Now, we want to develop a mathematical model, a mathematical representation for this narrative and then give it to Excel and ask Excel to solve it. Excel will return the best production plan. Before going through discussion of how to translate this narrative into a mathematical representation, let us take a second look at this problem. We have demand here, demand in different month, uneven demand, sometimes low, sometimes high. We have supply. We have two sources of supply, supply in regular time, which could be up to 200 units and supply in overtime, which could be up to 20 units. 200 units per month. 20 units per month. These numbers could have been different. We could have different numbers here as supply in regular time in different month, because sometimes in a month we may have some repair and maintenance, and the capacity in regular time may go down or sometime we may allocate some capacity to other things. Therefore, while we have assumed 200, 200, 200, 200, 200 and all of them are equal, in a mathematical representation, we don’t need these numbers to be equal. They can be any numbers. We can input them into our mathematical representation. So we have two sources of supply, supply in our regular time and supply in overtime, but supply in regular time costs us dollars. Supply in overtime costs us $13. Also we may produce in one month and carry to another month. That will cost us $5 per unit per month. On the other hand, we can produce in a future month to satisfy the demand of a previous month. In that case, it costs us $10 per unit per month. We want to formulate this problem as a linear programming and give it to Excel.

This is an overview of what I am going to do. These are my supply points, supply in regular time of month one, supply in overtime of month one, supply in regular time of month 2, supply in overtime of month 2, supply in regular time of month 9, supply in overtime of month 9. This is demand month 1, demand month 2, demand month 9. I would like to prepare a cost table like this to tell me what is the cost of, for example, producing overtime in month 2 to satisfy, for example, the demand of month 7. In a couple minutes we will discuss how to create such a cost table. Then these are my supplies. These are my demand. I would like to develop a production plan to tell me when to produce and when to consume. These are supply capacities. These are demand requirements. Cost table, production plan table. We will go through both of them, and we will learn how to solve this problem. So I clear the table to show my changing cells to show my production plan. This is these cells. I create a table like this. I explain this table first because it is easier to understand, then I will go and understand the other table.

These are my changing cells. Production regular time 1, production overtime 1; production regular time 5, production overtime 5, production regular time 9, production overtime 9. Demand 1, demand 2, demand 5, demand 6. Now, let me know what is the meaning of this cell. What is the meaning of this cell? What does it say? If I put a number in this cell what does it mean? If I put 5 over there, what does that 5 mean? That 5 means I produce 5 units in regular time of month 1 to satisfy the demand of month 1. Similarly, if I put a 10 here -- that is 10 not ID. If I put a 10 here, I produce 10 units in overtime of month 1 to satisfy the demand of month 1. If I put a 10 here, I produce 10 units in regular time of month 2 to satisfy the demand of month 2. If I put a 10 here, it means I produce 10 units in overtime of month 1 to satisfy the demand of month 2. 10 units in overtime of month 1 to satisfy the demand of month 2. If I put a 10 here, that means I produce 10 units in overtime of month 3 to satisfy the demand of month 4. If I put a 10 here, that means I produce 10 units in regular time of month 3 to satisfy the demand of month 2. So when demand in month 2 comes up, I will say okay wait for me I will produce in month 3, and I will give it to you in month 3. I will satisfy the demand of month 2 in month 3, and of course, as we said this is a cost involved. So now you are able to interpret all of these numbers. For example, this 15 means I will produce 15 units in overtime of month 7 to satisfy the demand of month 4. Okay?

Just put some numbers in different boxes and interpret their meaning. If you have any question, bring it up in our problem solving session. I think we are done with this table. The meaning of different elements in this table. This is our changing cells table. It will tell us how many units of product in each month, regular time or overtime should we produce to satisfy the demand at the minimal cost, minimal cost. Okay. Obviously the total numbers that I write in this row means the some of whatever I have produced in regular time of month 1 to satisfy the demands in different months. I will then put that summation here. So here I will define it as summation of these numbers. What that means, it means whatever I have produced to satisfy the demand of different months using the production capacity in regular time of month 1. Obviously what I have produced in regular time of month 1 to satisfy the demand of different months, and I put it over here, cannot exceed the capacity of production in regular time of that month. Therefore, what I have produced in one month cannot exceed the capacity of that month in regular time and in overtime. So here I define my right-hand side; the capacity of different month, regular time or overtime. I type those numbers in this column. Capacity, total capacity of production, using different resources at different months. That is what I have here. Then I add up these numbers and put it here. That is what I have produced for consumption in these months. This should be less than or equal to what I have, what capacity I have. What I have produced in overtime of month 1 to satisfy the demand of different months is summation, which is written here. It should be less than or equal to the overtime capacity of month 1. And then I copy down, up to here. What I have produced in different months to satisfy the demand of different months. Here are my demands in different months. I just copied them from the data. This column means what I have produced in different months to satisfy the demand of month 1 I can satisfy the demand of month 1 by producing in the same month or by producing in later months and accept back log cost. So I will have some numbers here, and that should be equal to 190. Later, smart students will learn that this sign could be equal or it could be greater than or equal. No matter if you put equal here or greater than or equal, Excel will produce 190 units, because if it produces more than 190 units, the total cost of production will go up. So no matter if you put equal or greater than or equal here, we can do that. We can put greater than or equal, but in both cases Excel will only produce 190 units, because if it produces more than 190 units – if it produces 191 units, there will be a cost involved for that one extra unit while you don’t need that extra unit in that month. However, you are not allowed to put less than or equal to, because if you put less than or equal there, then Excel will produce nothing, because if it produces nothing, the cost of production is 0. Unless you force Excel to produce something, Excel will not produce anything. Therefore, to be able to satisfy the demand, you should put here equal to 190 or greater than or equal to 190 but not less than or equal. So if all columns we will write the demand of that column, equal, greater than or equal, and here is the summation of the numbers that will appear in this column. This is some of the production in different months in overtime and regular time to satisfy the demand of month 3.

I will go to Excel, and I define this cell as summation of these cells. This is what I have produced in month 1, in regular time of month 1 to satisfy the demand of different months, and then I copied down. This row when added up here it says what I have produced in regular time of month 4 to satisfy the demand of different months. Similarly I will go here, and I define this cell as summation of the numbers in this column, and therefore this number would be sum of the production of different months to satisfy the demand of month 2. Here this was summation of these numbers.

So now I have declared that what I produce in different months cannot be greater than capacity of that month, and what I need in that month should be equal or as I said it could be equal or greater than or equal of what I need in that month. This is my changing cells table. Now I should go and define my costs. For example, what is the cost of this one? What is the cost of this one? What is the cost of this one? What does this cell mean? It says production regular time of month 3 to satisfy the demand of month 3. What is production cost in month 3? Production cost in month 3 in regular time is $6. So if I produce 1 unit here it costs us $6. What does this mean? It means producing in overtime of period 4 to satisfy the demand of period 5. Producing in overtime of period 4 costs us $13. That was given by the problem. So if it was here, it could have cost me $13, but it is not here. It is here. So I should produce at cost of $13, and I should also carry to the next month $13, one month carryover cost as given by problem is $5. So cost of one unit production overtime of month 4 to satisfy the demand of month 5 is $13 overtime production cost plus $5 carryover cost. What does this mean? Production in overtime of month 5. If it was here, production in overtime of month 5 to satisfy the demand of month 5 was $13, but it is not here. It is here. So I should produce at cost of $13 here, but I should also incur 1, 2, 3 back order costs. 1, 2, 3 back order costs because it is produced in month 5 to satisfy the demand of month 2. $13 production in overtime carried here that would be $13 + $10, $23 + $10, $33 + $10, $43, therefore, cost of producing one unit in overtime of period 5 to satisfy the demand of period 2 is 13 plus 10 plus 10 plus 10 equals 43. This is a table that I showed you before in a couple of previous slides. I showed the table here. This is cost table that I am now explaining to you. This is changing cells table that I have already explained to you. Now you can do this part. I am explaining this part.

Okay. We are here. We are talking about cost table. Look at this table. What does this number mean? Production in regular time of month 1 to satisfy the demand of month 1, and we know that cost is $6 per unit. This is production in overtime of month 1 to satisfy the demand of month 1. That is $13. It is given by the problem. So if in any month I produce in regular time to satisfy the demand of the same month, produce in regular time of month 7 to satisfy the demand of the same month, if that is the case, my cost is $6 per unit, $6 per unit. But if I am producing it in overtime, overtime of month 1 to satisfy the demand of month 1, that would be $13 per unit. $13, $6, $13, $6, $13, $6, all numbers here in this table are cost per unit not cost per how much production we have had, cost of per unit of product. Then I want to multiply this cost table by production plan table, and I can compute my total cost. So entering these numbers into Excel is piece of cake. Here, production in regular time of month 1 to satisfy the demand of month 1 costs me $6 per unit. But if I produce in month 1 in regular time of month 1 and carry to month 2 to satisfy the demand of month 2, that would be $6 production cost plus $5 inventory cost. So that would be 6 + 5. If I carried to period 3, that would be 6 plus 5 plus 5, would be 16. If I carry it to satisfy the demand of month 7 that would be 6 plus 5 plus 5 plus 5 plus 5 plus 5 plus 5, six 5s plus 6 is 36. If I produce in overtime of month 4 to satisfy the demand of month 5, that would be $13 production time. If I produce in month 4 to satisfy the demand of month 5, that would be $13 production cost + $5 inventory cost, which makes it $18. If I carry to month 6, that would be 13 plus 5 plus 5, which is 23. Therefore, let us fill this row and then the remainder is piece of cake. Produce 6 if you want to carry to next month 11, 16, 21, 26, 31, 36, 41, 46. If you produce in overtime and carry to the next month 18, 18 plus 5, 23. 23 plus 5, 28, therefore, what this number tells us is if you produce in overtime of month 1 to satisfy the demand of month 7, that will cost you $43. So now it is obvious that this is 11. This is 16, 21, 26, 31, 36, 41, and it is obvious that this is 18, 23, 28, 33 because each period that I carried forward, $5 will be added to it. Therefore, I can fill upper diagonal part of this table. Filling the diagonal part was easy. Upper diagonal part was easy. Filling the lower diagonal part is easy.

What does this number here mean? It means you produce in regular time of period 3 to satisfy the demand of period 2. In regular time it costs you $6, but because you are telling the demand to wait for one month, each month of waiting for one unit of product costs you $10, therefore, if you want to fill this number it would be 6 plus 10. If you want to fill this place it would be 6 + 10 + 10, which is 26. Exactly the same as inventory carryover, but inventory it carried forward. Back log is carried backward. It satisfied the demand of previous periods.

So if you produce in regular time of period 9 to satisfy the demand of period 9, it costs you $6, but if you want to satisfy the demand of period 8, it would be 6 + 10, which would be 16. If you want to satisfy the demand of period 7, it would be 6 + 10 + 10, which would be 26. If you produce in overtime of period 9 to satisfy the demand of period 4, that would be 13 + 10 + 10 + 10 + 10 + 10, which is 63; therefore, it is piece of cake to fill the whole table. If you have problems, then go back and listen to the lecture again.

Now, I have entered this data into Excel. The only other thing that I need to define is this cell. This is my objective function. These are my changing cells, my production plan, my solution. This is my objective function. I define my objective function as sum product of B2 to J19, which are these cells, B2 to J19, and these cells, B22 to J39. What does did say? It says the total cost of production is multiplication of these numbers which are cost per unit of product multiplied by these numbers, which are the number of units that I have produced in different months or in different boxes. So I go to Excel. I say my target cell is this cell, M42. I want to maximize it – no I want to minimize it. I click on minimize, not maximize. Maximize is usually up. But I click on min because this problem is not profit maximization. This is cost minimization.

So I’ll go and defined my changing cells. My changes cells are these cells. Then I will go and add constraint. I will say this and this and this and this one by one should be less than or equal to capacity. So I declare left-hand side for each row is less than or equal to right-hand side. What I produce in any month overtime or regular time is less than or equal to capacity of that month overtime or regular time. This is my supply constraint, what I produce cannot exceed my capacity. Then I should define my demand constraints. Demand constraints tells me what I produce in this month to satisfy the demand of that month should be equal to the demand of that month. As I said, we can put equal or alternatively we can put greater than or equal. In both cases Excel will not produce more than that, because the objective of Excel is to minimize the total cost, and therefore, if they produce more than what you need, you will have additional cost. You will not minimize your total cost. However, you cannot put less than or equal here. If you put less than or equal here, Excel will produce nothing and your total cost would be 0, and that is minimal cost. On the same line of reasoning, you cannot put equal here or equal here or equal here because in that case you are forcing Excel to take advantage of all your capacity, but why which Excel produce with maximum capacity when the demand is not there? If you put greater than or equal that is completely wrong, because you are presenting some knowledge to Excel which is in contrast of what is given in the narrative. In the narrative it says this is your maximum capacity. You cannot translate it to Excel that you can produce more than this. No you cannot. That is your mistake in translation. If you put greater than or equal here, that means you are wrongly translating the narrative into mathematics, but if you put equal here, that means you are forcing Excel to follow a wrong logic. You are forcing it to use all the capacities, incur all costs while the demand is not there. So all we need to tell Excel is that what you produce in different months should be less than or equal to your capacity, and what you need in each month is greater than or equal to demand. The data has been transferred to Excel. The data is correct. Min is over there. Target, changing cells, constraints. Assume linear model, assume non negative.

Okay. Everything is ready. Go ahead and solve the problem. Now Excel solves the problem, gives you your production plan. These red numbers are your production in different months to satisfy the demand of different months, and if you multiply this production plan by those cost numbers, the total is $15,070. That is the minimal cost. No one can come up with a production plan with a cost less than this. However, the problem has multiple optimal solutions. You may come out with a different production plan which costs the same. That is also correct. But you never come out with a production plan which costs less than this. This is optimal cost, minimum cost. For example, these numbers mean to satisfy the demand of month 6 you have produced 130 units in regular time of month 6, 20 units of overtime of month 6, and in months 7 you have produced 20 units to satisfy the demand of month 6. These numbers tell us in regular time of month 7 you have produced 20 units to satisfy the demand of month 6. You have produced 160 units to satisfy the demand of month 7, and you have produced 20 units to satisfy the demand of month 8. Production plan, objective function, optimal objective function. In some periods you have utilized your full capacity. In some periods you have not utilized it. The good news is if you come out with a different number than these numbers, and you come up with different numbers than these numbers, immediately you can put them into Excel and find a new solution. Even if these numbers change, you can immediately change this table and find a new optimal solution in a second.

Okay. So if you have understood this material, you are really fine, and you can enjoy your time. In order to check that you have understood this material, go to power points, look at the next problem, and try to solve it first. Don’t look at the solution. Look at the problem. Take it to Excel. Enter it. Solve it yourself. After you solve it, then you may go and look at the solution and find out if you have been correct or not. Alternatively whenever you encounter dead lock and you see you cannot go further, come back, listen to the remainder of my lecture, just up to the point where you can resolve your problem, then stop the lecture, go back and continue the solution. Listen, this is the way to learn the material, to try to exercise your central nervous system, not just to listen to what I say. That doesn’t work. So now go and solve the problem. When you encounter a difficulty, come review this lecture.

This is problem 2. Again, in order to be able to show all numbers in Excel sheet in a power point instead of going through 12 months, I have gone through 6 months. These are demand in the next 6 months. Demand is uneven, 500, 200. It can go from a low of 200 to a high of 500. The total demand is 6 periods is 1,800. My production capacity I have 15 workers. Each one can produce 20 units. So each month I have 20 times 15, which is 300 units production capacity.

Production in regular time costs me $2. If I do it in overtime it costs me $3. If I subcontract it, it costs me $6. $2 per unit, $3 per unit, $6 per unit, regular production, overtime production, subcontract, outsourcing. Inventory carryover cost is $1 per unit per month. Back order cost is $5 per unit per month. Beginning inventory 0. Ending inventory 0. I want you to formulate this problem in Excel, and then we can ask Excel to solve it. Okay?

We assume that overtime capacity is also 300 per month. And subcontracting capacity is also 300 per month, so I have 300 regular time production, 300 overtime production, and 300 subcontract production. So regular, overtime, outsource. I have 3 rows for each month. Month 1 I can produce regular, overtime, or outsource. Month 5 I can produce regular, overtime, or outsource. Capacity, regular 300, overtime 300, outsource 300. If you have another problem in which these numbers are different, you can just enter them. Only need to know the procedure. You don’t need these numbers equal to each other. This can be 300. This can be 100, 400, 200, 800, whatever. You just enter it. These are demands. You just enter them. So 6 months of demand. 6 months of supply, but in each month of supply, we can supply from 3 different sources. Regular production, overtime production, outsourcing production. Production capacity, supply capacity, demand capacity. As we said, we can easily fill the diagonal part, this part. It says if I produce in month 1 to satisfy the demand of month 1, if it is in regular time $2, overtime $3, outsource $6. If I produce in month 2 to satisfy the demand of month 2, regular time $2, overtime $3, outsource $6. So filling these numbers is a piece of cake. I enter regular time cost per one unit of product, overtime cost per one unit of product, outsourcing cost per one unit of product, then as soon as I enter these 3 I can copy, paste it, paste it, paste it. Now let’s fill these numbers. I have produced a cost of 2 in month 1. I want to satisfy the demand of month 2. So I should carry it for one month, and inventory cost for one month per unit of product is $1, so I add $1 per 2 and that is 3. Here I have produced at $3 in overtime in month 1. I want to carry to month 2. 3 plus 1, 4; 6 plus 1, 7. So I fill this one. Therefore, this one is easy. I add one unit to that. I add one unit to this. So if I have produced in month 2 to satisfy the demand of month 4, that would be $2 production in regular time. I should carry to month 3. That would be 2 + 1. And I should carry to month 4. That would be 2 + 1 + 1, which is 4. 3 production overtime in one month is 2 to satisfy the demand of month 4, $3 + 1 is $4 + 1 is $5. If I have out sources in month 2 to satisfy the demand of month 4, $6 + 1, 7 + 1, 8. Don’t forget we are not solving the problem. We don’t say where is our benefit to produce where it is not our benefit to produce. We are not saying that. We just translate our narrative into mathematical representation and pass it to Excel. We are not solving the problem. We are formulating the problem and then passing to Excel. Excel will solve the problem. We just translate our knowledge to Excel.

Let’s see how could I get these numbers? What do these numbers mean? I have produced in regular time of month 2 to satisfy the demand of month 1. Production regular time month 2 to $2. If I want to satisfy the demand of the previous month, I should add one backlog cost to it. And backlog cost was $5. So that is 2 + 5 which is 7. If I produce in regular time is 2 + 5 which is 7, and I produce in overtime it is 3 + 5 which is 8. If I outsource it is 6 + 5, which is 11. If I am going to understand the meaning of these numbers, let’s see what do they mean? It means I have produced in regular time of month 5, so cost of regular time in month 5 is $2. To satisfy the demand of months 2, so month 5 to satisfy the demand of month 2. This $2 would be added to $5 which become 7. This 7, the $5 that we add to it, it will become 12. This 12 add $5 to it, it becomes 17. On the same lines of reasoning, these 3 when $5 is added to it, it is 8. $5 added to it 13. $5 added to it 18. And the same for this 6 to be added to three $5, and it becomes 21. Now I should declare my constraint. What I have put in these places means how I have used the capacity of regular time in period 1. This is the total that should be less than or equal to my capacity. Now I go and define these rows. For example, this number is summation of all numbers in these columns. Summation of all of these is here. This is what I have provided using outsourcing in month 1 to satisfy the different months. This cannot with greater than the total outsourcing capacity of that month. This should be less than or equal to this one. Then I copy down and copy down. Here, this number is summation of whatever I have produced in different months to satisfy the demand of month 1. This is summation of all those numbers, which should be here. And that is equal or alternatively it could be also greater than or equal to this number. And this one is sum product of cost table into changing cells table into production table. I have not produced nothing yet in these changing cells, therefore it is 0. But if I go ask Excel to solve the problem, it will come out with this production plan in which all of production is either in overtime or regular time. Regular time, regular time, regular time, regular and overtime, regular and overtime, regular time. We have not done any outsourcing. We have not done any backlog. We cannot do any inventory carryover, and this 3,900 is our optimal objective function value. What can I say? Just take one of these problems, open Excel sheet and try to repeat it. Then that’s it.