Okay. Now we have solved a problem, and if you remember we said solving the problem differs from formulating the problem. Formulating the problem means transforming narrative or tabular, or pictorial representation into a mathematical representation to solve the problem we transform the mathematical representation to Excel and then ask Excel to solve. Now we have translated that information to Excel and have asked Excel to solve the problem. Excel has solved the problem. The solution is here. X1 and X2 values are here. And the value of the objective function for this specific solution for this specific product mix for this specific production plan, the value of objective function is here. We make 2 units of product 1, 6 units of product 2. Based on our initial assumptions profit of product 1 is 3, profit of product 2 is 5 per unit, and therefore we collect a total profit of $36, and no one, no one in the whole world can come out with a production plan with a product mix, with a solution and all these three names can be used interchangeably, no one in the whole world can come out with a solution which can create a profit more than this solution. So this is one piece of information we have. Solution and the value of the objective function. This table tells us what you need on this left-hand side. What you have is on the right-hand side. These two constraints are binding constraints, binding constraints. Binding constraint means left-hand side is equal to right-hand side. Also we have one constraint which left-hand side is not equal to right-hand side. So now if a person comes to you and says how much do you pay for one unit of resource 1? Nothing, absolutely nothing. Because we already have 4 units of resource 1, and we have not used all those units. We have only used 2 units out of those 4 units. And don’t forget our main assumption is everything that our company can do has already been formulated into this mathematical representation translated to Excel. All our world has been transformed here, translated here. Therefore, we don’t have such an opportunity to buy something, say for a cheap price and just sell it. It is not something in this problem. In this problem we have 3 resources. We need to transform them into a final product, and we can only sell the final product. We cannot sell the remaining of these resources. Of course it is a piece of cake for us to formulate that for example too, to come up with a problem in which we have the opportunity to sell our extra resources, but those assumptions makes our problem more complicated. If you are interested, you may think about a problem in which if a resource is not completely consumed you can sell it, and then you should have the sales price, and it is easy to formulate. Let’s stay with our basic assumptions, our initial assumptions and solve a very simple problem.

In this problem I have 4 units of resource 1, and I have consumed only 2 units, therefore, I don’t need any additional unit of resource 1. But what if that person come and say I have 1 unit of resource 2. How much are you willing to pay for it? What you can do is go to Excel under this situation 4, 12, and 18, your total profit is 36. You can go to Excel. You can replace 12 here, the 12 you have. You can replace it with 13. If you do that, and if you solve the problem once again, you will see that there will be a change in your solution. Now because you have more resource too, you can produce more product. We don’t know how that product mix will change, but we know that you can perhaps you can produce more product, and perhaps your profit goes up. If you go there, replace 12 by 13 and solve the problem, you will find that the objective function goes from 36 to 37 and a half. Go and do this experiment. Replace 12 by 13. Solve the problem. You will see that production plan, product mix will change, and profit will change to 37 and a half. Therefore, one additional unit of this resource will increase your profit from your net profit, free of all other costs, will change your net profit, your net contribution from 36 to 37 and a half. Therefore, the actual value of that resource for you is $1.5. Now if market price of that resource is less than $1.5, you go and buy. If the person who has come to your office tells you that I am willing to sell it for $1, we buy it, because you pay $1 to him, however, our total profit will go up by $1.5, and we have made 50 cents extra profit here. But if he says I am selling it for $2, we don’t buy it.

Now, if you change 13 back to 12, and if you go and change this 18 to 19 and solve the problem, your production plan will change, and your new objective function will be 37. Therefore, each unit of product 3 can increase your profit from 36 to 37, and that is $1, and if the market price of that product is less than $1, you buy it. This $1 is shadow price. It could be less than market price, or it could be greater than market price. See, interesting observations. However, when I solve the problem, if I click on sensitivity and say okay, so click on sensitivity and say okay, then Excel will generate another sheet. This sheet is sensitivity of report sheet. The bottom part of this sensitivity of report sheet is related to your resources, say resource 1, resource 2, resource 3, so this bottom part is related to resources. It says final value, final value means what you need. What you need is 2, 12, and 18. Left-hand side. But it also informs you of 3 interesting numbers, 0, 1.5, and 1. Do you remember what were they? They were the increasing the objective function due to 1 unit increase in the right-hand side of each of these 3 resources, one by one. So these numbers are shadow prices of those resources. Okay?

Tells you resource 1 has a shadow price of 0. Why it has a shadow price of 0? Because left-hand side is less than right-hand side. What we need is less than what we have. The other two have strictly positive shadow prices, non-0 prices because their left-hand side is equal to right-hand side, and if we change right-hand side of one of them by 1 unit, if we change the right-hand side of resource 2 by 1 unit, the profit will increase by $1.5. And if we change that number back to its shell value of 12 and now change this one to 19, it would increase the objective function by $1. Don’t forget when we are talking about 1 unit of product 1, 1 unit of product 2, and 1 unit of product 3, we are not talking about having all these 3 things together. In sensitivity analysis, we always talk about one change at a time. In sensitivity analysis, we always talk about one change at a time. Either first resource, second resource, or third resource, but no two resources together. So what would the impact of increasing this by one? 0. Shadow price is 0. If it remained the same, if the third remained the same but we increase the second resource by one, shadow price is 1.5. Profit will increase by 1.5. If we remain – retained this one at 4, retained this one at 12, but change this one to 19, then the objective function will go up by 1 unit. Shadow prices of each resources.

Okay. Now, suppose a person has come to you and tells you I have 10 units of resource 1. How much are you willing to pay for these 10 units? You will say 0, absolutely 0, because what I have is less than or equal to what I need. So I don’t need more. Shadow price of any resource in which which we need is less than what we have, is 0. That is a free resource. We don’t want that resource. We don’t need that resource. But if the person come and say I have 10 units of resource 2, how much are you willing to pay for it? What you will say? I have 10 units of resource two, how much are you willing to pay for it? How much? Perhaps your answer is $15. How did you get $15? Because you multiplied 1.5 by 10, and you got $15. And did you say I am willing to pay up to $15 because you want to have some money for yourself. So you say you are willing to pay say $14.5 or $14, then you pay $14 to him, and you collect $15 profit, still you put one extra box in your pocket.

Therefore, you expect when you change this 12 to 12 + 10, which is 22, you expect when this is 4, this is 18, and this is 22, you expect that if you go and solve the problem from the beginning, you expect that this 36 is increased by $15. So you expect to see an objective function of 51, which is 36 plus 50. Stop the recording. Go to that solver. Solve the problem, and let me know what is your objective function value.

If you go there and repeat this experiment, you will see that objective function will not go to 51. It will not go to 51. Why? Because that shadow price is not valid from 1 to infinity. It is valid from 1 to somewhere. After that shadow price will change. Perhaps if you buy 10 units of resource 2, then when you solve the problem your left-hand side would be less than your right-hand side. In that case, these resources will have a shadow price of 5. So shadow price will drop at some point from 1.5 to 0. Fortunately sensitivity analysis report will also provide us with that information. It says regarding resource 2 your left-hand side is 12. What you need is 12. Your shadow price is greater than 0, and it is 1.5, and your right-hand side is 12. Right-hand side is equal to left-hand side. Shadow price, greater than 0, but this value of shadow price is varied up to 6 unit increase, not up to 10 units, up to 6 unit increase. In other words, if you change this number to 13, 14, 15, 16, 17, and 18, for each unit increase in this number, you will get $1.5 increase in your objective function value. But after that, that 1.5 will not be valid. If you cannot follow what I am saying, just type the problem, solve it, and change the right-hand side of this constraint to 13, 14, 15, 16, 17, 18, and then 19, 20, 21, 22, and 23, and you will see the changes in the objective function. That 1.5 shadow price remains in effect for 6 units increase or 6 units decrease. So if you change 12 to 11, your objective function will go down from 6 36 to 34 and a half. If you change it to 10, your profit will go down from 36 to 33. Each unit decrease will also decrease the value of the objective function by that value. However, that decrease is also valid up to some point, 6 units decrease and it considers we have 6 here, 6 here, 6 here, and 6 here. There’s only 4 of these problems. These problems don’t have any other relationship. Here we could have 2, here 10, here 12, here 8 or something. So look at this information, this variable information. It says shadow price of resource 2 is 1.5, but it is 1.5 if you increase this 12 by 6 units, or if by up to 6 units or if you decrease this resource by up to 6 units. The same information is given for resource 3. It’s current value, it’s final value is 18. It’s right-hand side is 17; therefore, if I see this number without even looking at this number, I know it should be greater than 0. It is greater than 0, and it’s 1. That one is valid if I change the right-hand side by 6 units, add 6 units to it or subtract 6 units.

After that, that shadow price is not valid. Interesting observation, very interesting observation. Look, when I increase – when this is 4, and this is 18, and I increase this to 13, how objective function goes up from 36 to 37.5, because these 2 and 6 will change. Because I will produce more, and since I am producing more, I am making more profit, so when I change the right-hand side within the allowable range – when I change the right-hand side within the allowable range of increase or decrease, shadow price remains the same. Objective function – if shadow price is not 0, objective function will increase. Why it will increase? Because the production plan, the solution will increase. The solution will increase. Still it is multiplied by 5 and 3, and then it will create a new objective function value, therefore, the bottom table, the bottom table tells us what are the shadow prices? These shadow prices will remain the same as we are within allowable ranges. Since they will remain the same within allowable ranges, by changing the right-hand side, the objective function value will increase or decrease. Why it will increase or decrease? Because our production plan changes. Our production plan always changes unless the shadow price is 0. When shadow price is 0, production plan remains the same, and objective function will remain the same.

But we have more good news here. The bottom side of this table is related to constraints. The top side is related to coefficients of the objective function. The bottom parts is related to the right-hand side of the constraint. The top part is related to the coefficient of the objective function. The bottom part talks about 4, 12, 18. The top part talks about 3 and 5. The bottom part talks about constraints 4, 12, 18, the right-hand side. The impact on production plan on product mix, on solution, impact of the change on changes here, and impact of these changes on changes here. The top part talks about coefficient of the objective function, and their impact on here, and their impact on shadow prices. So the top part says solution product 1, solution product 2, final value 2 and 6. Two units of product 1, 6 units of product. Forget this column, forget this reduced cost. Objective coefficient 3 and 5, so this column is a repetition of this row. And this column is a repetition of this row, but it also has two additional columns which are not over there, allowable increase. It says this coefficient can increase by 4.5 units. So it can go from 3 to 7.5, or it can decrease by 3 units. It can decrease by 3 units. So this can increase by 4.5 units, and can decrease by 3 units. It can go from 0 to 7.5. And as long as the coefficient of this product, which is 3 now – as long as it is between 3 minus 3, which is 0, and 3 + 4.5, which is 7.5, as long as the coefficient of this product is between 0 and 7.5, the production plan remains the same. The production plan remains the same. As long as the profit of that product is between 0 and 7.5, we always produce 2 units of product 1, and 6 units of product 2. If you don’t believe me, go and check it. However, if this one remains as 3, and the other one changes, the other one, I am producing 6 units of the second product, its coefficient is 5, as long as it is within allowable range, and allowable range is 3 units decrease and if you see 3 year and 3 year, this is coincidental. There is no relationship between this 3 and that 3.

This product I am producing 6 units. Its coefficient is 5. As long as it is within allowable range, and allowable range is 3 units decrease. Therefore, as long as it is from 2, 2 allowable increase whenever you see this E, and a positive value in front of it, if you see E and a positive value, you may assume that is positive infinity, very large positive number. If you see E and a negative number in front of it, that means a very small value, negative infinity. So as long as the coefficient of this product is within 2 and + infinity, our production plan remains constant, remains the same. This 2 and 6 remains the same, but because these – either this one or this one has increased or decreased, then this objective function value will increase or decrease.

Okay, my friends, if you followed my discussion, you are in a good track. If you didn’t, then this is what you should do. Open Excel spreadsheet, type this information over there, and try to repeat what I said, to play with the right-hand side, to play with the left-hand side, and see the impact. Okay? I really recommend you do this before going on to the next session.