Now we will go to Chapter 6, Linear Programming. Suppose you have a 3, 4, 5 year old brother or niece or nephew, and you have provided him or her with 8 small brick Legos and 6 large brick Legos, and you are asking him or her – let’s make it nephew and use him for the remainder of the discussion. And you have asked him to manufacture a chair and table. Suppose these are designs of chair and table, and you are asking him to make chair, which is composed of 2 small bricks and one large brick or table, which is formed by 2 small bricks and 2 large bricks. For each chair he manufactures, you will give him 15 cents, and for each table you will give him 20 cents. He is a profit maximization seal, and we want to know how many tables and how many chairs he should produce to maximize his profit. So we have 2 scarce resources, small brick and large brick. We have 2 activities. We want to allocate these resources or activities, so this is a resource allocation program. We want to allocate our resources to our products in order to maximize our profit. These resources are available. We have all of them paid for, and manufacturing system for each chair, if they produce, they will make 15 cents and for each table they will make 20 cents.

If you want you can stop the discussion here and think what is the best strategy, how many chairs should you produce and how many tables should you produce?

What is the best strategy? What is the product mix? How many chairs, how many tables to maximize your objective function, to maximize your profit? If you want you can stop it here and answer this question, and we can continue.

2 chairs and 2 tables is the best strategy. With this strategy you make 70 cents profit. For two chairs you need 4 small bricks, and for 2 tables you need 4 small bricks, so you utilize all your small bricks. For 2 chairs you need 2 large bricks and for 2 tables you need 4 large bricks, therefore, you have also consumed all your large bricks. All your large bricks were consumed. All your small bricks were consumed. 2 chairs, 2 tables, a total profit of 2 times 15 plus 2 times 20, which is 70 cents. When I say solve the problem, I don’t mean find the solution. I mean to translate the problem into a mathematical representation. Here in this slide we have transformed an additive representation into a pictorial representation into a schematical representation. I am asking to translate that narrative or that schematic or that pictorial representation into a mathematical representation. The first question that I should ask myself is: what is the decision that I should make in this problem? What is the decision that your nephew should make in this problem? Look, the number of small bricks or the number of large bricks are not a decision that your nephew makes, because you have already provided him with 8 small bricks and 6 large bricks. Resources are given. That is not a decision he should make. How much profit he can make on each chair and each table is not a decision he can make. Each table is sold for 20 cents. You give him 20 cents for each table he gives you. Each chair is sold for 15 cents. You give him 15 cents for each chair he manufactures. Profit is not a decision he can make yet. Number of resources, the volume of each resource is not a decision he can make. What is his decision? I think the main thing that he should think about is how many chairs to produce and how many tables to produce. Okay? As soon as he identifies how many chairs to produce and how many tables to produce, then he can compute the total profit that he can make. Correct? Therefore, the decision he should make is how many chairs to produce and how many tables to produce. He makes this decision in the direction of maximizing the total profit. How many tables to produce? How many chairs to produce? We call them decision variables. He should make these decisions. As soon as he makes these decisions, he will know the total profit, and he wants to make these decisions in order to maximize the total profit, in order to maximize the objective function. So decision variables, how many tables to produce, how many chairs to produce, objective function, total profit. The goal to maximize the objective function. What I will do is come and define the number of chairs as X and the number of tables as Y, but I don’t know X is equal to is Y is equal to what. I don’t know that, but I know that is what I want to identify. In linear programming, we usually don’t show our variables with X and Y, because then we have X, Y, Z, T, Q, we don’t have too many variables. To have too many variables we usually show by X1, X2, X3, X4, X5, X2000, X3000. In there program, because we only have 2 decision variables, we only have 2 products, therefore, I say X1 is the number of product 1. X2 is the number of product 2. X1 is equal to the number of chairs. X2 is equal to the number of tables. Don’t forget what I’m trying to do is not to solve the problem, is not to find out that the best solution for this problem is 2 chair and 2 tables. No. What I am trying to do is translate this narrative or pictorial representation into a mathematical representation. X1 is the number of chairs that I produce. X2 is the number of tables that I am producing, and I want to identify X1 and X2 in order to maximize my total profit, in order to maximize my objective function. Objective function our problem could be profit, which we want to maximize it, or it could be cost, which we usually want to minimize it. In this problem, it is profit, and we want to maximize our total profit.

If you like, you can stop and think about it. In the remainder of our discussion, I don’t say stop and think about it, whenever you think that I am asking a question, stop, think about it and see if you can answer the question. And don’t forget exercising your central nervous system is the best way to get an A in midterm and final and in general in this course. Here I have 2 resources, large brick and small brick, two resources. And, therefore, I will have 2 constraints, large brick constraint and small brick constraint. Large brick constraint tells me you have 6 large bricks. How many large bricks do you need? You have 6 large bricks. How many large bricks do you need? Okay. You are producing X1 unit, chair. For each chair you need one large brick, correct? You are producing X1 chair, therefore, you need 1 times X1 large brick to produce your chairs, but for each table you need 2 large bricks, and you produce X2 tables; therefore, the number of large bricks that you need to produce X2 tables is 2 times X2. 1 times X1 + 2 times X2 is the total number of large bricks that you need. 6 a total number of large bricks that you have. The total number of large bricks that you need should be less than or at most equal to the number of large bricks that you have. Therefore, large brick constraint tells us 1 times X1, which is the number of chairs, + 2 times X2, which is the number of tables, 1X1 + 2X2 should be less than or equal to 6. Did you get it? If not, stop the record, think about it for a minute and then as soon as you understand it, write in the constraint for small brick is a piece of cake.

I produce X1 units chair. For each chair I need 2 small bricks. So 2 times X1. I produce X2 units, table. For each table I need 2 small bricks, so 2 times X2 is what I need of small bricks. The total number of small bricks is 8. Therefore, 2X1 + 2X2 which be less than or equal to 8, and that is small brick constraint. 2X1 + 2X2 is less than or equal to 8. Constraint 1. Constraint 2. 2 resources. Now I should say how much profit – how much is my profit? And my intention is to maximize that profit. If I am producing X1 units of product 1, X1 units of chair, each chair is sold for 15 cents; therefore, my profit is 15 times X1. If I produce X2 tables, for each table I make 20 cents profit, therefore, 15 X1 + 20X2 is my objective function, which I want to maximize it. Objective function, constraint, constraint. Decision variable, decision variable. Decision variable, decision variable, constraint, constraint, objective function. However, I cannot produce negative number of chairs or negative number of tables, therefore, I also need non-negativity constraints, which tells me number of chairs that you produce and number of tables that you produce could not be less than 0. Decision variable, objective function, functional constraint, non-negativity constraint. So when I ask you to solve a linear programming problem, I mean formulate it. Translate it into another language. Translate it from pictorial representation or from schematic representation or from narrative representation into mathematical representation, which is the highest level of representation and best way of representation. Now read this example and see if you can solve it.

In this program, we have 3 resources; resource 1, resource 2, and resource 3. And we have 2 products; product 1 and product 2. So product 1 and product 2 compete with each other in consumption of resource 1, resource 2, and resource 3. Product 1 needs one hour of resource 1, nothing of resource 2, and 3 hours of resource 3. So, for example, in the previous one our resources were bricks, Lego bricks. Here it may be a manufacturing plant, a human being, or a machine, or a combination of these things, but in general terminology we have 2 products which compete with each other in using 3 resources. Product 1 needs one hour of resource 1, nothing of resource 2, and 3 hours of resource 3. Or we can say 1 unit of resource 1, no units of resource 2, and 3 units of resource 3. That is what our product 1 needs. Product 2 needs nothing from resource 1, 2 hours of resource 2, and 2 hours of resource 3. So product 2 needs 0 hours of resource 1, 2 hours of resource 2, and 2 hours of resource 3. Now we know how much is consumption of each unit of each product from each resource. But how much of these resources are available? We have 4 units of resource 1, 12 units of resource 2, and 18 units of resource 3. The other piece of information we need is how much profit to we make? How much is the contribution of product 1 in profit and product 2 in profit? Contribution margin of product 1 and 2 is $300 and $500, therefore, each unit of product 1 free of all its costs can produce $300 profit and each unit of product 2 free of all its costs can produce $500. Now I am asking you to formulate this problem. As soon as you formulate the problem I will show you a way to enter that formulation into Excel and ask Excel to solve the problem and give you the optimal solution. The optimal solution is solution which maximizes your profit or a solution which minimizes your cost. In this problem we are concerned with profit, therefore, the optimal solution here is a solution that gives us the maximum profit. The optimal solution is defined based on how many product 1 do I produce, how many product 2 do I produce? It is what we call a solution. When I put those numbers into the objective function, into the profit function, then that becomes objective function value. So when I say the solution, I mean X1 is X to 1. X2 is equal to 1. When I say objective function value, it is what is the objective function value when I put the corresponding X1 and X2 into the objective function.

Okay. Formulate this problem. Put this narrative representation into a mathematical representation. Before you do that, let me first put this narrative representation into a pictorial representation to make it easier to understand. So I am producing product 1, and I am producing product 2. Each unit of product 1 creates $300 profit. From now on to make it easier I will just write 3. That means 300. Each unit of product 2 creates $500 profit, and from now on I say $5 profit. Product 1 needs 1 unit of resource 1, and 3 units of resource 3. Product 2 needs 2 units of resource 2 and 2 units of resource 3. 1 unit of resource 1, nothing of resource 2, 2 units of resource 2, nothing of resource 1, 3 units of resource 3, 2 units of resource 3. $3 profit, $5 profit. 4 units of resource 1 is what I have. 12 units of resource 2 is what I have. 18 units of resource 3 is what I have. I transformed an additive representation into a pictorial representation. Now I am asking you to translate this narrative representation or pictorial representation into a mathematical representation.

You can spend a few minutes to see what you can do. Our decision variables are how many product 1 to produce, how many product 2 to produce. Of course we are willing to produce infinite number of product 1 and infinite number of product 2, but our constraints do not allow us. For example, constraint 1 does not allow me to produce more than 4 units of product 1. And constraint 2 does not allow me to produce more than 6 units of product 2. So the maximum number of units of product 1 that I can produce is 4. The maximum units of product 2 that I can produce is 6. But we are not asking you to solve the problem. All I am asking you to transform is the narrative representation or pictorial representation into mathematical representation the decision that you should make is how many units of product 1, how many units of product 2 in order to maximize your total profit under these specific available volume of resources and these consumption rates. Your decision variables are how many product 1, how many product 2. X1 number of units of product 1, X2, number of units of product 2. As soon as you have reached this point and have a clear and correct understanding of your decision variables, 50 percent of the problem is done. I will formulate the problem and by that we mean to put other representations into mathematical representations.

X1 number of units of product 1, X2 number of units of product 2, therefore, the objective function value is for each number of unit of product 1 I am making $3 and for each unit of product 2 I am making $5. X1 unit of 1 so that would be 3 times 1, X2 units of 2 would be 5 times X2. 3 times X1, the number of units of product 1 + 5 times X2, the number of units of product 2, that is my total profit. So the objective function Z, which I want to maximize it is 3X1 + 5X2. I should write my constraints.

I should write my constraints with respect to constraint 1, which is resource 1, product 1 each unit needs one unit of resource 1. Product 2 does not need any resource 1; therefore, one times X1 is what I need. 4 is what I have. What I need should be less than or equal to what I have. For resource 1, X1, 1 times X1 should be less than or equal to 4. Resource 2, product 1 does not need resource 2. Product 2, each unit of product 2 needs 2 units of resource 2, and I produce X2 units of product 2. X2 units of product 2 each needs 2 units. Total number of units 12. Resource 2, 2 times X2 is what I need. 12 is what I have. What I need should be less than or equal to what I have, and that is resource 2 constraint.

Now we go to resource 3. Each unit of product 1 needs 3 units of resource 3. Each unit of product 2 needs 2 units of resource 3. X1 unit, 2X units. Each units 3, each units 2, the total number 18. So that would be quite straight forward. 3X1 + 2X2 less than or equal to 18.

I should also apply non-negativity constraint. A non-negativity constraint tells me X1 and X2 must be greater than or equal to 0. Decision variable, decision variable, objective function, objective function, functional constraints, functional constraints, non-negativity constraints. So up to now we were able to translate a narrative representation or a pictorial representation into a mathematical representation, and it is what we would call formulation. When I say formulate this problem I mean translate a narrative or pictorial representation into a mathematical representation. As soon as we can do this, then we can pass this problem to Excel and ask Excel to solve it. Solve it problem as an assignment and bring it to class. Don’t forget here you have 3 decisions to make, how many product 1, how many product 2, how many product 3; therefore, you will have decision variables and X1 you lose it for product 1, X2 for product 2, X3 for product 3. Profits are known. Available knowledge of resources is known, very easy to formulate the problem.

Okay. Before going into Excel, let’s look at this quiz that you may see in the test. So here is a problem with two decision variables, and these are the constraints. Correct? 2 decision variables, objective function, functional constraints, non-negativity constraint. What combination of X1 and X2 could be the optimal solution? Don’t forget we refer to optimal solution as value of X1 and value of X2, variable of decision variables. The objective function value is when we put this X1 and X2 into the objective function. Okay. The problem is asking what combination of X1 and X2 could be the optimal solution? Don’t forget optimal solution should satisfy all constraints. It cannot violate any constraint. If a solution satisfies all constraints, it is called feasible solution. If it violates even a single constraint it is called infeasible solution. A feasible solution could be optimal, and infeasible solution could not be optimal because infeasible solution violates a constraint and when violates a constraint means it needs some resources that we don’t have, or needs requirements that we don’t have. So the solution must be feasible to be optimal, and it should be better than any other feasible solution to be optimal. Okay.

Look at this problem. It says which combination A, B, C, D, or E could be the optimal solution? We should first check to find out if a solution is feasible or not. If it’s not feasible, then we ignore it. Among those which are feasible, we should find out which one creates the maximum value of the objective function, and that solution could be optimal. Look at the first one, X1 equal to 4, X2 equal to 4. Is this feasible? When we look at constraint one X1 equal to 4 is less than or equal to 4 and therefore doesn’t violate constraint 1. When we look at constraint 2, X2 is 4, 2 times 4 is 8, 8 is less than or equal to 12. So this solution doesn’t violate this constraint or this constraint. But when we put this solution into this constraint it is 3 times 4 plus 2 times 4. 12 plus 8, 20 is not less than or equal to 18, therefore, this solution is not feasible. It is infeasible because it violates constraint 3. Look at this solution. Is it feasible or not? Negative 3 is less than 4, so it is feasible. 2 times 6 is less than or equal to 12, 12 is equal to 12, so it doesn’t violate. Here look, 3 times negative 3 is 9, negative 9 + 2 times 6, which is 12. This one is not also violated, but from the beginning, we really didn’t need to go through these constraints because it is negative, and we know non-negativity constraints does not allow negative values. So this is infeasible because it violates non-negativity constraints.

C. X1 is equal to 3. X2 is equal to 4. We know it doesn’t violate constraint 1 and 2, but what about constraint 3? 3 times 3 is 9. 2 times 4 is 8. 9 + 8 is 17. It doesn’t violate it. This solution is all value of decision variables are positive, so it also does not violate non-negativity constraint; therefore, this is a feasible solution. It’s a feasible solution. It doesn’t violate any constraint, so I can put it into the objective function and find its objective function value. That is 3 times 3 + 4 times 5. That is 9 + 20, which is 29; therefore, this solution is feasible and value of its objective function is 29.

Let’s look at this solution, X1 0 and X2 7. It does not violate this constraint, but it violates this constraint. It does not violate constraint 3, but it violates constraint 2. Therefore, infeasible because it violates constraint 2.

X1 equals 2, X2 equals 6. X1 equals 2 does not violate this constraint. X2 equals 6 does not violate this constraint. 2 times 3 + 6 times 2 is 18, does not violate this constraint, therefore, this one is also feasible. I should compute the objective function value with respect to this feasible solution. 2 times 3 is 6. 6 times 5 is 30. 30 + 6 is 36. 36 is greater than 29, therefore, what combination of X1 and X2 could be the optimal solution? This could not be, this could not be, this could not be, because it is smaller than this one. This could not be, therefore, this one could be, and the answer is E.