the order is variable.  Whenever my inventory has reached a specific level, I will order.  Reorder point is defined in terms of quantity, inventory on hand.  So you may think of it as a virtual two-bin system.  Whenever the first bin gets empty, I order enough products to fill this first order.  While I am waiting to get this product, I am using the inventory of this bin.

A perpetual inventory system is like a two-bin system in which when one bin gets empty I order enough to fill it up.  The other one was a single bin at the end of each period I look to see how much I need and I order that much.  So the benefit of this system is that it keeps track of removal from inventory continuously.

ABC analysis – the question is which type of inventory counting system is preferred?  Is it periodical or perpetual?  Perpetual is always better but more expensive because we need an automated system to continuously count our inventory.  Therefore, I may conduct an ABC analysis.   Here are my 12 parts (see Figure 3).  Here is the annual demand of each part.  Here is the unit cost of each part.  If I multiply them, I will get annual value of all items in my inventory system in my warehouse.  If I sort them in non-increasing order, I will see that two items, which is 2 divided by 12, which is something between 15 to 20 percent of items, form 67 percent of the annual value. And here, 7 items divided by 12 is a little more than 50 percent, say 55 percent of items, form 60 percent of the value.  These are group C.  These are group A and obviously these are group B.  For group A, I use perpetual, and group C, I may use periodical, and for group B, one of the two options.

Now we reach to our basic inventory model, which we try to develop a mathematical formula for a portion of real world, and this model is called economic order quantity, or EOQ.  In this model, we assume that we only have one product; only a single product.  Demand is known, and demand is constant throughout the year. So for example, I know that I need 5,000 units of product per year, and if a year is 50 weeks, then 1/50 of this number, I need every week.  If a week is 5 days, 1/5 of whatever I need per week I need per day, so demand is known and it is constant.  Every day, every minute, every hour, I have the same demand as any other minute, hour, or day.  Each order is received in a single delivery.  When I order, I should wait for a lead time, one day, 2 days, 3 days, it is known, and it is fixed.  After lead time, I receive the inventory that I have ordered.  Lead time is fixed.  It is 1 day, 2 days, 3 days, but it is always that.  If it is 3 days, it is always 3 days.  Therefore, as soon as inventory is on hand, reach to demand during lead time as soon as my inventory is equal to what I need for lead time and if lead time is three days, as soon as my inventory reaches a level that I need for 3 days, then I order.  During those 3 days, because lead time is fixed, after 3 days I will get the product.  And because demand is fixed and constant, at the second I get the product, my inventory reaches 0, and then I replenish.

There are only two costs involved in this model: ordering cost, cost of ordering and receiving the order; and holding or carrying costs, costs to carry an item in inventory for one year; cost to count one item in inventory for one year.  Unit cost of product does not play any role in this model because we do not get quantity discount.  No matter if we order one unit or one million units, the price is the same.

So let me give you an example and this will clarify the topic I am going to discuss. Annual demand for a product is 9,600 units (D), so I need 9,600 units per year.  In every minute of a year, I need the same number of units as another minute.  So D, which I will use capital D for demand being equal to 9,600.  Annual carrying cost per unit of product is $16 per unit per year (H). What does it mean?  If I have one unit of inventory in my warehouse and if I keep it for one year, it costs me $16.  That includes, for example, financial cost, physical cost of holding this inventory, and obsolescence cost.  That is, $16 per unit per year, or Inventory carrying cost.  And we show it by H, and that is equal to 16.  Ordering cost per order is $75 (S).  Each time I place an order, it costs me $75.  S is for ordering cost, and it is equal to 75.  Now we need to answer this question, how much should we order each time to minimize our total cost?  That is one question.  Should I order 9,600 at the beginning of the year or should I order twice a year each time 4,800 units, or say 96 times a year each time 100 units.  What is the economic order quantity?  So like many other business problems here, we have two different cost structures.  One goes up if ordering cost goes up and the other goes down as ordering quantity goes down.  Then the other questions I should answer are how many times should we order?  What is the length of order cycle if we have 288 working days per year?  What is the total cost of this system?  As I said, we don’t consider purchasing cost because it is constant, and it does not depend on our strategy.  In EOQ—in mathematical problems—if you get fractional numbers don’t worry about it.  Just put it as is.

So let’s look at ordering cost.  If I order one time a year, if I order 9,600 units at the beginning of the year, I have one order and therefore, one ordering cost.  But if I put an order for 100 units each time, then I have 96 orders and 96 ordering costs.  In this case, it is 96 times 75 because each order costs me $75.  That is my ordering cost, which is very high.  Alternatively, if I order at the beginning of the year, then ordering cost is 1 times 9,600.  So if D is demand in units per year, Q is what I order each time per order.  Therefore, the number of orders per year is equal to what?  This is all I need.  This is what I order each time.  Therefore, to find out how many times do I order is: what I need divided by what I order each time.  So that would be D divided by Q.  Q I order each time.  D is all I need per year.  We said we don’t have quantities gone.  It doesn’t matter what Q is.  Whatever I order, the price is still the same. If I order 100 units each time price is the same; the same as if I order 9,600 units at the beginning.  S is order cost per order.  This is the number of orders.  Order cost per order is S; therefore, annual ordering cost is S multiplied by the number of orders.  I think this is clear. Therefore, if I order 50 units each time because I need 9,600 units per year, I will have 192 orders per year.  If I multiply 192 by 75, I get this number, 14,400, a large number.  If each time I order 100 units, I will have 9,600 divided by 100, which is 96 orders.  Then I multiply by 75 and I get 7,200.  So if I increase my ordering size and order 500 units each time, I will order 19.2 times.  19.2 times in this period means I order 20 times in this year, but 19.2 of it is for this year and 0.8 will go to next year.  So if you don’t get integer numbers here, don’t worry.  In the first year, you have 27 orders and one other order, which 0.4 of it is for this year and 0.6 of it is for next year.

Don’t forget all these assumptions are valid for the next year and next year after that.  This is the most basic inventory model.  We can develop all types of inventory models, but they are more difficult to develop, and I am sure you don’t want me to go through very difficult models.  So let’s first try to understand this basic model.  Then the pathways to others are not really difficult.  You only need interest and time.  So here if I order 800 units, I should order 12 times, and my cost is 900.

So in summary, as order size goes up, number of orders goes down.  Because we have fixed cost per order, total ordering cost per year goes down.  This is number of orders per year.  This is ordering cost per year.  The curve looks like this—S times D divided by Q looks like this: It decreases as ordering size increases.  So it is a benefit to order as much as we can, and the best thing is to order all we need at the beginning of the year.  One order a year for 9,600 units at the cost of $75, but unfortunately the story has another side and that is inventory carrying cost.  When I order, when I get this Q, whatever it is, if it is 100 you need or 9,600 at the beginning of the year, when I receive the order, this will be my quantity in hand.  Then I start using it with a constant rate.  So that is why it is a straight line coming down.  Usage rate is constant.  It comes down, down, down.  I am consuming it, consuming it, consuming it.  Then inventory reaches 0.  Before inventory reaches 0, suppose this is my lead time.  Suppose this is 4 days.  If this is my model, this is 1 day, 2 day, 3 day, 4 days.  I will go see demand in 4 days is how much.  So as soon as my inventory in hand reaches this much, I place an order. As soon as my inventory on hand reaches demand in lead time, what I need in lead time, then I place an order.  Because demand rate is fixed, usage rate is fixed.  And because lead time is fixed, exactly at the same time my inventory reaches 0, I will get the next order so I will not be out of stock.  Then I consume it. Therefore, this is the story.  I get the inventory, I consume it.  As soon as it gets to 0 I get the next batch.  I consume it.  As soon as I get 0 inventory, I get the next batch.  I will consume it.  Next inventory and next one. Suppose this is one year (see Figure 7).  I have 4.2 orders per year.  Correct?  Q, order quantity at the beginning of the period we get Q.  No matter what Q is 100, 9,600, or whatever, at the beginning of the period I get Q units.  At the end of the period I have 0 units; therefore, Q units in one course, 0 units in another course.  Your GPA is Q + 0 divided by 2, which equals to Q divided by 2. Therefore, if each time you get Q, and if that Q goes to 0 at the end of the period, this is like this throughout the period and your average inventory was Q divided by 2.  One day, you have a salary of Q dollars.  The other day, you have a salary of 0.  Your average salary during these two days is Q + 0 divided by 2, which is Q divided by 2.  Therefore, if each time I ordered Q and I don’t have any safety stock and my inventory reaches 0 and I get the next order.

Look at this story. Average inventory per period and average inventory per year.  This is a period, beginning of the period I have Q units, end of the period I have 0 units.  Average inventory is Q + 0 divided by 2, Q divided by 2.  This is my average inventory.  Average inventory per period is whatever I ordered divided by 2.  That is what I do have throughout the period.  Average inventory per period is also known as cycle inventory, but what is average inventory per year? Suppose this is a year (see Figure 8).  In this year, I have 4 periods.  Average inventory per period is Q divided by 2.  What is average inventory per year?  So I have 9,600 total demand.  I have ordered 4 times, that is 9,600 divided by 4, which is 2,400.  Each time I ordered 2,400.  It goes to 0, 2,400, 0, 2,400, 0, 2,400, and 0.  In general, Q, 0, Q, 0, Q, 0, Q, 0.  Average inventory per period is Q + 0 divided by 2, which is Q divided by 2.  In this period is Q divided by 2.  In this period is Q divided by 2.  In this period is Q divided by 2.  Therefore, average inventory in this period is Q divided by 2.  This period is Q divided by 2, Q divided by 2, Q divided by 2, therefore, average inventory per year is multiplied by 4.  That is Q divided by 2, Q divided by 2, Q divided by 2, Q divided by 2; therefore, throughout the period it is Q divided by 2 throughout the year.  Each time I order Q throughout the year, I have Q divided by 2 average inventories.  So Q is equal to order quantity in units per order.  Average inventory per year, each time I order Q is Q divided by 2.  H is inventory carrying cost for one unit per year; therefore, for one unit per year I have H cost if I am carrying 2 units per year, then my carrying cost is H times Q divided by 2.

Look here. If Q goes up, annual carrying cost goes up.  In the other scenario, cost structure I had, in ordering cost, when Q goes up, ordering cost comes down, but in this one, when Q goes up, cost also goes up.  We said we are system analysts.  We try to make the optimal solution for everybody and not for only inventory counting cost.  If we are talking about inventory counting costs, it tells us order as little as you can.  If I am talking only about ordering cost, it tells me order as much as you can.  Order as little as you can where it says order as much as you can.  I can’t listen to none of them.  I should see what the benefit for the whole system is.

In this carrying cost, if order size goes up, then average inventory is here.  This order size—half of it is average inventory. Order size 600, average inventory 300.  Order size 800, average inventory 400.  Order size 900, average inventory 450.  So this is average inventory.  I should multiply by 16, and I will get this number.  As I go up, inventory carrying cost goes up.  And I have a curve like this.  H time Q divided by 2 increases as Q increases.  So I have order size, number of orders goes down as order size goes up, ordering cost goes down.  Average inventory goes up as order size goes up, carrying cost goes up, total cost goes down and then goes up.  It’s like this. This is carrying cost.  This is ordering cost.  This is total cost.  It reached a minimum and then goes up.  The best strategy for me is to order this much.  This is what minimizes my total cost.  That is what we call EOQ, economic order quantity. Total cost is inventory carrying cost plus ordering cost.  You can use calculus computed derivative of TC with respect to Q and compute EOQ.  If you do that, you will find that EOQ is at the intersection of both costs.  EOQ is where ordering cost intersects with carrying cost.  Therefore, these two are equal at EOQ, and therefore, we can solve this equation and therefore we can find EOQ is equal to square root to 2D times S divided by H: This is 2; this is demand per year, order cost per order, carrying cost per unit per year.  Now we go back to our previous example, and now we are in a good position to solve this problem.  So we know that annual demand is 9,600.  Ordering cost is 75, carrying cost is 16; how much should we order each time to minimize total cost, which is to order EOQ?  EOQ 2DS divided by H.  I put the numbers over there.  D is 9,600, H is 16, S is 75.  EOQ is 300. The best strategy is to order 300, not 100, not 9,600, 300.  That is what minimizes my overall cost: both carrying cost and ordering cost.  And as we said we don’t include purchasing cost because purchasing cost plays no role in this play because no matter we order 100 units, 200 units, 300 units, or 9,600 units each time, we don’t get quantity discount.  We will buy it for the same price. How many times should we order?  Annual demand is 9,600 units. How much do I order each time?  300.  Each time I order EOQ, therefore, how many times should we order?  I need 9,600.  Each time I order 300.  That is D divided by EOQ, which is 9,600 divided by 300, which is 32. 9,600 units are required for 288 days.  300 units are enough for how many days?  Each day, I need 9,600 divided by 288.  Each day, I need that much.  Then if I divide 300 by that number, it gets me the number of days.  Alternatively, I can say 9,600 are for 288 days, and 300, I divided by 9,600, which gives me a fraction of a year.  Then I multiply by 288 and get the number.  Just think about it.  You don’t need to memorize any formula.  Just manage it.  You have 288 days.  Each time you order 300 units, all you need during that 288 is 9,600.  How much is the length of each cycle?  It is 9 days. You can go through other different ways to come up with the same 9 days.  What is the optimal total cost? Total cost formula is Q divided by 2 times H + D divided by Q times S.  The economic quantity is Q.  All other things unknown.  We put them over there.  300 divided by 2 times 16 plus 9,600 divided by 300, times 75 gives me 2,400 plus 2,400.  Remember one test to know that your computations are correct is these two costs should come out equal at EOQ. Before EOQ, ordering cost is higher.  After EOQ, carrying cost is higher.  At EOQ, they are equal. Total cost is 4,800.  This is the optimal policy that minimizes the total cost.

Now, I solve another problem.  I mean, I really don’t need to solve it.  You can solve it yourself.  You have Centura Hospital.  Demand in this hospital is 31,200 units per year.  In our current strategy, we order 6,000 units each time.  Ordering cost is $130, carrying cost is $0.90.  A year is 52 weeks.  What is the average inventory or cycle inventory?  Here, Q is 6,000.  It reaches 0 and goes up to 6,000. Therefore, average inventory is Q divided by 2, which is 3,000.  What is total annual carrying cost?  This is average inventory per period and average inventory per year.  Therefore, on average I have 3,000 units in inventory and each one costing $0.90; therefore, carrying cost per units times average inventory is 2,700.  How many times do we order?  Each time I order 6,000.  All I need is 31,200.  31,200 divided by 6 means 5.2.  We place 6 orders.  5 orders are completely utilized in this period and the 6th one, 20% of it is utilized this year and the remainder is utilized next year.  But we simply stay with this 5.2.  The number of order per year is 5.2 What is the total annual ordering cost?  You order 5.2 times, ordering cost per each time you order is $130.  It is S times D divided by Q, which is 130 times 5.2, which equals $676.  I would like to ask a bright student a question without any other computation.  Could you tell me if EOQ is less than 600 or greater than 600?  Which one?  Who should be able to answer this question?  I don’t answer it here.  I just leave it to you.  But if you want to see if you are an A- or A student, you should be able to answer this question.  Is EOQ 6,000?  Is EOQ greater than 6,000?  Is EOQ less than 6,000?  Which one?  Without any other computation.

Okay.  These are now your assignment.  This is one assignment, assignment 4A.  You will solve it, and you will come to class next session, and we will go through this assignment and one or two other assignments, and then we are done.