This is our second lecture on forecasting in which we will be introduced to exponential smoothing. We learned moving average in which we take the most recent end-pieces of data, add them up and divide them by N. Alternatively we may say we divide each one by N and then add them up, or we can say we multiply each one by 1 over N and then add them up. For example, in a 4-period moving average in period T I take At, At-1, At-2, At-3 and divide them by 4. Or I can divide each one by 4, and dividing by 4 is the same as multiplying by .25; therefore, in a 4-period moving average, I multiply At by .25, At-1 by .25, At-2 by .25, At-3 by .25. So the weight of all pieces of data are equal. In a weighted moving average, the weights are not equal. They still sum up to 1, but they are not equal. For example, the most recent piece of data might get .4; the oldest piece of data might get a coefficient of .1. However, the summation of all the coefficients should come out equal to 1.

We will later show that exponential smoothing is a kind of weighted moving average. Let’s look at exponential smoothing formula. Forecast for the next period is equal to forecast for this period plus a fraction of the difference between actual and forecast in this period. I have my forecast for this period. I see my actual forecast for this period. I find the difference between that actual and forecast, and I incorporate a fraction of that difference and assume it forecast for the next period. We can write this formula line a different way. I might multiply alpha by At and alpha by Ft, then I will get this relationship, and then I might put Fts together. In that case I get Ft +1 is equal to (1-a)Ft+alpha times At. These two formulas are exactly the same. However, in the test, I usually give you this formula.

Let’s go through this example. In the first period, I assume forecast for period 1 is exactly the same as actual for period 1. Alternatively, I may add all pieces of data average them, and assume that as forecast for period 1. However, in this course we stay with the simpler assumption that forecast for period 1 is equal to the actual for period. Therefore, forecast for period 2 is equal to forecast for period 1 + alpha times actual of period 1-forecast of period 1. This is 100, 100, this is 100. The difference is 0. Alpha is equal to .2; therefore, F2 comes out equal to 100. In other words, F2 is equal to A1, and we put it over here. Now we wait until the end of period 2 and realize that A2 is equal to 150. We computed this, but this one was given. Our sales or marketing department have informed us actual demand for period 2 was 150. Now we compute F3, which is 1-alpha times F2 + alpha times A2. F2 is here. A2 is there, or if you want to go with the other formula, F3 is exactly F2 + alpha, which is .2 times the difference between A2 and F2. A2 is 150. F2 is 100. This difference is 50. 50 multiplied by .2 is 10. F2 was 100. 100 + 10 is 110, and that is F3. Or using the other formula again, we get 110. I put 110 over there as my forecast for period 3.

But look at an interesting observation, and this is why I try to set the stage for the exponential smoothing using the weighted moving average. F3 is equal to (1-alpha) F2 + alpha A2; therefore, I can claim that F3 is computed using F2 and A2. How did I compute F2? F2 was computed based on A1. F3 was computed based on A2 and F2. Therefore, I can say that F3 was computed based on A2 and A1. F2 was computed based on A1. F3 was computed based on A1 and A2.

Now let’s compute F4. I wait until the end of period 3. I realize that the demand for period 3 is 120 while my forecast was 110. F3 110, A3 120, alpha .2; I get 112, which is my forecast for period 4. This is F4 formula. F4 was computed based on F3 and A3, and I have already proved that F3 was computed based on A2 and A1; therefore, F4 was computed based on A3, A2, and A1. If you continue the same logic, you will see that F5 is computed based on A4, A3, A2, and A1, and you will see that F10 is computed based on A9 to A1. And we will see that Ft is computed based on At-1 and At-2 all the way to A1. Indeed forecasting using exponential smoothing takes into account all pieces of data but with different ways.

Now let’s go through this example where alpha is equal to .1. We have data for first 8 periods, and we want a forecast for period 9. For period 1 actual is 200 and forecast is 200, therefore, because these are equal to each other, my forecast for the next period is 200. Now I have actual of 250. This is At, and forecast of 200. Alpha is equal to .1. Alpha, 1-alpha. 1-alpha times F2. Alpha times A2, the result is 205. Here I have forecast for period 3 equal to 205 and actual has come out 175. Forecast is multiplied by 1-alpha, which is .9. Actual is multiplied by alpha, which is .1. .1 Times 175 + .9 times 205, and that would come out 202. And then I can continue up to there using the same procedure. When I reach period 8, I can compute forecast for period 9. This is actual. It should be multiplied by .1. This is forecast. It should be multiplied by .9. This is 19 + 198, which is equal to 217, and that’s my forecast for period 9. F9 is equal to 217. Can you tell me what is the relationship between alpha and N in N period moving average? What large alpha means? What small alpha means? The formula was Ft + 1 is equal to Ft, my previous forecast + alpha times At-Ft. When alpha goes up, I will give you more weight to the recent deviation. When alpha goes down, I will give less weight to the recent deviations. In moving average when N is small, I am more reactive. When N is large, the curve is more smooth. The larger the N the smaller the alpha. This is the relationship between exponential smoothing and moving average.

Let’s use alpha equal to .4. If you followed the same computations you would come to these results. This is alpha equal to .1. This is alpha equal to .4, and this is my actual data. As alpha goes down the curve becomes more smooth. It is closer to a horizontal line. When alpha goes up it becomes more reactive to the recent changes. In moving average when periods are small, we are more reactive in exponential smoothing. When alpha is large, we are more reactive. In moving average when N is large we have a more smooth curve. In exponential smoothing when alpha is small we have a more smooth one. But which is better? An exponential smoothing with a larger value of alpha, or one with smaller value of alpha? We don’t know. It is the same as moving average. We need to compute the MAD, and MAD will tell us which one is better. So here I have my data. These are my forecasts using alpha equal to .1, and these are my forecasts using alpha equal to .4. Look at period 2 to period 8. Actual is 250. Forecast is 200. Absolute deviation is 50. Demand 175, forecast 205, absolute deviation 30. I add up all absolute deviations and divide it by 7; 46.73. For alpha equal .4 I will add up all absolute deviations and divide it by 7, and I will get 51.38. I compare these two. This MAD is lower. For this specific set of data, alpha equal to .1 is a better coefficient.

Let me use this example to show how the forecasting changes when alpha goes up or comes down. Here first I have alpha equal to 1. If you put alpha equal to 1, Ft + 1 is equal to 1-alpha times Ft + alpha times At. When alpha is 1, here I have 1, and here 1-1 is 0. So 0 is multiplied by Ft and Ft + 1 is equal to At, so it is exactly like naïve technique. This is my forecast, which is the same as actual for the previous period. The same is here forecast equal to actual for the previous period. Forecast equal to actual for the previous period.

Now let’s see what will happen when alpha goes down, .9, .8, .7, .6, .5, .4, .3, .2, .1, 0. When alpha is 0, I have a straight line. I have no reaction toward what is happening in reality, but when alpha goes up, I become more reactive. More reactive. More reactive. .4 I’m here, .5 other there, .6, .7, .8, .9, 1. In exponential smoothing you always take forecast of the previous period an actual of the previous period and combine those two to come up with forecast for this period. There are only 2 values, actual and forecast. We combine them using alpha and come up with the forecast for next week. But forecast for the previous period has all the history in his stomach, and when we add it to the actual for the previous period, the whole history of data can be calculated. Now let’s take another look at the exponential smoothing formula.

Forecast for period T is a function of actual in the previous period and forecast in the previous period while actual is multiplied by alpha and forecast is multiplied by 1-alpha. Each time we get a new piece of data, we multiply it by alpha; however, Ft-1 is not just Ft-1. It has a history of previous data inside its stomach. If we only look at one period earlier, Ft-1 is a function of At-2 and Ft-2. Where At-2 is usually is multiplied by alpha, Ft-2 is multiplied by 1-alpha, however if I replace Ft-1 by the equation of Ft-1, then I will get this new equation in which each actual piece of data is multiplied by alpha, and I know that because in the formula I get an actual and multiply by alpha. But look at this, a piece of data which is one period old is only multiplied by alpha, and it is not multiplied by 1-alpha. A piece of data which is 2 periods old is multiplied by alpha and also is multiplied by 1-alpha. Now if I replace Ft-2 by its equation, then I will get the new Ft. So this formula for Ft and this formula for Ft are equivalent, and the second formula shows what is inside the stomach of Ft-1. As I can see in this formula, all pieces of data are multiplied by alpha. So alpha is there, but the piece of data which is one period old is not multiplied by anything else. The piece of data which is two periods old it is multiplied by 1-alpha. The piece of data which is 3 periods old is multiplied by 1-alpha to the power of 2, which is one less than 3. And if I continue I will get this general formula for Ft. All pieces of actual data are multiplied by alpha, but each of them is also multiplied by 1-alpha to the power of something. Here is nothing. Here is 1-alpha to the power of 1. Here is 1-alpha to the power of 2. So the piece of data which is 3 periods old is multiplied by 1-alpha to the power of 3 minus 1. The piece of data which is 2 periods old is multiplied by 1-alpha to the power of 2-1. The piece of data which is one period old is multiplied by 1-alpha to the power of 1-1. 3-1 is 2. 2-1 is 1, and 1-1 is 0. Anything to the power of 0 is equal to 1. If I want to find the coefficient of At-6 in my equation, At-6, as we said all pieces of information are multiplied by alpha, and because this piece of information is 6 periods old and it is also multiplied by 1-alpha to the power of 5, suppose alpha is equal to .5, then 1-alpha to the power of 0 is 1. 1-alpha to the power of 1 is .5. 1-alpha to the power of 2 is .5 squared, which is .25. 1-alpha to the power of 5 if alpha is equal to .5, that would be .5 to the power of 5, which is something around .01 or .02, very small, therefore exponential smoothing takes into account all pieces of data, but as the data gets older, its coefficient gets smaller. Because alpha is 1 or less than 1, it’s between 0 and 1, therefore 1-alpha is also between 0 and 1, and something between 0 and 1 when it gets to the power of 2, to the power of 3, to the power of 4 it gets smaller and smaller and smaller. So exponential smoothing is a weighted moving average which takes into actual pieces of data, but the coefficient of each piece of data gets smaller as that piece of data gets older.

When we talk about an N period moving average, the newest piece of data is 1 period old. The oldest piece of data is N periods old. On the average, data is 1 + N divided by 2 periods old. We can mathematically show that the age of data in exponential smoothing is 1 over alpha. So if you want these two techniques to be close to each other to represent almost the same thing, we could equate these two statements 1 + N divided by 2 is equal to 1 over alpha and then if you solve this equation you would get alpha is equal to 2 divided by N + 1. For example, in a 6 period moving average, the newest piece of data is 1 period old. The oldest piece of data is 6 periods old. The data on average is 3 and a half periods old. If you want to find an exponential smoothing which is almost equivalent to this we said 3.5 equal to 1 over alpha. Alpha is almost .28. It doesn’t mean that the two techniques would be exactly the same, but they are more or less close to each other.

Here we want to do exponential smoothing using Excel, and we want to also introduce a couple of Excel features. Here is my data. Suppose I also had demand in period 0, and I have used that demand of period 0 as my forecast for period 1. Who suppose using any other technique I have covered for the forecast for period 1, and I want to use that forecast and continue from there using exponential smoothing. So what I will do, I will go here, forecast for period 2 is equal to 1-alpha, and I make it absolute by pushing F4 times forecast + alpha. I make it absolute times actual. This is my forecast for period 2. This is my actual for period 2. I copy this one. I have forecast and actual for period 2 to come up with forecast for period 3. Forecast of period 2 times 1-alpha + actual of period 2 times alpha, and then I can copy down for all periods. Now I have actual and I have forecast, actual minus forecast, and I can copy them and down. So this column is the difference between actual and forecast, and that is my arrow for my deviation. Now I need to compute equal to absolute ABS of this, and I can copy down. Now I want to compute MAD. I can write this one is equal to absolute deviation divided by number of observations which is here. I should write equal to summation of these two divided by 2. That’s correct, but if I copy this down it is not correct anymore. What this cell says is it only adds these two numbers and divided by 6. Why? I need to add from here to here and divide by 6. The same is true for this one. In order to avoid this mistake, when I do this summation I put my cursor over there and push F4 to make the first element absolute. Now when I copy down, when I look at this number from there to here, divided by, and then I can continue to the end. For example here is the summation of all these numbers divided by 24. This is my MAD. Here it is equal to this one. Now I should do the same for deviation including the signs. This is equal to this. This one is equal to summation from here to here, but I will go over there, push F4 to make the first one absolute. Now if I copy down, this one is summation of all these numbers. In order to compute the tracking signal, I should divide this one by this one, enter. And I should copy down, and that would be tracking signal. Here I use MAD equal to .5. Is .5 better, or .4, or .1, or .2? So I type .1 and .2, then I right click on it fill the settings. I make it like this. Then I type MAD here. Note that if your data is here, MAD should be one column to the right, one row to the north. So that is exactly where it goes. Now after, I go to data. What it if analysis is data table? This is one dimensional data table because I don’t have 2 dimensions. I only have these columns. So my column report is alpha, and alpha is over there. These are my MAD values. I reduced the decimal point, and here I type equal to mean of these numbers. Then I look over there and find out the minimum is here for alpha equal .3. Maybe it is difficult to go and find which one is minimum. So instead of that I will go here and click on conditional formatting and highlight cell equals. If I say this cell is equal to this cell, then make it colored. Then I add that format back to all of them and I save the third one.