

Chapter 3

Throughput Analysis

Throughput first, Inventory Second, Living in Cost World Last – First Leave the Cost World

I do not know what I may appear to the world; but to myself, I seem to have been only like a boy playing on the seashore, and diverting myself in now and then finding a smoother pebble or a prettier shell than ordinary, while the great ocean of truth lay all undiscovered before me. Isaac Newton, 1643-1727

The key problems of this chapter can be accessed at

<http://www.csun.edu/~aa2035/CourseBase/Throughput/ThroughKeyProblems.pptx>

The recorded lectures – if any- and the excel worksheets- if any- are embedded inside the PowerPoint slides. By clicking on them the mp3/mp4 files and excel pages will open

In this chapter, we will discuss Throughput and review Capacity. We will also review concepts such as Bottleneck and Utilization, and introduce Capacity Waste Factor (CWF).

Throughput is the number of flow units that pass through the process per unit of time. We assume that we produce on demand and Throughput is equal to Demand. In a synchronized system, Throughput = Demand, that is $R=D$.

Steps in computing Throughput.

1. Observe the process for a number of periods.
2. Measure the number of flow units that are processed per unit of time.
3. Compute the average number of flow units per unit of time.

An indirect way of computing Throughput: $R = I/T$.

Throughput is the average flow rate. Capacity is the maximum sustainable flow rate. In periods of heavy congestion, Throughput is equal to Capacity for a short period, in all other times, Throughput < Capacity. Throughput cannot be equal to Capacity for a long period, i.e. Utilization cannot be equal to 1 except for short periods.

Imagine a freeway where all the cars are driving exactly 65 mph, and the distance between each car is only 1 inch. As long as everyone maintains a speed of exactly 65 mph, that is fine. However, can they do that? What happens if just one car hits the breaks? How long does it take to clean the freeway? Do cars move on the freeway easier when Utilization is 1 and they are moving bumper to bumper, or when 50% of the freeway is empty $U = 0.5$, or when $U = 0.25$? How much may traffic jam accidents create in each of these situations?

Cycle Time is defined in relation to Capacity and is a measure of internal capability.
 $CT = 1/R_p$.

Takt Time is defined in relation to Demand or Throughput and is a measure of external demand. Takt means pace in German. If demand = 12/hour, then we assume the Throughput = 12/hour.

Takt Time = $1/\text{Demand} = 1/\text{Throughput} = 1/R$.

Takt Time = $1/12$ hour or $60(1/12) = 5$ minutes.

$R = \min \{R_a, R_p\}$, since $R_a < R_p$, \rightarrow Takt time = $1/R_a > \text{Cycle Time} = 1/R_p$

Average inter-arrival time = $T_a = 1/R_a = 1/R$ = Average inter-departure time

Sometimes, a manager may state that his machines have takt time of X minutes. This statement is incorrect. Machines have cycle time. Capacity is related to cycle time. Takt time is a measure of external demand; it has nothing to do with the internal capacity. In a synchronized system, Takt time is the time each station has to send one flow unit to the next station. Cycle time is the time the bottleneck (s) need to send the next product out. TT is always greater than CT since R_p is always greater than R .

Chapter 5 is on **flow time minimization**. Chapter 4 is on **throughput maximization**.

Chapter 4 and 5 are both **"time" minimizations**. Why?

By trying to increase throughput, we will minimize Takt time. Therefore, Chapter 5 is on **flow time minimization**. Chapter 4 is on **takt time minimization**.

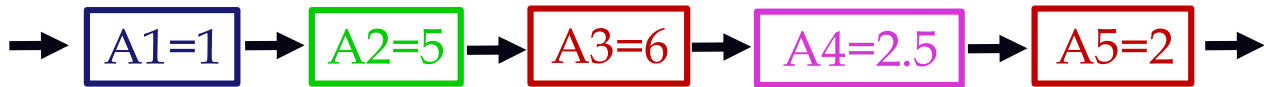
Problem 1. On the last column of the left table below, we have work content (activity time) of five sequential activities (T_p of each activity).

Activity Time or work content of an activity (T_p). The amount of time the activity needs to be completed once on a specific resource. This includes a share of all distractions such as maintenance, repair, setup, etc. They are all inside T_p ; it is the effective unit load, not the theoretical unit load.

Activity	Resource	Work Content (mins)
Mailroom	Mailroom Clerk	1
Data Entry	Data-entry Clerk	5
Initial Processing	Claims processor	6
Inspection	Claims Supervisor	2.5
Final Processing	Claims processor	2

Resource	Unit Load(min)
Mailroom Clerk	1
Data-entry Clerk	5
Claims processor	8
Claims Supervisor	2.5

The third and fifth activities are done on a single resource, therefore regarding the resources, we have four Tp values.



Unit Load of a Resource Unit (Tp) – The amount of time the resource works to process each flow unit. That is a summation of the work content (activity time) of all activities assigned to the resource. This includes a share of all distractions such as maintenance, repair, setup, etc. It is an effective unit load, not the theoretical unit load.

Unit Load of Resource-3 = Work Content of Activity-3 + Work Content of Activity-5

Tp of Resource-3 = Tp Activity-3 + Tp Activity-5

Tp of Resource-3 = Unit Load of Resource-3 = 6+2 = 8

Effective Capacity of a Resource Unit is computed as $R_p = 1/\text{unit load} = 1/T_p$ in the same unit of time as Tp is defined.

If in a **Resource Pool** we have two resource units, then the Effective Capacity of a Resource Pool is $1/T_p + 1/T_p = 2/T_p$. If in a resource pool we have c resource units, then the Effective Capacity of a Resource Pool is c/T_p . If there are c Resource Units in the Resource Pool then the capacity of the resource pool is $R_p = c/T_p$, per the time unit in which Tp is defined.

Resource	Unit Load: T_p min/claim	Effective capacity of Resource Unit: $R_p = 1/T_p$ claims/min	Number of the Resource Units in the Resource Pool: c	Effective capacity of Resource Pool: $R_p = c/T_p$ claims/min
Mailroom clerk	1	1	1	1
Data-entry clerk	5	0.2	8	1.6
Claims processor	8	0.125	12	1.5
Claims supervisor	2.5	0.4	5	2

Unit load of claim processor is 8 min. Capacity is $1/8 = 0.125$ units per minute. If we multiply 0.125 by 60 (minutes per hour), the capacity is then 7.5 per hour. The capacity of a single resource unit is $1/T_p$ per the time unit as T_p stated. If each resource unit has a capacity of $1/8$ and if we have 12 resources units ($c=12$) in our resource pool of claim processor, then the capacity of the resource pool is $c/T_p = 12/8 = 1.5$ per minute or 90 per hour. Following the same computations, capacity per minute of all four resource pools is shown in the fourth column.

Bottleneck – The resource pool with the minimum effective capacity.

The effective capacity of a process: The effective capacity of the bottleneck; i.e. 1 flow unit per minute or 60 per hour.

The bottleneck is the mailroom clerk. One way of increasing capacity is to cross-train Claims Supervisor to help Mailroom clerk! An excellent example of how companies can increase capacity, especially in peak periods, by cross-training, is UPS.

This is done at UPS headquarters whereas all managers have been trained to work at the line in peak times during high seasons.

100% Utilization is a High Risk. Suppose the throughput of the system is 400 per day, (capacity is 480 per day). All the components of the previous table remain the same in the following table. Utilizations are computed in the last column.

Resource Pool	Unit Load: T_p min/claim	Effective capacity of Resource Unit: $1/T_p$ claims/min	# of Resource Units in the Resource Pool: c_p	Effective capacity of the Resource Pool: $R_p = c/T_p$ claims/min	Scheduled availability min/day	Effective capacity of Resource Pool: claims/day	Utilization
Mailroom clerk	1	1	1	1	480	480	83%
Data-entry clerk	5	0.2	8	1.6	480	768	52%
Claims processor	8	0.125	12	1.5	480	720	56%
Claims supervisor	2.5	0.4	5	2	480	960	42%

Theoretical Flow Time = $1+5+8+2.5 \rightarrow FT = 16.5$

Capacity = 1/min. \rightarrow Cycle Time = 1/1 minute $\rightarrow CT = 1$.

Demand = 5/6 per minute \rightarrow Throughput = 5/6 per minute.

Takt Time = 1/Demand = 1/Throughput = 5/6 per minute

Takt Time = $1/(5/6) = \rightarrow TT = 1.2$ minutes



If you have difficulty in understanding this problem, you may first try to see if you can solve these problems [Assignment Waiting Line Problems Set 0](#) . You may then access the PowerPoints slides of the above pages at [Throughput](#) and its recorded lecture at [Throughput.Recorded](#). The first 25 minutes of the lecture and its corresponding slides cover Problem 1. The last 25 minutes of the lecture and its corresponding slides cover Problem 2.

Problem 2. Suppose all the assumptions remain the same. However, there are two types of billing: Physician claims 60%, Hospital claims 40%. Unit loads of Physician claims are shown in column 2 and Hospital claims are shown in column 3. Unit load of a prototype flow unit is 0.6 (Physician claim unit load) + 0.4 (Hospital claim unit load). Therefore, the unit load of the data entry clerk is $0.6(5) + 0.4(6) = 5.4$ minutes. Unit loads of the remaining resources are computed following the same way. Now the effective capacity of the process is reduced to 400 per day. 400 units of a product which of 60% Physician claim and 40% Hospital claim; that is 240 Physician claims and 160 Hospital claims per day.

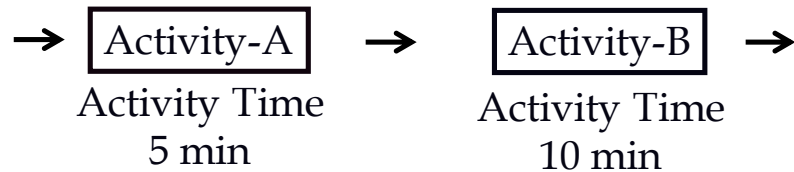
Resource Pool	UL- Physician min/claim	UL- Hospital min/claim	UL (60%- 40%) mix min/claim	Effective capacity of Resource Unit: $1/Tp$ claims/min	# of Resouce Units in the Resource Pool: cp	Effective capacity of the Resource Pool: $R_p = cp/Tp$ claims/min	Effective capacity of the Resource Pool: $R_p = cp/Tp$ claims/day
Mailroom clerk	1	1.5	$0.6(1)+0.4(1.5) = 1.2$	0.83	1	0.83	400.00
Data-entry clerk	5	6	5.4	0.19	8	1.48	711.11
Claims processor	8	8	8	0.13	12	1.50	720.00
Claims supervisor	2.5	4	3.1	0.32	5	1.61	774.19

$R = R_a \rightarrow$ Takt time = $1/R_a$

Average inter-arrival time = $T_a = 1/R_a = 1/R$ = Average inter- exit time

In a synchronized system, Takt time is the time each station has to send one flow unit out to the next station.

Problem 3. Capacity Waste Factor (CWF). Reconsider the following example where Activities A and B are handled by resources 1 and 2 respectively.



T_p of Activity-A referred to as work content of activity-A, is 5 min., and T_p of Activity-B referred to as work content of Activity-B, is 10 min. T_p of Resource-1 referred to as unit load of resource-1, is 5 min., and T_p of Resource-2 referred to as unit load of Resource-2, is 10 min.

Now suppose Resource-A has a Capacity Waste Factor (CWF) of 40%, and Resource-B has a Capacity Waste Factor (CWF) of 25%. That means out of what we have as T_p of Resource-1, 25% of it is waste, and the rest is the Theoretical Unit Load.

Unit Load $(1-CWF)$ = Theoretical Unit Load.

Theoretical unit load of Resource-1 = $ThUL = T_p(1-CWF) = 5(1-0.4) = 3$ minutes.

Theoretical unit load of Resource-2 = $ThUL = T_p(1-CWF) = 10(1-0.25) = 7.5$ minutes.

Alternatively, if we had 3 and 7.5 as theoretical unit loads of Resource-1 and Resource-2, their unit load, T_p , could have been computed as

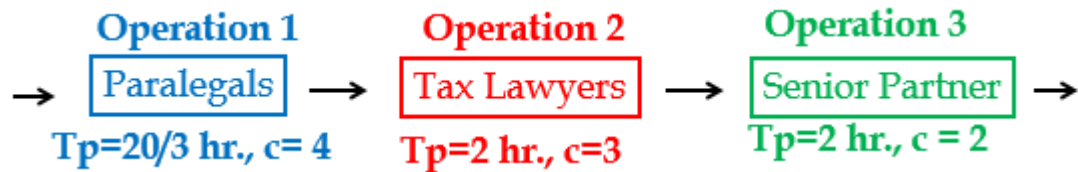
$T_p = ThUL / (1-CWF)$

Resource-1 $T_p = 3 / (1-0.4) = 5$ mins.

Resourc-2 $T_p = 7.5 / (1-0.25) = 10$ mins.

Note that if we do not consider the CWF, then we are talking about the theoretical capacity. If we take CWF into account, then we are talking effective capacity or simply capacity.

Problem 4. A law firm processes shopping centers and medical complexes contracts. There are four Paralegals, three Tax lawyers, and two Senior Partners. The unit loads of the resources to handle one standard contract is given below. Assume 8 hours per day and 20 days per month.



It takes a Paralegal 20 hours to complete 3 contracts. That is $20/3 = 6.667$ hours to complete a contract. It takes a Tax lawyer 2 hours to complete a contract. It takes Senior Partner 2 hours to complete a contract.

a) Compute the Flow Time of a contract.

$6.667 + 2 + 2 = 10.667$. This is not the flow time. It is the Theoretical Flow Time.

Flow Time = Theoretical Flow Time + Waiting times

Flow Time = 10.67 + Waiting times. We do not know the waiting times yet.

Compute the Capacity of each of the three Resource **Pools**.

A Paralegal can complete 1 contract in $20/3 = 6.667$ hours.

How many contracts in one hour? $1/6.667 = 0.15$

How many contracts can all the Paralegals complete in one hour?

There are 4 Paralegals: $c = 4$

Four Paralegals $4(0.15) = 0.6$ contracts per hour.

We could have also said that $T_p = 6.6667$ hours.

The capacity of one resource unit is $1/T_p$.

The capacity of one resource unit is $1/6.667 = 0.15$.

Capacity of all resource units: $R_p = c/T_p$ where $c = 4$ and $T_p = 6.667$

$R_p = 4/6.667 = 0.6$ per hour

Capacity of the resource pool is **0.6 contracts per hour**.

It is $8(0.6) = 4.8$ contracts **per day**

A Senior Partner can complete 1 contract in 2 hours.

How many contracts in one hour? $1/2 = 0.5$

How many contracts can all the Senior Partners complete in one hour?

There are two Senior Partners: $c = 2$

Therefore, Senior Partners $2(0.5) = 1$ contract per hour

We could have also said that $T_p = 2$.

The capacity of one resource unit is $1/T_p$.

The capacity of one resource unit is $1/2 = 0.5$.

Capacity of all resource units: $R_p = c/T_p$ where $c=2$ and $T_p = 2$

$R_p = 2/2 = 1$ per hour

The capacity of the resource pool is **one contract per hour**.

It is $8(1) = 8$ contracts **per day**

c) Compute the capacity of the process.

	Unit Load 50%SH 50%MD	Capacity of a Resource Unit /hr	# Of Resource Units	Capacity of the Resource Pool/hr	Cap of the R-Pool / day
Paralegal	6.667	0.15	4	0.6	4.8
Tax lawyer	2	0.5	3	1.5	12
Senior partner	2	0.5	2	1	8

d) Compute the cycle time?

4.8 units in 8 hours.

Cycle time = $8/4.8 = 1.67$ hours

Alternatively, $1/0.6 = 1.67$ hours

d) Compute the average inventory.

Let us look at the utilization of the three stations

Station	Capacity	Throughput	Utilization
Station 1	4.8	4.8	$4.8/4.8 = 1$
Station 2	12	4.8	$4.8/12 = 0.4$
Station 3	8	4.8	$4.8/8 = 0.6$

Chapter 4. Throughput

On average one person with a resource in Station 1, 0.4 people with a resource in Station 2, and 0.6 people with a resource in Station 3

Inventory with the processors is $1 + 0.4 + 0.6 = 2$

On average there are two flow units with the processors; Inventory in the processors (I)

Now let us look from another angle; from the Little's Law point of view

$RT = I \rightarrow R = 4.8$ per 8 hours or 0.6 per hour

$T = 10.67$ hours $\rightarrow I = 0.6(10.67) = 6.4$

6.4 vs 2? Where is my mistake?

$$1(4) + 0.4(3) + 0.6(2) = 6.4$$

e) There are 150 cases in November. Can the company process all 150 cases?

$150/20 = 7.5$ per day $\rightarrow 4.8$ (Capacity) < 7.5 (Demand).

f) If the firm wishes to process all the 150 cases available in November, how many professionals of each type are needed?

The capacity of a paralegal is 0.15 units per hour, therefore the daily capacity of this resource unit is $0.15(8) = 1.2$ units. Similarly, the daily capacity of a tax lawyer and a senior partner are $0.5(8) = 4$ units.

of paralegals required = $7.5/1.2 = 6.25$

of tax lawyers required = $7.5/4 = 1.875$

of senior partners required = $7.5/4 = 1.875$

These could be rounded up to 7, 2 and 2

We need 7, 2, and 2. We have 4, 3, and 2. We may hire three additional paralegals.

Alternatively, we may hire just two and for a total of six paralegals.

How much overtime for 0.25 paralegal who works eight hrs. /day?

$0.25(8) = 2$ hours total overtime.

There will be 6 paralegals; overtime pf each = $2/6 = 1/3$ hour

Alternatively, 20 minutes per paralegal. PLUS some safety Capacity.

Now suppose Throughput is 3.6 contracts per day.

Compute the average inventory.

Station	Capacity	Throughput	Utilization
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Chapter 4. Throughput

Station 1	4.8	3.6	$3.6/4.8 = 0.75$
Station 2	12	3.6	$3.6/12 = 0.3$
Station 3	8	3.6	$3.6/8 = 0.45$

Managerial Observation: Note that the utilization of bottleneck resource is not necessarily 100%.

On average

0.75 person with a resource in Station 1

0.3 person with a resource in Station 2

0.45 person with a resource in Station 3

Station	Rp	R	U	c	Ip
Station1	4.8	3.6	0.75	4	$4 * 0.75 = 3$
Station2	12	3.6	0.3	3	$3 * 0.3 = 0.9$
Station3	8	3.6	.45	2	$2 * 0.45 = 0.9$

$$I = 3 + 0.9 + 0.9 = 4.8$$

Let us check it through the Little's Law

$$\text{Theoretical Flow Time} = 6.66667 + 2 + 2 = 10.66667$$

$$RT = I \rightarrow 3.6(10.66667/8) = I \rightarrow I = 4.8$$

$$(3.6/8)(10.66667) = 4.8$$

Now suppose there are 16.8 contracts waiting in different waiting lines? What is the Flow Time?

$$RT = I \rightarrow 3.6 * T = 16.8 + 4.8 = 21.6 \rightarrow 3.6T = 21.6 \rightarrow T = 6 \text{ days.}$$



If you have difficulty in understanding this problem, you may access the PowerPoints slides of the problem at [Throughput Part 2a-Problem](#) and the recorded lecture at [Throughput Part2a-Problem.Recorded](#). The first 25 minutes of the lecture, and the last slides, covers Problem 1, and the last 5 minutes covers an extension of the problem. We encourage you to watch, at least, the last 5 minutes.

Capacity Waste Factor (CWF). The effective capacity of a resource unit is $1/T_p$. Unit load T_p is an aggregation of the productive as well as the wasted time.

T_p includes share of each flow unit of capacity waste and detractors such as

Resource breakdown

Maintenance

Quality rejects

Rework and repetitions

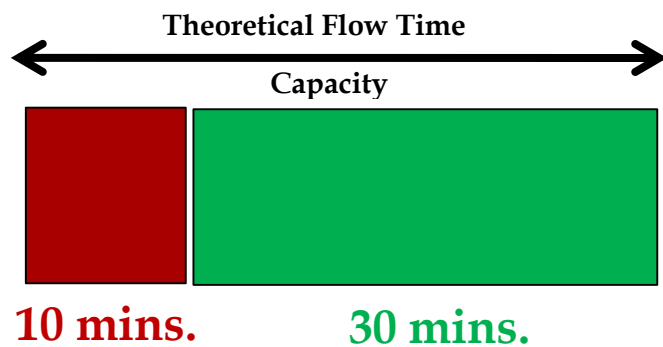
Setups between different products or batches

We may want to turn our attention to waste elimination and segregate the wasted capacity. Theoretical capacity is the effective capacity net of all capacity distracts.

Activity time

Capacity is computed based on the Unit Load

Theoretical Flow Time is computed based on Activity Time



Then what is the Flow Time?

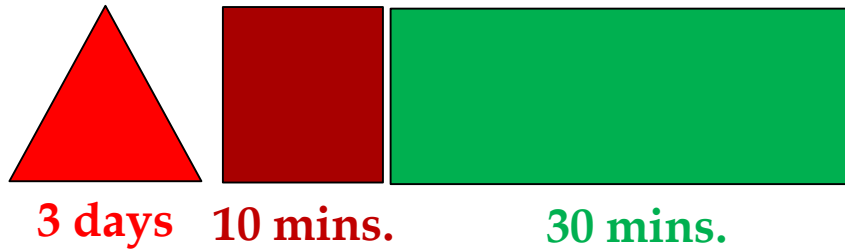
Unit Load, Activity time, Capacity, Theoretical Flow Time

Flow Time $T_i + T_p$

Flow time includes time in buffers

Capacity does not care about buffer times





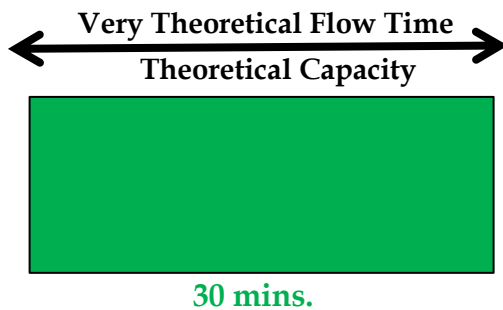
Flow Time Capacity does not care about buffer times

Theoretical Unit Load, Theoretical Activity Time

Theoretical Capacity is computed based on the Theoretical Unit Load (ThUL)

Theoretical Flow Time is NOT computed based on Theoretical Activity Time

Very Theoretical Flow Time is computed based on theoretical Activity Time ThUL
(1+CWF) = Unit Load (Tp),



Theoretical Unit Load, Theoretical Activity Time, Theoretical Capacity,

Very Theoretical Flow Time

An operating room (a resource unit) performs surgery every 30 min, $T_p = 30$ min. T_p includes all the distracts. We also refer to it as the Unit Load.

Effective capacity is $1/30$ per min or $60/30 = 2$ per hour.

On average, $1/3$ of the time is wasted (cleaning, restocking, changeover of nursing staff and fixing of malfunctioning equipment).

Capacity Waste Factor (CWF) = $1/3$.

Theoretical Unit load = $T_p \cdot (1 - \text{CWF}) = 30(1 - 1/3) = 20$ min.

$T_p = \text{Unit Load} = \text{ThUnit Load} / (1 - \text{CWF}) = 20 / (1 - 1/3) = 30$

Theoretical Capacity = $c / \text{ThUnit Load}$

Effective Capacity = Capacity = $c/\text{Unit Load}$.

Theoretical Capacity = 1/20 per minute or 3 per hour.

Effective Capacity = Theoretical Capacity (1-CWF)

Problem 5. A law firm processes (i) shopping centers and (ii) medical complexes contracts. The time requirements (unit loads) for preparing a standard contract of each type along with some other information is given below. In November 2012, the firm had 150 orders, 75 of each type. Assume 20 days per month and 8 hours per day. CWF at the three resources is 25%, 0%, and 50%, respectively.

	Unit Load Shopping (hrs /contract)	Unit Load Medical (hrs /contract)	No. Of Professionals
Paralegal	4	6	4
Tax lawyer	1	3	3
Senior partner	1	1	2

a) What is the effective capacity of the process (contracts / day)?

Paralegal: Theoretical Unit Load (50%Sh 50% Med): $0.5(4) + 0.5(6) = 5$ hrs.

Theoretical Capacity = 1/5 per hr.

Capacity Waste Factor (CWF) = 0.25

Unit Load = $T_p = 5/(1-0.25) = 20/3$ hrs.

Effective Capacity = Capacity = $1/(20/3) = 3/20$ per hr.

Tax Lawyer: Theoretical Unit Load $0.5(1) + 0.5(3) = \text{two hrs.}$

CWF = 0

Theoretical Unit Load = $T_p = 2$ hrs.

Theoretical Capacity = 1/2 per hr.

Effective Capacity = Capacity = 1/2 per hr.

Senior Partner: Theoretical Unit Load $0.5(1) + 0.5(1) = \text{one hrs.}$

Theoretical Capacity = $1/1 = 1$ per hr.

CWF = 0.5

Chapter 4. Throughput

Unit Load = $T_p = 1/(1-0.5) = 2$ hrs.

Effective Capacity = Capacity = $1/2$ per hr.

	ThUnit Load SH (hrs)	ThUnit Load MD (hrs)	Unit Load 50%SH 50%MD	Theoretical Capacity of a Resource Unit /hr	CWF	Unit Load 50%SH 50%MD	Capacity of a Resource Unit /hr	# Of Resource Units	Th Capacity of the Resource Pool/hr	Capacity of the Resource Pool/hr	Th Cap of R- Pool / day	Cap of the R- Pool / day
Paralegal	4	6	5	0.2	0.25	6.667	0.15	4	0.8	0.6	6.4	4.8
Tax lawyer	1	3	2	0.5	0	2	0.5	3	1.5	1.5	12	12
Senior partner	1	1	1	1	0.5	2	0.5	2	2	1	16	8



The PowerPoints slides of this problem are on the second part of at [Throughput Part 2a-Problem](#). We encourage you also to open the following problem, put it in slideshow mode, and solve it [Throughput Part 2b](#).