THE DREADED MATH QUIZ

Please complete the problems below is best you can. Please show your work.

1) Frank works in an apple orchard, picking and packing apples. One day he packed 24 crates of apples, with 36 apples in each crate. How many apples did he pick altogether?

_____

2) Mary is building a deck around a rectangular pool, which is 8 feet wide by 15 feet long. If the deck will be 3 feet wide all around the pool, what will be the area of the deck?

_____

3) Divide these fractions. Please write each quotient in lowest terms:

$\frac{8}{9} \div \frac{2}{7}$ $\frac{1}{4} \div \frac{5}{6}$ $\frac{7}{3} \div \frac{1}{9}$

4) Please factor the following polynomials:

$X^2 - 16$ $3x^2 + 22x + 24$

The Mathematics Survey: A Tool for Assessing Attitudes and Dispositions

The need for positive attitudes and dispositions permeates the teaching and learning of mathematics. What students believe about mathematics influences what they are willing to say publicly, what questions they are likely to pose, what risks they are willing to take, and what connections they make to their lives outside the classroom (Borasi 1990; Whitin and Whitin 2000). Unless students have a realistic sense of mathematical applications in real-life contexts, they are unlikely to see themselves pursuing courses in advanced mathematics or choosing mathematics-related careers (Picker and Berry 2001).

Principles and Standards for School Mathematics (NCTM 2000) delineates a range of attitudes and beliefs about mathematics that contribute to productive problem solving and communication. For example, perseverance, curiosity, confidence, and flexible thinking are related to learners' investment in challenging problem solving and investigations involving complex patterns and relationships. Confidence, open-mindedness, a willingness to share one's own successes and failures, and the ability to shift perspectives are hallmarks of meaningful communication. Resourcefulness and reflective analysis are important dimensions in learners' ability to use various forms of representation to generate, clarify, and express thinking.

Given the ways in which mathematical attitudes, skills, knowledge, and strategies are intertwined, assessing students' attitudes and beliefs can provide valuable information, especially at the beginning of the school year. The results can be used to guide the development of a classroom environment conducive to growth in positive attitudes and in addressing misconceptions and counterproductive beliefs (Picker and Berry 2001; Rock and Shaw 2000). This article describes the development and implementation of a mathematics attitude survey designed to meet this need. Examples of fourth-grade children's responses to the survey over a four-year period, the ways in which the results guided teaching and learning, and an analysis of post-assessments illustrate the process.

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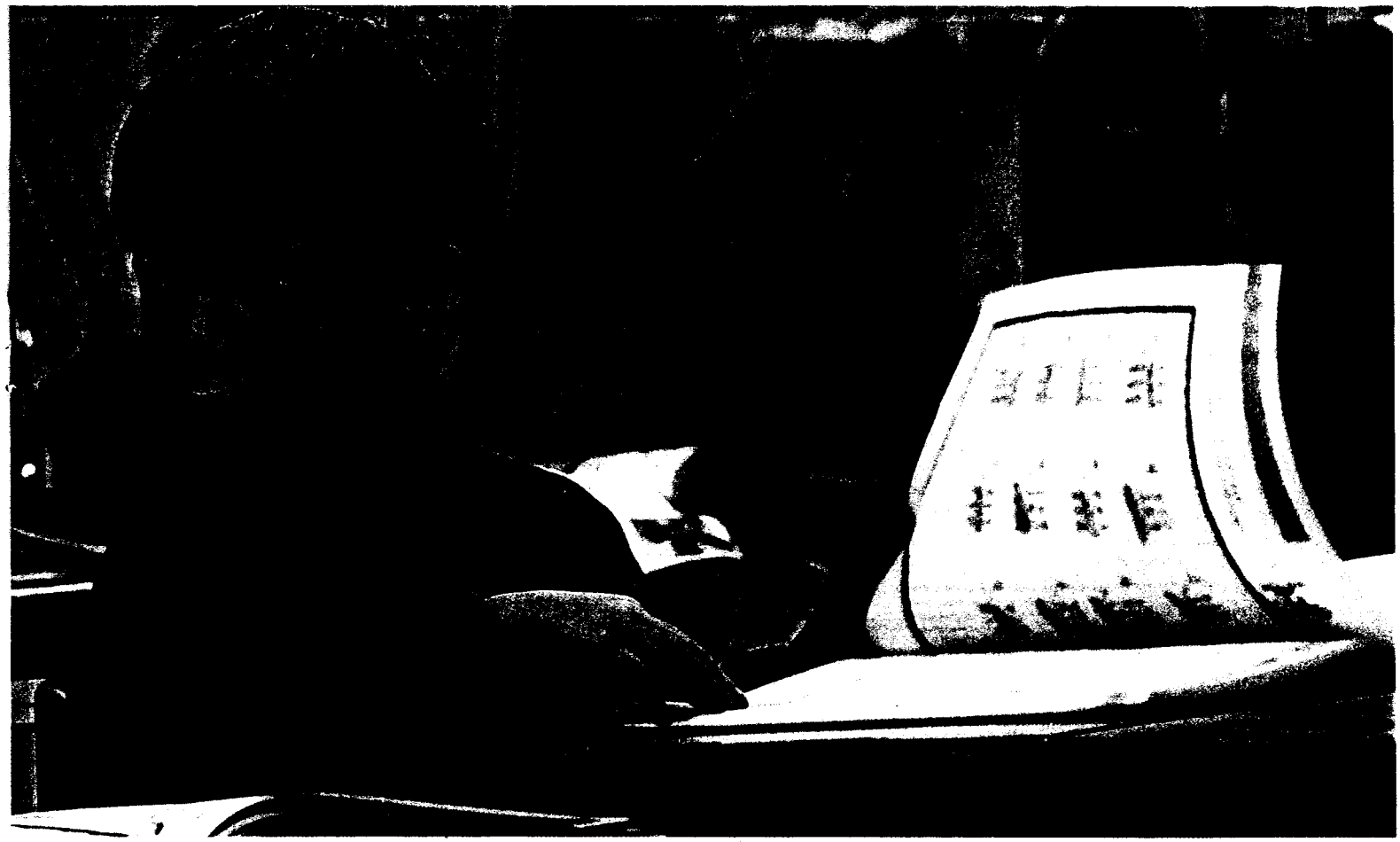
The Mathematics Survey as a Tool for Assessment

The mathematics survey (**fig. 1**) is composed of six sentence-completion prompts designed to elicit children's perceptions of what constitutes mathematical knowledge, ways of thinking, and usefulness in everyday life. It was adapted from the Burke Reading Survey (Goodman, Watson, and Burke 1987) and correlates with attitudes identified by NCTM (2000). Prompts 1, 2, 3, and 5—"To be good in math, you need to ... because ..."; "Math is hard when ..."; "Math is easy when ..."; "The best thing about math is ..."—are included to encourage responses that reflect the degree of students' confidence, curiosity, flexible thinking, and their views of student-teacher and student-student relationships. The fourth prompt—"How can math help you?"—addresses students' perceptions about the usefulness of mathematics and real-life applications. The final prompt—"If you have trouble solving a problem in math, what do you do?"—is intentionally somewhat ambiguous so that the students can reveal their definitions of, attitudes toward, and strategies for problem solving.



The following questions guided the analysis of student responses:

- What do these responses reveal about students' perceptions regarding the teacher-student relationship? Regarding student-student relationships? Do references to the teacher imply that the teacher is the sole source of knowledge? Is there evidence of student autonomy? Of collaborative thinking? Are other students mentioned? If so, in what ways?
- What do these responses reveal about students' perceptions regarding mathematical content and applications? Do students cite examples of functional applications of mathematical ideas? Do they view mathematics as valuable and relevant in both the present and the future?
- What do these responses reveal about students' perceptions regarding processes of engaging in mathematical investigations (e.g., planning, reasoning, using strategies in a flexible manner, making connections, representing ideas in multiple ways, collaborating, discovering patterns and relationships)? Do the students view challenge as rewarding?

Examples of typical responses collected from the four fall surveys illustrate the assessment process (see **fig. 2**). In these examples, words and phrases about listening, paying attention, and studying suggest a belief that mathematics is a silent and solitary endeavor. The responses suggest that these children view mathematics class as a period of teacher-student interchanges in which the teacher poses questions or problems and evaluates answers. The children do not usually mention their peers except in a negative sense, such as "talking to other people" rather than "paying attention." To be a successful mathematics student, one must listen to the teacher, follow directions, and study. Responses such as this to prompt 1—"study real hard because you need to know the problems in a flash"—also imply that speed is a universal measure of mathematical success. References to extended investigations are absent. Similarly, each year about half the responses to prompt 6—"If you have trouble solving a problem in math, what do you do?"—suggest

Figure 1

Mathematics survey

1. To be good in math, you need to ... because ...
2. Math is hard when ...
3. Math is easy when ...
4. How can math help you?
5. The best thing about math is ...
6. If you have trouble solving a problem in math, what do you do?

Tell anything else you want about math.
On the back of your paper, draw a picture that shows what math means to you.

Figure 2

Typical student responses from the fall mathematics surveys

1. **To be good in math, you need to ... because ...**
   "study and do all your papers"
   "concentrate and don't play around because if you don't concentrate you will do bad"
   "study and listen because you might not know what to do"
   "study real hard because you need to know the problems in a flash"
2. **Math is hard when ...**
   "it's 2-digit division"
   "you don't listen, not pay attention, or talking to other people"
   "you don't concentrate"
   "you don't follow directions"
   "you can't understand what the teacher says"
3. **Math is easy when ...**
   "you work good by listening to the teacher"
   "when you have studied really hard"
   "when it's times or plus"
4. **How can math help you?**
   "when you get good grades and not get on restriction"
   "when you go to the store"
   "when you get a job"
   "when you get an A"
   "If you saw two numbers and if you wanted to do something with the numbers."
   ...
6. **If you have trouble solving a problem in math, what do you do?**
   "Raise your hand and ask the teacher."
   "Ask for help."
   "I skip it and do the next one."

that students depend on the teacher for direction (e.g., "raise your hand," "ask the teacher"). Other responses such as the one suggesting skipping the problem imply time management. Only rarely do students describe devising an alternative method, collaborating, or using various forms of representation as problem-solving strategies.

In response to prompt 4—"How can math help you?"—the children's statements indicate that the rewards of engaging in mathematical activity are extrinsic—for example, getting good grades and maintaining good relationships with parents. Almost exclusively, students do not mention the intrigue or challenge of investigations as rewarding or "fun." Few students mention mathematics as useful in the present; most give vague references to college or employment in the distant future. Although some children cite using money as a helpful part of mathematics, others mention computational activity that is devoid of any context, such as the student who suggested seeing "two numbers" and "wanting to do something" with them.

The patterns that emerged from the survey analysis illuminate a range of attitudes and beliefs that could interfere with students' mathematical growth. Identifying these trends served to guide plans for structuring the environment, designing instructional activities, and delineating the teacher's role. **Figure 3** shows a summary of the trends from the fall surveys; productive attitudes, dispositions, and beliefs about mathematics (NCTM 2000); and plans for teaching and learning.

Using the Results of the Survey

Changing the view of mathematics as a solitary endeavor entailed structuring group problem-solving tasks, promoting collaborative conversations, and encouraging the children to view their peers as resources. In the case of these fourth graders, the teacher decided that beginning the year with a noncomputational activity, such as pentominoes or classification with attribute blocks, could help expand the children's limited views of mathematics as computation. Further, a puzzle or game format could highlight for the children that all mathematical activity is not done "in a flash" or by following a prescribed procedure. As pairs or small groups of children worked together to solve puzzles, their conversations laid an important foundation for writing and visual representation (Huinker and Laughlin 1996; Whitin and Whitin 2003; Wickett 1997). At the completion of the task, the teacher conducted a reflective conversation that specifically addressed the targeted dispositions. She posed questions such as these: "What was going through your mind when you first started the puzzle?" "How did your group's ideas help you?" "What did you do if your first idea didn't work?" Many children were surprised to find that their peers encountered frustration or that the teacher did not regard their building off a classmate's suggestion as "cheating."

Following the conversation, the teacher asked the children to record their discoveries about their learning processes in writing and to name specific children whose thinking helped them—for example, "Catherine helped me when she said, 'Switch them around.'" Public acknowledgment of collaborative efforts and written reflections about problem-solving processes continued throughout the year (Whitin and Whitin 2000). **Figure 4** shows one student's representation of the value of mathematical conversations. The first box shows a red and a blue circle, representing two children's ideas. During the conversation, the two ideas begin to blend (second box) until finally they merge into one purple circle in which "the class works together and

the ideas get mixed." The student's summary—"I think math is easier when our class puts our ideas together"—demonstrates a marked change in attitude from such presurvey comments as "Math is easy when it's times or plus."

To build the students' confidence and self-reliance, the teacher needed to shift attention away from herself as dispenser of knowledge. To achieve this goal, she carefully examined the implicit messages conveyed through her interactions with the students. She made conscious efforts to respond to the children's questions and comments in ways that invited revisiting or extending a problem (Schwartz 1996). If the students asked, "Is that right?" the teacher, to encourage them to revisit their thinking process and either confirm or revise their solution, would respond, "How can you be sure?" or "Explain your thinking." When the children shared a conjecture—for example, "When you add two odd numbers, you get an even number"—the teacher invited further investigation by asking, "Does that always work?" If a child raised a question during a class discussion, the teacher developed the habit of turning the question over to the group. In addition, she learned to carefully choose words that conveyed resourcefulness and curiosity—for example, *invent* or *discover* rather than *find* or *use*.

Regularly using student-authored problems for homework and a morning "problem of the day" were additional ways to increase the children's confidence. Over time, the teacher strove to feature as the "problem of the day" an original problem written by every child in the class. As part of the ritual, the student-author led the discussion of the problem's solution (Whitin and Whitin 2000). On other occasions, the children expanded on entries in their mathematics journals and "published" their findings for parents and other classes. In these ways the children had opportunities throughout the year to view one another as resources, develop their confidence, and feel recognized and rewarded for their learning.

Instituting a mathematical forum where students shared their problem-solving strategies served to develop the children's flexible thinking. When one child demonstrated his use of the distributive property, for example, the teacher suggested that the other students work in groups to apply his strategy to a variety of problems and later share their discoveries. The teacher also introduced the students to problem posing (Brown and Walter 1990). Sometimes she planned problem-posing explorations in advance (Whitin forthcoming), and on other occasions she built on children's spontaneous questions, such as, "If $4 \times 3 = 3 \times 4$, why doesn't $34 \times 3 = 33 \times 4$?" This question inspired an investigation of the commutative property in multiplication, as well as the problem-posing extension, "Are there similar multiplication problems that *do* yield the same product? What are the attributes of the problems that do 'work'?" In pursuing these extensions, one child found that 22×3 equaled 33×2. Following the pattern of using a two-digit number with the same number of tens and ones, she tried 33×6 and 66×3. When she discovered that the strategy worked, she wrote: "When I wrote down the problems I didn't know if I would get the same answer or not. I used these numbers because $3 \times 6 = 18$ and $6 \times 3 = 18$." Her statement conveys her willingness to take risks, her resourceful thinking, and her sense of personal accomplishment. These

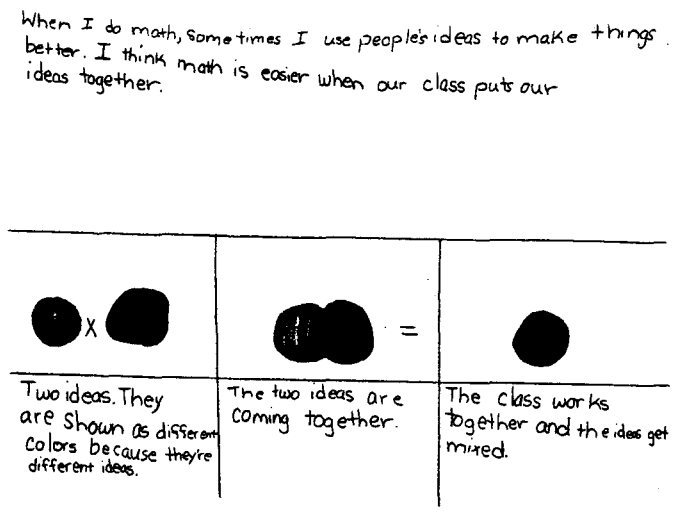
Figure 3

Using the analysis of survey results to guide instructional plans

Trends Implied by the Surveys	Positive Attitudes, Dispositions, Beliefs	Instructional Plans for Change
1. Mathematics is a solitary, silent endeavor.	1. Collaboration and communication contribute to mathematical understanding.	1. Structure group tasks; make children's strategies public; encourage children to note others' contributions to their learning
2. The teacher is in charge of imparting knowledge. The rewards for developing mathematical expertise are external and are often postponed until the future.	2. Mathematics involves learners in constructing meaning for themselves. The rewards for developing expertise are intrinsic.	2. Encourage interaction, revisiting, extending (Schwartz 1996); involve students through student-authored problems, mathematics journals, mathematics "publications"
3. Problems are solved in a swift, prescribed manner.	3. Problems are solved through flexible use of multiple strategies. The time required to solve problems depends on the complexity of the problem.	3. Encourage strategy sharing, problem-posing investigations, extended explorations, mathematics journals (Whitin and Whitin 2000)
4. Mathematics is unrelated to other subjects.	4. Mathematics has real-life application across the curriculum and in contexts outside school.	4. Emphasize content-related problems (e.g., science), problems inspired by children's literature, student-authored problems

Figure 4

A student's visual depiction of the value of sharing ideas in mathematical problem solving



and other problem-posing explorations also helped dispel the notion that all mathematical proficiency is universally equated with the speed of generating a solution.

Finally, problem-solving opportunities arose in contexts outside mathematics class. Students gathered, represented, and analyzed data to make decisions about the ideal seed mixture to place in the class bird feeders (Whitin and Whitin 1999). While studying geology in science, the children used nets to build models of crystals, a process that afforded them the opportunity to apply geometric principles and terminology to the natural world (see **fig. 5**). In addition, the teacher regularly read aloud mathematics-related children's literature to demonstrate mathematical connections within a wide variety of contexts, initiate investigations, and inspire the children's writing. The changes in children's views about the usefulness of mathematics, as well as other attitudes and dispositions, were later reflected in their end-of-the-year assessment.

The Survey as Post-Assessment: Reflection and Evaluation

At the end of each year, the children completed the survey as a post-assessment. Usually the same survey format was used for both the pre-assessment and the post-assessment, but in the final year the post-assessment survey was slightly modified. Question 4—"How can math help you?"—was changed to "To think mathematically means ..." so that the children would state more directly their definitions of mathematical activity. Two new prompts were added: "When you write and draw about math ..." and " 'What if' in math. ...." The first would ensure that the children address multiple forms of representation and communication, while the second referred to problem-posing experiences (Brown and Walter 1990). Both questions were included as a means to evaluate the effectiveness of the instructional modifications made to address the needs identified on the fall surveys. The examples in **figure 6** illustrate trends in the end-of-the-year assessments.

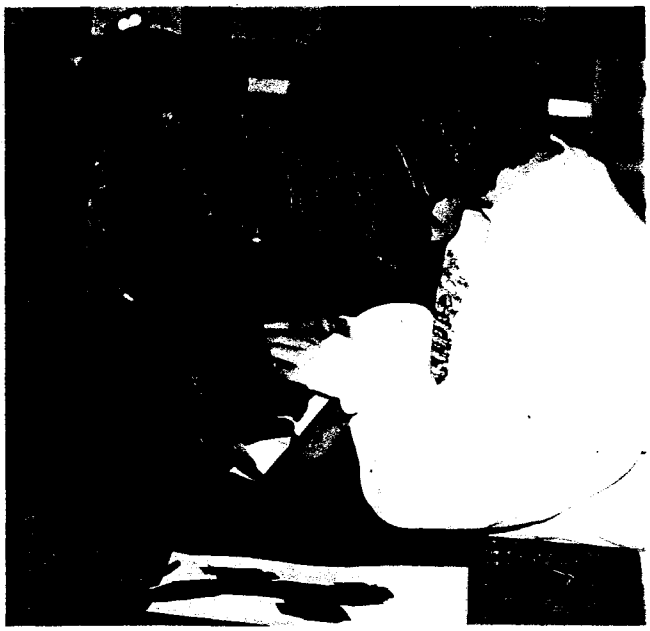
These examples of responses show a wider variety than those from the fall surveys. This range suggests that over the course of the year, the children developed more individualized mathematical identities as well as the confidence to express themselves. One of the sharpest contrasts with the initial surveys is that the later surveys contain almost no references to the teacher. For a student facing a difficult problem, asking "someone to help me" implies peers as well as adults. This student also shows responsibility by adding, "I'll do the rest." In this case, "help" does not mean that "someone else will do the work for me." The student who mentioned the teacher directly included other alternatives as well. Responses such as "use your own or someone else's strategy" and "Math is easy when there are groups or partners" also show an appreciation for collaborative thinking. In fact, as a whole, the children used the pronoun "we" in phrasing their answers to various prompts. This subtle shift in language from the fall surveys further implies a collaborative spirit.

In addition to showing less dependence on the teacher, the students showed more resourcefulness in their attitudes about problem solving. The response "make it into an easy math problem" suggests the strategy of simplifying the problem, while "trying another problem to help solve that problem" and "relate other math to it" show students' awareness of connections among mathematical ideas. The child who noted "I write down what I think" is aware that writing is a tool for reflection and discovery.

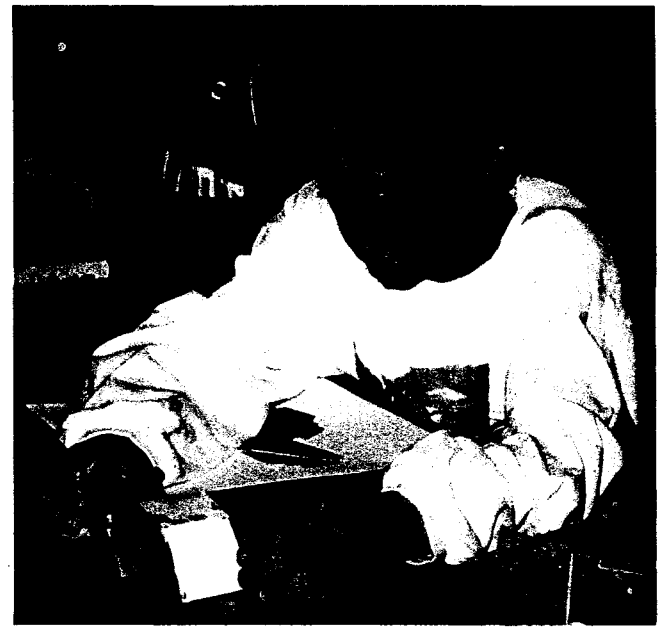
Sample responses to two of the revised and new questions are shown in **figure 7**. The collection of statements implies active construction of mathematical meaning. Thinking mathematically incorporates strategic thinking ("math strategies"), the ability to assume multiple perspectives or pose

Figure 5

While studying geology in science, the children used nets to build models of crystals, a process that afforded them the opportunity to apply geometric principles and terminology to the natural world.



5a. Building a model of a calcite crystal



5b. Comparing a calcite crystal with its model

problems ("say a lot of things about one question"), ownership ("make up creative problems"), and responsibility for one's own learning ("be energetic and serious about your attitude and thinking").

Some responses to the new question " 'What if' in math ..." revealed an appreciation for engaging in mathematical investigations—for example, "everything would be more of a challenge and a mystery, math would be even more fun." Interestingly, the child who initially responded to the prompt "How can math help you?" with the abstract example of "wanting to do something with 2 numbers" commented in the spring that "you can use math with almost everything." In contrast to earlier comments about "getting an A" and benefiting from mathematics "in college," these comments convey a sense of mathematical activity as intrinsically rewarding. Thus, analysis of the post-assessment surveys provided documentation of individual children's growth as well as trends in the classroom community. This feedback was valuable in the ongoing process of refining teaching throughout the four years of this study.

Children's attitudes and dispositions play a vital role in mathematics classrooms. The survey described here suggests one way to gain a window into children's existing beliefs. Given that information, teachers can better make instructional plans to help their students become more confident, enthusiastic, and autonomous learners.

References

Borasi, Raffaella. "The Invisible Hand Operating in Mathematics Instruction: Students' Conceptions and Expectations." In *Teaching and Learning Mathematics in the 1990s,* 1990 Yearbook of the National Council of Teachers of Mathematics (NCTM), edited by Thomas J. Cooney, pp. 174–82. Reston, VA: NCTM, 1990.

Brown, Stephen, and Marion Walter. *The Art of Problem Posing.* Hillsdale, NJ: Lawrence Erlbaum, 1990.

Goodman, Yetta, Dorothy J. Watson, and Carolyn L. Burke. *Reading Miscue Inventory: Alternative Procedures.* New York: Richard C. Owen, 1987.

Huinker, DeAnn, and Connie Laughlin. "Talk Your Way

Figure 6

Typical student responses from post-assessment mathematics surveys

1. **To be good in math, you need to ... because ...**
   "try hard and correct yourself because you would have a hard time if you don't"
   "have your own or someone else's strategy"
   "be able to solve puzzles and be able to make puzzles because it means you know what math is"
   "study on what you do good because then you will do a lot better"
2. **Math is hard when ...**
   "you don't have a strategy and you can't find a strategy"
3. **Math is easy when ...**
   "you really understand it"
   "you have had experience"
   "there are groups or partners"
   ...
6. **If you have trouble solving a problem, what do you do?**
   "If it is times I do plus. If it is division I do times."
   "I try my hardest and write down what I think."
   "I relate other math to it and work from there."
   "I ask someone to help me a little and I'll do the rest."
   "Think slow so you will not be confused and you might need help from your teacher or use paper."
   "You can look at a hard problem and make it into an easy math problem."

Figure 7

Typical student responses to sample revised and new survey questions

When you write and draw about math ...
"if you draw a picture, then you will find the answer"
"you can understand it better"
"it puts new thoughts in your head"
"you are showing how you think in math"
"I have a feeling I will make lots of connections"
To think mathematically means ...
"to think hard about your math and make up creative problems"
"you use math strategies to solve a problem"
"you can say a lot of things about one question"
"to be energetic and serious about your attitude and thinking"

into Writing." In *Communication in Mathematics K–12 and Beyond*, 1996 Yearbook of the National Council of Teachers of Mathematics (NCTM), edited by Portia C. Elliott, pp. 81–88. Reston, VA: NCTM, 1996.

National Council of Teachers of Mathematics (NCTM). *Principles and Standards for School Mathematics.* Reston, VA: NCTM, 2000.

Picker, Susan W., and John S. Berry. "Your Students' Images of Mathematicians and Mathematics." *Mathematics Teaching in the Middle School* 7 (December 2001): 202–8.

Rock, David, and Jean Shaw. "Exploring Children's Thinking about Mathematicians and Their Work." *Teaching Children Mathematics* 6 (May 2000): 550–55.

Schwartz, Sydney L. "Hidden Messages in Teacher Talk: Praise and Empowerment." *Teaching Children Mathematics* 44 (March 1996): 396–401.

Whitin, David J., and Phyllis Whitin. "Building a Community of Mathematicians." *Teaching Children Mathematics* 9 (April 2003): 464–69.

Whitin, Phyllis. "Meeting the Challenges of Negotiated Mathematical Inquiry." *Teaching and Learning: The Journal of Natural Inquiry and Reflective Practice* (2006–7) (forthcoming). www.und.nodak.edu/dept/ehd/journal/.

Whitin, Phyllis, and David J. Whitin. "Mathematics Is for the Birds: Reasoning for a Reason." In *Developing Mathematical Reasoning, K–12*, 1999 Yearbook of the National Council of Teachers of Mathematics (NCTM), edited by Lee Stiff, pp. 107–14. Reston, VA: NCTM, 1999.

———. *Math Is Language Too: Talking and Writing in the Mathematics Classroom.* Urbana, IL: National Council of Teachers of English and Reston, VA: National Council of Teachers of Mathematics, 2000.

Wickett, Maryann. "Uncovering Bias in the Classroom—A Personal Journey." In *Multicultural and Gender Equity in the Mathematics Classroom: The Gift of Diversity*, 1997 Yearbook of the National Council of Teachers of Mathematics (NCTM), edited by Janet Trentacosta and Margaret J. Kenney, pp. 102–6. Reston, VA: NCTM, 1997. ▲

Reflect and Discuss:

The Mathematics Survey: A Tool for Assessing Attitudes and Dispositions

Reflective teaching is a process of self-observation and self-evaluation. It means looking at your classroom practice, thinking about what you do and why you do it, and then evaluating whether it works. By collecting information about what goes on in our classrooms and then analyzing and evaluating this information, we identify and explore our own practices and underlying beliefs.

The following questions related to "The Mathematics Survey: A Tool for Assessing Attitudes and Dispositions" by Phyllis Whitin are suggested prompts to aid you in reflecting on the article and on how the author's ideas might benefit your own classroom practice. You are encouraged to reflect on the article independently as well as discuss it with your colleagues.

- **What is the relationship between attitudes and dispositions and mathematical proficiency?**
- **How do your students' comments and actions in the classroom suggest their attitudes and dispositions? Share your observations and analysis with your colleagues.**
- **What strategies do you use to foster students' development of positive attitudes and dispositions?**
- **Children's views of geometry or data analysis are sometimes very different from their views of number or algebra. What questions would you use to assess students' attitudes and dispositions toward the next mathematics unit you are scheduled to teach? How might the data change how you go about teaching the unit?**
- **Teachers, particularly novice teachers, tend to rely on lesson planning formats to consider the important elements of the lesson. How might one modify the lesson plan format to remind teachers that attention to issues of attitudes, confidence, and collaboration are important?**

You are invited to tell us how you used "Reflect and Discuss" as part of your professional development. The Editorial Panel appreciates the interest and values the views of those who take the time to send us their comments. Letters may be submitted to *Teaching Children Mathematics* at **tcm@nctm.org.** Please include "Readers' Exchange" in the subject line. Because of space limitations, letters and rejoinders from authors beyond the 250-word limit may be subject to abridgement. Letters also are edited for style and content.

Hot Topics

Teaching Children Mathematics is interested in publishing articles that relate to the following "hot topics" and meet the needs of both prospective and "seasoned" teachers:

● Multiple representations ● Differentiated instruction ● Assessment strategies ● Teachers' exploration of mathematics for themselves ● Linking mathematics with other subjects ● Using calculators and technology in the classroom ● Intervention strategies ● Teachers' knowledge of mathematics ● Role of administrators in mathematics education ● Using children's literature in mathematics

If you have interesting ideas, research, or classroom-tested approaches concerning any of these topics, please write them up and share them with this journal. Submit your manuscript to *TCM* by accessing tcm.msubmit.net. For more information, refer to the "TCM Writer's Packet" at www.nctm.org/publication/write.htm.

For the Success of Each Learner

Educational Leadership

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Improving Mathematics Teaching


February 2004

James W. Stigler and James Hiebert

The TIMSS video studies provide a picture of what happens in mathematics classrooms in the United States and in other countries.

How does mathematics instruction differ from country to country? What do these international comparisons tell us about how to improve mathematics achievement?

We have been working for 10 years on a research program aimed at answering these questions. The TIMSS video studies document typical teaching practices in various countries. These studies employ the video survey, a novel methodology that combines two research traditions: qualitative classroom research and large-scale survey research. The video studies capture close-up pictures of the classroom processes used by national samples of 8th grade mathematics teachers in different countries. These teachers are not necessarily experienced or effective. They are ordinary teachers, teaching lessons that they routinely teach.

Why would we want to study a random sample of ordinary lessons? First, these lessons together represent what average teaching looks like in different countries. If we want to improve student learning, we must find a way to improve teaching in the average classroom. Even slight improvements in the average can positively affect millions of students. This concept represents a new way to formulate the question of how to improve teaching.

Second, studying a national sample of classroom lessons can help us discover whether policy initiatives have influenced classroom practice. All reform efforts to improve teaching and learning must pass through a final common pathway: the classroom. Most reforms get stopped short at the classroom door; all available evidence suggests that classroom practice has changed little in the past 100 years.

Finally, studying lessons from different cultures gives researchers and teachers the opportunity to discover alternative ideas about how we can teach mathematics. Watching lessons from other countries prompts questions about the assumptions that guide common practices in our own country. It is often a startling experience to journey back and forth, looking first at foreign videos and then back at our own.

The First TIMSS Video Study

The TIMSS 1995 video study (Stigler & Hiebert, 1999) examined national samples of 8th grade mathematics lessons from three countries: Germany, Japan, and the United States. Several findings from the first study provide important background information.

Lack of a Shared Language to Describe Teaching

The lack of a shared language for describing teaching makes it very difficult to generate and disseminate professional knowledge. Even before beginning the video studies, we suspected that this problem existed; indeed, that suspicion was one of the reasons we chose to document teaching through videotapes instead of questionnaires. As the tapes started to arrive and we discussed what we saw on them, it became obvious that different people saw different things and described what they saw in different ways.

For example, we tried in the first study to mark where on the video each mathematics problem started and where it ended, a process that we thought might simplify our task by enabling us to analyze each problem separately. We could not agree on what a problem was (although we did manage to do so in the later study). Some observers would only count an activity as a problem if it involved students in sustained thinking over a long period of time. Others might count as a problem a brief exercise that students could solve quickly by recalling a solution that they had previously been taught. The word "problem" clearly means different things to different people. Other words and phrases, such as "develop concepts" or "teach for understanding," pose similar challenges.

Slippage Between Policy and Classroom Practice

In part because we lack a shared language, attempts by policymakers to change what happens in classrooms often achieve either no results or unintended results as reform efforts get filtered through the weak communication channels we rely on to disseminate policy (Elmore, 2000). In our first video study, we asked teachers whether they had read mathematics education reform documents (for example, those published by the National Council of Teachers of Mathematics) and whether they implemented the documents' recommendations in their classrooms. Most teachers said that they had read such documents and that they used the reform ideas in their classrooms. However, the videos revealed great unevenness in how teachers interpreted the reforms and showed little evidence that classroom practices actually reflected the goals of the reforms.

The Cultural Nature of Teaching

We concluded from our first study that teaching is a cultural activity: learned implicitly, hard to see from within the culture, and hard to change. We were struck by the homogeneity of teaching methods observed within each country and by the striking differences in methods we observed across Germany, Japan, and the United States. Even in the United States, a country with great diversity in language, ethnicity, and economic conditions and an education system controlled by local governing boards, the nationwide variation in 8th grade mathematics teaching was much smaller than we had expected.

The 1999 TIMSS Video Study

The TIMSS 1999 video study expanded on the first study. In addition to the United States, we included Australia, the Czech Republic, Hong Kong,[1] Japan, the Netherlands, and Switzerland. Each of these countries performed significantly higher than the United States did on the TIMSS 1995 mathematics achievement test for 8th grade.

The design of the 1999 video study was simple. We selected a random sample of 100 8th grade mathematics classrooms from each country and videotaped them at some point during the school year. We digitized, transcribed, and translated the tapes into English, after which an international team of researchers analyzed them. Coding and analysis focused on the organization of lessons, the mathematical content of lessons, and the ways in which the class worked on the content as the lessons unfolded. Here are some of the most interesting findings.

Effective Teaching Takes Many Forms

In both the 1995 study and the 1999 study, teaching methods in Japan differed markedly from

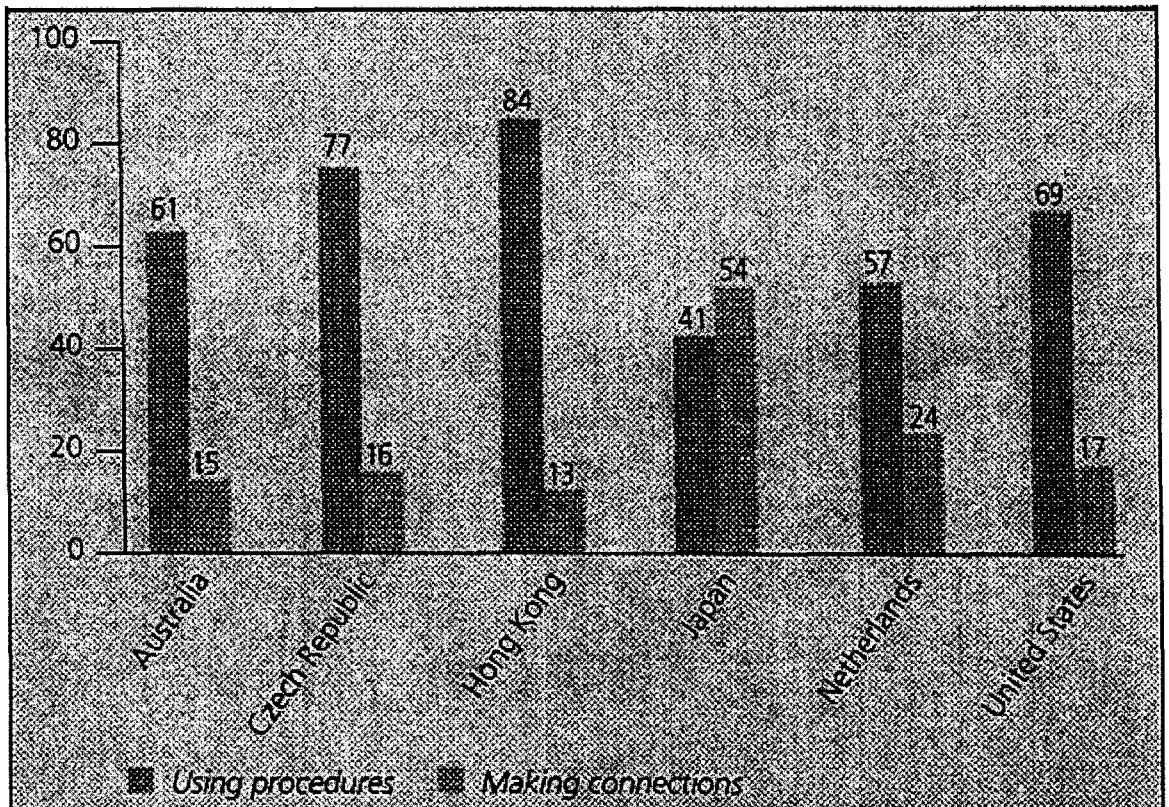
what we observed in all of the other countries. Japanese students, for example, spent an average of 15 minutes working on each mathematics problem during the lesson, in part because students often were asked to develop their own solution procedures for problems that they had not seen before.

Because Japan was the only high-achieving country in the first video study (as indicated by students' performance on the testing component of TIMSS), many researchers assumed that the United States would need to copy Japanese methods to produce the levels of learning displayed by Japanese students. The 1999 study, however, makes it clear that despite the well-crafted nature of the Japanese lessons, high achievement does not necessitate a Japanese style of teaching. Other countries posted high scores with lessons that looked decidedly un-Japanese. For example, in contrast to the relatively long time spent on each problem in Japan, every other country in the study spent only up to five minutes on the average problem.

The videotapes from each country reveal a unique combination of features. Many teaching methods that are hotly debated in the United States vary among the six higher-achieving countries. For example, the Netherlands uses calculators and real-world problem scenarios quite frequently. Japan does neither. Yet both countries have high levels of student achievement.

As another example, consider the debate over what kinds of problems students should work on during the mathematics lesson: basic computational skills and procedures (*using procedures* problems) or rich mathematical problems that focus on concepts and connections among mathematical ideas (*making connections* problems). Figure 1 shows the percentage of each kind of problem observed in six of the seven countries.

Figure 1. Types of Math Problems Presented



The percentage of math problems that focused on *making connections* varied greatly among high-scoring TIMSS countries.

Note: Switzerland was not included in this analysis because this feature of teaching was coded only by English speakers and English transcripts of the Swiss lessons were not available.

Japan is an outlier; 54 percent of the problems observed in the country's classrooms were *making connections* problems. But note that Hong Kong, one of the highest-achieving countries in the study, is at the opposite end of the continuum, with only 13 percent of problems coded as *making connections*. Classrooms in all of the countries spend time both on problems that call for *using procedures* and on those that call for working on concepts or *making connections*. The percentage of problems presented in each category, however, does not appear to predict students' performance on achievement tests.

Implementation Is Important

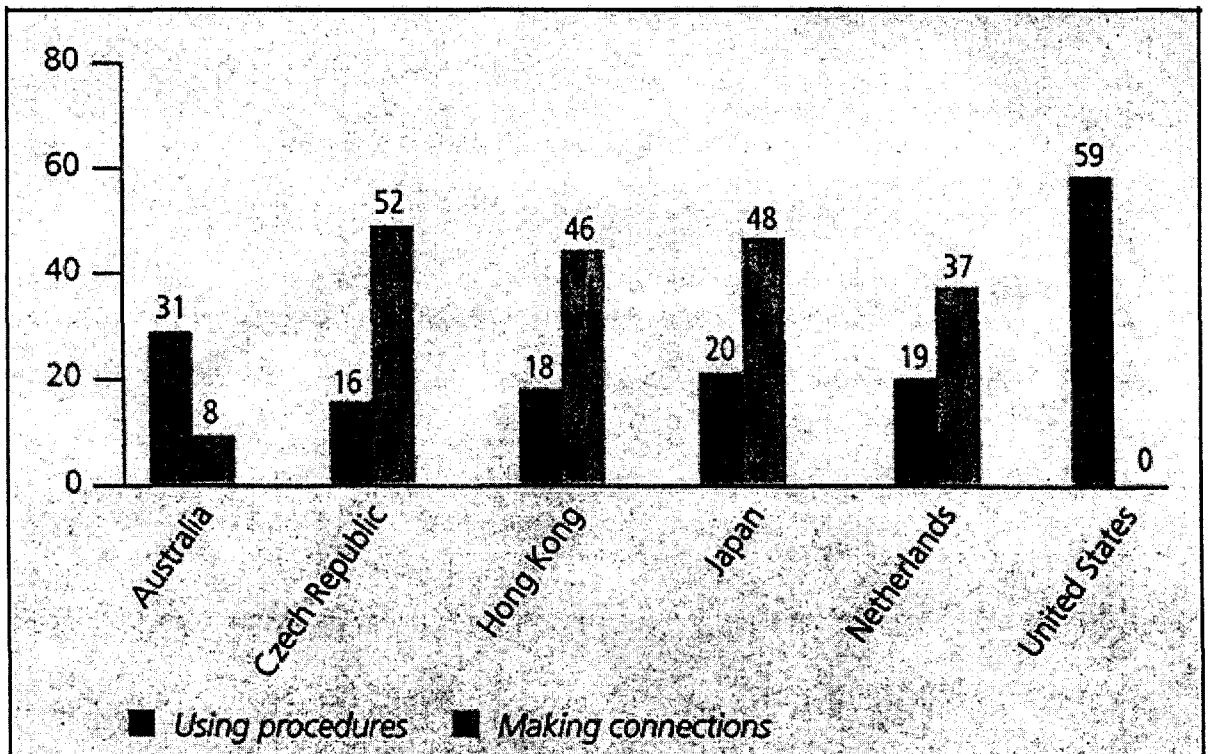
What, then, do the higher-achieving countries have in common? The answer does not lie in the organization of classrooms, the kinds of technologies used, or even the types of problems presented to students, but in the way in which teachers and students work on problems as the lesson unfolds.

In the 1999 video study, we coded each problem twice: once to characterize the type of problem and the second time to describe how the problem was implemented in the classroom. The teacher could implement a *making connections* problem as a *making connections* problem, or the teacher could transform it into another type of problem—most commonly, a *using procedures* problem. For example, a teacher might transform a *making connections* problem designed to have students figure out a method for calculating the area of various types of triangles into a *using procedures* problem by giving students, at the outset, the formula (1/2 Base × Height) and telling students

to simply plug in the relevant values.

Figure 2 shows how the teachers in the study actually implemented the *making connections* problems in the classroom: the percentage implemented as *making connections* and the percentage implemented as *using procedures*. Unlike Figure 1, this analysis reveals a pattern in which the highest-achieving countries resemble one another. Hong Kong and Japan, the two countries that differed most in the percentage of *making connections* problems presented, show a new similarity. In both countries, the majority of *making connections* problems are implemented as *making connections* problems; a much smaller percentage are transformed into lower-level *using procedures* problems. Here is the most striking finding of all: In the United States, teachers implemented none of the *making connections* problems in the way in which they were intended. Instead, the U.S. teachers turned most of the problems into procedural exercises or just supplied students with the answers to the problems.

Figure 2. How Teachers Implemented *Making Connections* Math Problems



High-scoring TIMSS countries implemented a higher percentage of *making connections* problems as *making connections* problems. U.S. teachers tended to turn these problems into procedural exercises.

Note: Switzerland was not included in this analysis because this feature of teaching was coded only by English speakers and English transcripts of the Swiss lessons were not available.

The debates over mathematics education in the United States often pit two views against each other. One group believes that U.S. classrooms do not focus enough on concepts and understanding. The other group believes that U.S. classrooms overemphasize concepts at the expense of basic skills, thus holding back student achievement (Loveless, 2003).

Our research indicates that the lower achievement of U.S. students cannot be explained by an overemphasis on concepts and understanding. In fact, U.S. 8th graders spend most of their time in mathematics classrooms practicing procedures. They rarely spend time engaged in the serious

study of mathematical concepts.

Improving Teaching

On the basis of this brief tour of the TIMSS videotape studies, three broad ideas can inform our efforts to improve classroom teaching of mathematics in the United States.

Focus on the Details of Teaching, Not Teachers

Most current efforts to improve the quality of teaching focus on the teacher: how the profession can recruit more qualified teachers and how we can remedy deficiencies in the knowledge of current teachers. The focus on teachers has some merit, of course, but we believe that a focus on the improvement of *teaching*—the methods that teachers use in the classroom—will yield greater returns.

The TIMSS video studies reveal that teaching is cultural; most teachers within a culture use similar methods. Indeed, within our study, teachers with strong mathematical knowledge showed the same cultural patterns of teaching as teachers with weaker knowledge. We must find a way to improve the standard operating procedures in U.S. mathematics classrooms—to make incremental and continuous improvements in the quality of the instruction that most students experience.

A focus on teaching must avoid the temptation to consider only the superficial aspects of teaching: the organization, tools, curriculum content, and textbooks. The cultural activity of teaching—the ways in which the teacher and students interact about the subject—can be more powerful than the curriculum materials that teachers use. As Figure 2 shows, even when the curriculum includes potentially rich problems, U.S. teachers use their traditional cultural teaching routines to transform the problems and reduce their instructional potential. We must find a way to change not just individual teachers, but the culture of teaching itself.

Become Aware of Cultural Routines

We can only change teaching by using methods known to change culture. Primary among these methods is the analysis of practice, which brings cultural routines to awareness so that teachers can consciously evaluate and improve them. A recent study by Hill and Ball (in press) of a large-scale professional development program found that analysis of classroom practice was one of three factors predicting growth of teachers' content knowledge.

Analysis of classroom practice plays several important roles. It gives teachers the opportunity to analyze how teaching affects learning and to examine closely those cases in which learning does not occur. It also gives teachers the skills they need to integrate new ideas into their own practice. For example, by analyzing videotaped examples of other teachers implementing *making connections* problems, teachers can identify the techniques used to implement such problems, as well as the way in which teachers embed these techniques within the flow of a lesson.

Attempts to implement reform without analysis of practice are not likely to succeed.

Build a Knowledge Base for the Teaching Profession

Finally, educators must find a way to inject new knowledge into the system of improvement and to share that knowledge with future generations of teachers (Hiebert, Gallimore, & Stigler, 2002). As John Dewey pointed out long ago, one of the saddest things about U.S. education is that the wisdom of our most successful teachers is lost to the profession when they retire.

What kind of knowledge do teachers need? They need theories, empirical research, and alternative images of what implementation looks like. U.S. teachers who want to improve their implementation of *making connections* problems, for example, will run up against a formidable challenge: They might never have seen what it looks like to implement these problems

effectively.

Teachers need access to examples, such as those collected in the TIMSS video studies. They need to decide how they can integrate these examples into their own practice. They need to analyze what happens when they try something new in their own teaching: Does it help students achieve the learning goals? Finally, they need to record what they are learning and share that knowledge with their colleagues.

Teachers have a central role to play in building a useful knowledge base for the profession. Enabling teachers to learn about teaching practices in other countries and to reflect on the implications of those practices holds great promise for improving the mathematics instruction provided to all students.

References

Elmore, R. F. (2000). *Building a new structure for school leadership*. Washington, DC: Albert Shanker Institute.

Hiebert, J., Gallimore, R., Garnier, H., Givvin, K. B., Hollingsworth, H., Jacobs, J., Chui, A. M. Y., Wearne, D., Smith, M., Kersting, N., Manaster, A., Tseng, E., Etterbeek, W., Manaster, C., Gonzales, P., & Stigler, J. W. (2003). *Teaching mathematics in seven countries: Results from the TIMSS 1999 video study* (NCES 2003-013). Washington, DC: U.S. Department of Education.

Hiebert, J., Gallimore, R., & Stigler, J. W. (2002). A knowledge base for the teaching profession. *Educational Researcher, 31*(5), 3–15.

Hill, H., & Ball, D. (in press). Learning mathematics for teaching: Results from California's Mathematics Professional Development Institutes. *Journal for Research in Mathematics Education*.

Loveless, T. (2003, February). *Trends in math achievement: The importance of basic skills*. Presentation at the Secretary's Summit on Mathematics, Washington, DC. Available: www.ed.gov/inits/mathscience/loveless.html

Stigler, J. W., & Hiebert, J. (1999). *The teaching gap*. New York: Free Press.

Endnote

[1] This article refers to the Hong Kong Special Administrative Region of China as a country, along with the six other participants in the TIMSS 1999 video study, for the sake of consistency.

Author's note: The TIMSS 1999 video study was funded by the National Center for Education Statistics and the Office of Educational Research and Improvement of the U.S. Department of Education, as well as the National Science Foundation. It was conducted as a component of the Trends in International Mathematics and Science Study (TIMSS), under the auspices of the International Association for the Evaluation of Educational Achievement (IEA). Each participating country provided the services of a research coordinator who guided the sampling and recruiting of participating teachers. In addition, Australia and Switzerland contributed financial support for data collection and processing of their respective samples of lessons.
For complete details of the TIMSS 1999 video study, see Hiebert et al. (2003). This report and a four-CD set with 28 complete mathematics lessons for public release, four from each country, are available at www.lessonlab.com. Ronald Gallimore, codirector of the TIMSS 1999 video study, contributed to the ideas presented in this article. The views expressed in this article are the authors' and do not necessarily reflect those of the International Association for the Evaluation of Educational Achievement, the funding agencies, or any of the individuals who contributed to the studies.

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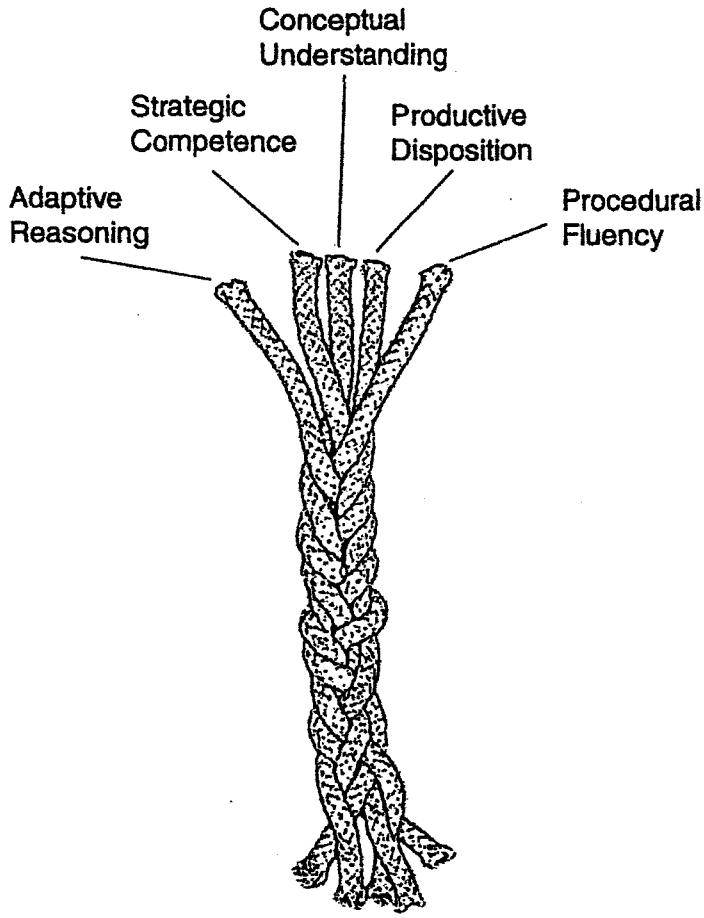
Intertwined Strands of Proficiency



Kilpatrick, J., Swafford, J., & Findell, B. (2001).
Adding It Up: Helping Children Learn Mathematics.
Mathematics Learning Study Committee, Center for Education, Division of Behavioral and Social Sciences and Education, National Research Council.

QUESTIONS TO HELP STUDENTS DEVELOP NUMBER SENSE AND CLARIFY THINKING

Tell how you did that?
What went on in your mind when _____?
When have you done something like this before?
What would be your criteria for ____?
What do others think about what_____said?
Do you agree? Disagree? Why or why not?
Does that make sense? Why or why not?
Does that always work? Why or why not?
Is that always true? Explain.
Do you see a pattern? Explain
Can you predict the next one? What about the last one?
How did your prediction compare with your results?
How can you find out?
How did you know _____?
What might you do next?
What's another way you might approach this?
How might you be able to use this in other situations?
What do you think would happen if_____?
What would it look like if?
How does this relate to _____?
Have we ever solved a problem like this one before?
What is alike and/or different about the solutions?

ARTICLES

The Pathways of Math's Ways

Dr. Mel Levine

Students face a hefty challenge as they struggle to climb through the pathways leading to an academic summit called success in mathematics. Once they get there, the view is great. En route there are seemingly endless obstacles. The pitfalls are especially troubling for students who have differences in learning that impede their ability to think with numbers. Let's look at some of the challenges and the ways they might not be met. We can divide these into: knowing what you're doing; remembering what to do and what you're doing, and becoming a good problem solver.

Knowing What You're Doing
Different students have different levels of understanding when they engage in mathematics. Some go through the mathematical motions, while others understand in depth such concepts as place value, factoring, and circumference. Students with weak concept formation are apt to over-rely on rote memory.
HELPFUL HINTS: They need help mapping out the important concepts in diagrams and getting a chance to explain them in their own words before applying them. They also can use hands on experience applying the concept(s) outside of school in practical ways.

Some kids have trouble understanding the technical language of math. Terms like exponent, hypothesis, and denominator can confuse them. Also, they are likely to become confused with word problems and verbal explanations of the processes.
HELPFUL HINTS: These kids may benefit from keeping a personal dictionary of key terms. They need practice looking at word problems and just identifying what process (such as subtraction) will be needed – without having to solve the problem. To help with problems understanding verbal explanations, teachers should give these kids correctly solved problems (demonstration models) to analyze and talk about.

Remembering What to Do
Lots of students go to pieces over the memory load imposed by mathematics, which is one of the most cumulative subjects kids face; things keep on depending upon what you've learned in the past, and that adds up to a colossal drain on memory. Some kids have trouble recalling facts. Some do okay with math facts, but have a hard time recalling how to do things (like long division). Others have trouble remembering what they're doing while they're doing it (a so-called active working memory deficit). Still others fail to recognize and respond well to the many different recurring patterns, patterns such as hexagons, phrases (like "is the equivalent of"), and symbols (such as %, +, =).
HELPFUL HINTS: Try to figure out which memory part isn't working. Then design drill games to use for 10 minutes a night just before the child goes to bed until the recall becomes fast and accurate (we call this automatization). Remember, long-term memory works best right before someone goes to sleep.

Becoming a Good Problem Solver
Good work in math depends upon a systematic stepwise approach to problem solving. Some kids try to do everything at once or they work too quickly or they don't consider alternative strategies, trying only the first approach that comes to mind. Often they don't proofread or focus enough on the details. These shortcomings are especially common in kids with attention deficits when they approach mathematics.
HELPFUL HINTS: Help kids like this pace themselves. Reward them for working slowly. Give them proof reading exercises, opportunities to find errors in the work of others. Encourage them to talk their way through problems – step by step. Have them describe how they will solve a problem before they begin their work. Also, have them explain the steps they used once they have completed a problem.

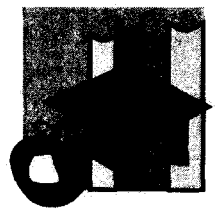
Taming Math
Math can be intimidating for kids. They need practical arithmetic experiences that are fun. Examples would be computing sports statistics, doing craft projects that entail calculations, and playing number games. Most important, however: don't ever let a child get more than six months behind in mathematics. Catching up after that can be nearly impossible, and affected students are apt to develop paralyzing mathematics phobias.

If we understand the nature of a child's strengths and weaknesses, we can help any child to achieve and feel good in mathematics. It's a matter of finding the best itinerary and the best route through the subject's many possible pathways.

CMC Research Notes:

Mathematics throughout History; Preparing Students for a Changing World

by Jack Carter, CSU East Bay



Mathematics throughout History

Furinghetti (2000) reviewed arguments supporting the history of mathematics as a link between teaching and providing students with flexibility, open-mindedness, and motivation toward mathematics. The review emphasized the role of historical mathematics as a magnifying glass for analyzing critical points and difficult concepts through the words of past authors. Furinghetti found that this use of the history of mathematics avoided the presentation of a polished theory and thereby allowed students to follow the paths of those who struggled with mathematical problems underlying the theory, and to gain a deeper appreciation of the theory studied. Results also showed that mathematics throughout history not only implied that all kinds of students should know mathematics for its cultural value, but also that teachers should use the functionality of history in the learning/understanding of mathematics.

Gulikers and Blom (2001) presented a framework for bringing structure to bridging the gap between history and mathematics education, between historians and teachers, and between past mathematicians and present day students. Arguments for applying mathematics throughout history to teaching related to (1) the desirability and necessity of teaching and learning mathematics along the lines of its historical development and (2) the wish to influence teachers' attitudes and enrich teachers' methodological repertoires. Arguments for applying mathematics throughout history to students related to (1) enhancing knowledge of how mathematical concepts have developed, (2) helping students learn in nonlinear ways, and (3) setting programs that balanced learning obstacles and smooth progress. Gulikers and Blom concluded that teaching mathematics in historical perspective developed resources that were in accordance with students' natural development, but that a gap existed between historical research in mathematics and teachers' practical needs.

Fleener, Reeder, Young, and Reynolds (2002) explored the influence of a mathematics education curriculum focused on understanding through explorations of historical topics. Participants in the study were elementary education majors enrolled in university-level mathematics content or methods classes. Prior to enrolling in the classes, the future teachers exhibited an overwhelming focus on technical aspects of mathematics, and after the classes, the researchers expected these students to express more emancipatory ideas about mathematics. Fleener et al. were surprised to learn that, despite several semesters of opportunities to expand their ideas about mathematics throughout history, students maintained a predominantly technical interest perspective of mathematics. The researchers concluded that the students' past pervasive emphasis on technical skills and demonstration of competencies in K–12 mathematics prevented the research from having an impact on students' approaches to teaching of mathematics.

Bosse and Hurd (2002) explored some of the historical factors leading to the relatively small number of women employed in mathematics throughout history. The researchers generated a large database and biographical commonalities were combined to create a generalized biography of the lives of historic woman mathematicians. The generalized biography demonstrated four findings:

1. Every woman in the study was hindered in a significant manner in her pursuit of mathematics.
2. Nearly all mathematically successful women were supported in their mathematical pursuits by male mathematicians.
3. Alliances with male university professors produced mentoring relationships that were symbiotic to both parties and subservient on the part of the women.
4. The role assigned to women by male mentors entailed the communication of mathematical ideas to others, and women

Continued on page 26

worked in the realm of mathematics education rather than mathematics research.

Bosse and Hurd also found evidence that most of the historic women mathematicians were from affluent families where heads of households were either aristocrats or occupationally wedded to universities. The researchers concluded that the women studied were consistently given responsibilities that held one intrinsic commonality—the communication of mathematical ideas.

Preparing Students

Lesh, Zawojewski and Carmona (2003) studied the nature of the mathematical understandings that are likely to have the greatest impact on issues of equity and opportunity and to prepare students for a changing world. Key to the study were construct-eliciting activities that put students in situations where they were able to judge the usefulness of alternative ways of mathematical thinking and thereby recognize what was needed for success beyond school. Results showed that the constructs and conceptual tools that students developed also molded the shape of the changing world that students envisioned. Representational fluency was also at the heart of what students needed to prepare themselves for the world beyond schools. Results showed that this fluency was a chief mathematical construct for understanding complex, changing systems. Lesh et al. also found that mathematical topics most useful in preparing students for a changing world were those where the emphasis was on multimedia displays, iterative and recursive functions, and dynamic systems.

Schwartz and Martin (2004) compared instructional activities that promoted student invention to prepare for future mathematics learning with tell-and-practice classroom activities. The experimental activities supported many different paths of interaction and invention while the tell-and-practice activities relied upon direct instruction and individual practice. Findings showed that the inventing activities prepared students for a changing world and that students were able to access that preparation for subsequent mathematics learning. Schwartz and Martin concluded that the results indicated that one way to prepare students to learn in a changing world involves letting them generate original productions. Particularly, the opportunity to produce novel structures in changing material, symbolic, and social environments constituted a powerful mechanism for developing new ideas.

Sfard and Lavie (2005) observed conversations between children and their parents about quantitative comparisons to probe children's arithmetical understanding and its implications for preparing them for a changing world. By assuming that thinking can be viewed as a special case of communicating and that arithmetic can be thought of as specialized form of discourse, Sfard and Lavie found that children's discourse diverged from that of grown-ups' discourse along three dimensions—cardinality, purpose, and consolidation. In each dimension the researchers found that the children started as ritualized participants in the arithmetical routines of grown-ups, but gradually the rituals shifted to become genuine explorations. Sfard and Lavie concluded that the condition for seeing the arithmetical world in mature ways is forgetting the image of the world as it was at the outset.

References

Bosse, M.J. and D.A.C. Hurd. "Woman Communicating Mathematics: The Historic Role of Women." *Mathematics and Computer Education* 36 (3)(2002): 218–26.

Fleener, M.J., S.L Reeder, E. Young, and A.M. Reynolds. "History of Mathematics: Building Relationships for Teaching and Learning." *Action in Teacher Education* 24 (3)(2002): 73–84.

Furinghetti, F. "The History of Mathematics As a Coupling Link between Secondary and University Teaching." *International Journal of Mathematical Education in Science and Technology* 31 (1)(2000): 43–51.

Gulikers, I. and K. Blom. "'A Historical Angle', A Survey of Recent Literature on the Use and Value of History in Geometrical Education." *Educational Studies in Mathematics* 47 (2)(2001): 223–258.

Lesh, R., J.S. Zawojewski, and G. Carmona. "What Mathematical Abilities Are Needed for Success Beyond School in a Technology-Based Age of Information?" In *Beyond Constructivism, Models and Modeling Perspectives on Mathematics Problem Solving, Learning, and Teaching*, edited by R. Lesh and H.M. Doerr, 205–222. Mahwah, NJ: Lawrence Erlbaum Associates, 2003.

Schwartz, D.L. and T. Martin. "Inventing to Prepare for Future Learning: The Hidden Efficiency of Encouraging Original Student Production in Statistics Instruction." *Cognition and Instruction* 22 (2)(2004): 129–184.

Sfard, A. and I. Lavie. "Why Cannot Children See as the Same What Grown-Ups Cannot See as Different?—Early Number Thinking Revisited." *Cognition and Instruction* 23 (2)(2005): 237–309.

X = Karin (Johnny) > 95%

What does it take to learn algebra? First you have to master the fundamentals.

By Karin Klein

JOHNNY PATRELLO was a greaser. I was a dork. And yet, despite our rigidly stratified school culture, we came together in the spring of 1968 at Walt Whitman Junior High School, where I tutored Johnny in algebra.

I thought about Johnny again as I read The Times' series this week on L.A. dropout problem. Algebra, the reporters found, is an insurmountable stumbling block for many high school students.

What struck me was that the reasons why Johnny can't do algebra in L.A. today are remarkably similar to why Johnny Patrello couldn't do algebra almost four decades ago in Yonkers, N.Y.

Johnny and I were brought together by Mrs. Elizabeth Bukanz, the algebra teacher. Mrs. Bukanz wore her sandy hair in a frizzy French twist and her glasses on a chain. But she was gentle and smiling, and she had passion — at least for what she called "the beauty of algebra." I, too, loved its perfect logic and tidy solutions, so unlike my messy teenage life.

But Johnny was deaf to algebra's siren song. He was flunking, and Mrs. Bukanz hoped that if I used my study halls to tutor him, he might score at least 65% on the New York State Regents exam. Passing the exam allowed even failing students to move on to high school, which started in 10th grade; otherwise, Johnny would be left behind.

Johnny wore his leather jacket in class despite the spring warmth, and he habitually tilted his face toward the floor so that when he looked up at me, he seemed embarrassed. Yet for such a cool guy, he was surprisingly friendly and committed to giving this a try.

Things looked pretty hopeless to both of us those first couple of sessions, as Johnny stumbled through algebra problems while I tried to figure out exactly what he didn't understand. Then, as we took it down to each step of each little calculation, the trouble became clear: Johnny somehow had reached ninth grade without learning the multiplication tables.

Dork: Karin Klein

Greaser: Johnny Patrello

Because he was shaky on those, his long multiplication was error prone and his long division a mess. As Johnny tried to work algebraic equations, his arithmetic kept bringing up weird results. He'd figure he was on the wrong track and make up an answer.

This discovery should have made us feel worse. How could we possibly make up for a dearth of third-grade skills and cover algebra too?

But at least we knew where to start.

We spent about half of those early sessions on multiplication drills. Seven times eight, eight times seven — Johnny could never remember. As an adult, in memory of Johnny's struggles, I would rehearse my kids at an early age in that one math fact. Get that 56 down, I would tell them, and the rest of multiplication is a snap.

Today's failing high school students, though plagued by more poverty and upheaval than Johnny or I ever knew, bring the same scanty skills to algebra class, according to The Times' series. They never quite grasped multiplication tables, but still they moved on to more complicated math.

Who can focus on the step-by-step logic of peeling back an equation until "x" is bared when it involves arithmetic that comes slow and slippery, always giving a different answer to the same calculation?

Yet in all these decades, the same school structure that failed Johnny goes on, dragging kids through the grades even if they don't master the material from the year before. This especially makes no sense for math, which is almost entirely sequential.

Leaving children back isn't a solution; it simply makes them feel stupid. They learn, like Johnny, to look at the floor. The floor can't embarrass them.

What I learned from Johnny — aside from the fact that greasers could be sweet-natured and very, very smart — is that schools are structured to help administrators feel organized, not to help children learn.

Young children's skills are all over the map, yet we corral them into second grade, third grade and so forth, where everyone moves at one pace in all subjects. Better to group them according to their skills in each subject, without the "grade" labels, and let them move on to the next skill when they have mastered the one they were on. If they're not getting it, give them extra tutoring, but don't push them forward until they're ready. This way, there is no failure — only progress.

It requires a sea change in thinking, but it's not impossible or even all that hard. Back before standardized tests put classes in lockstep, some progressive schools already were using team teaching to do this in math as well as reading and writing.

Johnny finally nailed seven times eight, then with amazing quickness worked his way through basic "x" problems up to multiple variables and beyond. Still, I couldn't quite catch him up to a year's worth of work in a couple of months. And on a sweltering June day, with humidity that neared 100%, the regents exam came, faster than we felt ready for it.

A couple of weeks later, I saw Johnny in the hall. He shot me a dazed look and broke the news — 95%! That moment has wiped from memory my own regents score. But I won the algebra award at the graduation ceremony. Johnny cheered, apparently undaunted by the fear of appearing uncool.

We lost touch in high school. I was college-prep, he was voc-ed. We would pass occasionally in the halls, and he would glance up from the floor and say, "Hi, teach!"

I know he received his diploma because I see his picture in my old yearbook, wearing a suit and tie instead of his leather jacket. His eyes still look up cautiously from his slightly downcast face, as though he is a bit surprised to be there.

BEFORE I USED Johnny's full name in a story that would reach more than a million readers, it was only right to try to contact him for permission. Directory assistance found one John Patrello, not too far from Yonkers.

The phone was answered by his wife, Joann. It was the same Johnny, but he had died a year and a half ago of a massive stroke, leaving behind Joann and four children.

As she and I talked, both of us in tears at times, it was amazing how much of what I remembered about Johnny continued throughout his life — the tough outer look, the sweetness a millimeter underneath, the quick mind, the habit of tilting his face toward the floor. His eldest is a doctor; the second, a teacher. His teenage daughter wants to be a journalist, and I'll see what I can do to help her along the way.

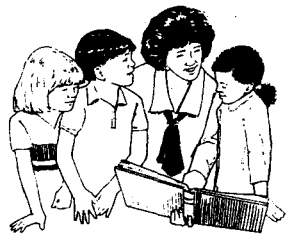
Johnny became an auto mechanic. ("He loved math, and you know auto repair involves a lot of math," Joann said. Yes, it does.)

Another thing Joann told me about Johnny: He was incredibly fast at multiplication.

KARIN KLEIN *is an editorial writer for The Times.*

Going Back in Time

by Christina Myren, Conejo Valley USD, cmyren@aol.com; and Fran Threewit, Kenwood Elementary, fthreewit@comcast.net



At our last *ComMuniCator* panel meeting we were browsing through children's textbooks from the last century. The books, *A Child's Book of Number* and *First Days in Number*, were written in 1903 and 1924 respectively. At first glance, the books appeared quite different from today's four-color, two-page spread editions. However, as we read further we were struck with the similarity in content of what children use today. Because of the theme of this issue, Mathematics Throughout History, we decided it would be interesting to share some of the similarities with you.

Our first example is the opening two paragraphs from *First Days in Number*:

> It is not the purpose of this book to *teach* number facts to the primary pupil. Education is development, and especially is this true of the best primary work. The mind of the child just entering school is bent upon investigation, exploration, and discovery. It is the privilege of the primary teacher to guide this investigation, furnish proper and fruitful environment for exploration, and teach the child the use of the best written and spoken language in which he may tell of his discoveries.
>
> There is no broader field for the free development of the child mind than is to be found in number work. The child needs only to be invited and he will discover many facts concerning his home, birds, animals, flowers, fruits, and forms; in short, he will discover the world in which he lives.

As we read in *A Child's Book of Number*, we were again taken by the instructions to the teacher that emphasized connecting the work in number to the child's world. The emphasis on playful learning and daily connections to number are numerous:

Continued on page 28

MAKING RHYMES ABOUT TWO

Some children made rhymes about adding two. Here is one of them. See if you can make a better one.

Two little boys,
Sitting in the door.
Two more join them,
Then there are ——.

Four little boys,
Picking up sticks.
Two more help them,
Then there are ——.

Six little boys,
Swinging on the gate.
Two more get on,
Then there are ——.

Tell how many:

2	6	5	8	4	3	7	1
2	2	2	2	2	2	2	2

[9]

AN OUTDOOR GAME

These children have made up a number game. They form a circle as in "Drop-the-handkerchief". One child runs around the circle saying:

"Hink-i-ty, pink-i-ty, pank-i-ty, poo,
I had —— pennies and now I have two.
Hink-i-ty, pink-i-ty, pank-i-ty, poo,
Who knows how many I lost? Do you?"

As she says "Do you?" she taps some one. The one tapped answers, and tries to catch her before she can get around the circle.

If she is caught, she has to go around again. If not, the other child goes around and taps some one in the same way.

They use any number from two to ten where the blank is.

[19]

That the child may delight in his work, see that it is useful to him in his play and in his little daily affairs, and wish to know more of number from day to day, there are many little rhymes, number stories, and simple descriptions of games. These are not given for the sake of the drill they contain, but to secure and hold the child's interest in his study of number. For it is expected that many or all of them will suggest similar things that the child will wish to do with number, and thus they will suggest real "projects" in the true sense of that much-used and overused term.

These rhymes, number stories, and descriptions of games also furnish valuable material in silent reading, for they are so simple and interesting that the children will wish to read them without their being assigned for study.

Examples of rhymes, games, and connections to number stories are seen on the pages from the book that are shown on page 27.

We are sure you will agree with us that each of the above examples, with some editing of style and color, could be part of a current primary textbook series.

The passages and pages from these books remind us again that we need to go back to the tried and true ideas of connecting mathematics to the child's world by the use of rhymes, number stories, and simple games. At the same time, we need to stay focused on the developmental needs of primary children.

References

Stone, John C. *A Child's Book of Number*. Chicago: Benj. H. Sanborn & Company, 1924.

VanAmburgh, Della. *First Days in Number*. New York: Silver Burdett and Company, 1903.

Timeless Activities from a Well-Worn Book

by Christina Myren, Conejo Valley USD, cmyren@aol.com, and Fran Threewit, Kenwood Elementary School, fthreewit@comcast.net

CONCEPT: Number
SKILLS: Working with three addends, finding tens, adding nine to a single digit number, finding the missing addend
MATHEMATICS STANDARDS: Grade 1: NS 1.4, 2.0; Grade 2: NS 1.0, 2.0
GRADES: 1–2
MATERIALS: Chalkboard or paper, appropriate writing tool

BACKGROUND

As we perused the old textbooks referred to in the article *Going Back in Time* on pages 27–28 we found several timeless activities worthy of note for primary students. We have selected two activities from the book *A Child's Book of Numbers* that are appropriate for first and second graders. The first activity reinforces facts to 9 and the second emphasizes facts up to 18 with 9 as one of the addends.

DIRECTIONS

Because of their charm and clarity we are presenting the directions for both of the activities as they originally appeared in the 1924 textbook. The intention of the second activity is that it be taught at school and then played at home to continue the reinforcement of the facts.

We have included a parent letter worksheet on page 48 that you may use as a follow-up homework assignment for your students to complete with a family member. Please refer to the K–2 Student Problem on page 59 for further directions.

Reference

Stone, John C. *A Child's Book of Number*. Chicago: Benj. H. Sanborn & Company, 1924.

A GAME WITH NINE

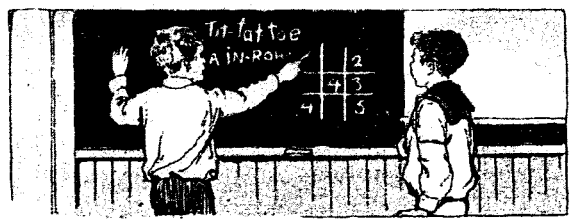
These children are playing a game. The one in the center says, "I am 9. Who will help me make 16?" Some child runs in the ring and says, "I am 7. I will help you make 16."

The child that helped make 16 then says, "I am 9. Who will help me make 14?" Some one runs in the ring and says, "I am 5. I will help you make 14."

The first child in the ring to help 9 make some number stays and asks a question. The one in the center is always 9. He never asks for a number larger than 18.

[53]

A HOME GAME FOR TWO



Jack and Billie have made up a number game. They call it "Tit-tat-toe; three-in-a-row." It takes two to play the game. They take turns in writing some number not larger than 9. The one that finishes a row of three numbers that make 10 wins one point. When all the spaces are filled, they draw new lines and begin another game.

Sometimes they agree to write numbers that will make some other number instead of 10.

See what these make:

5	3	2	5	7	4	6	2
3	4	6	2	1	4	4	8
7	8	7	8	7	7	5	5

[60]

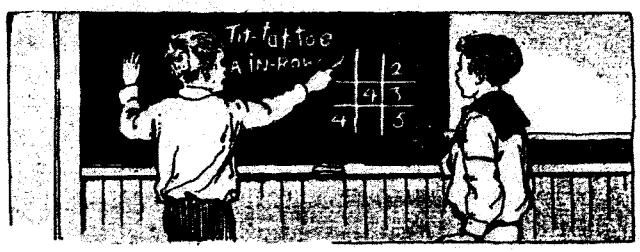
Dear Family,
Your child's homework tonight is to play the game that is explained below. The game comes from a child's textbook called *A Child's Book of Numbers* that was written in 1923.

As you play this game, ask questions to help your child make predictions. You may want to ask such questions as:

- ✔ What number might make sense here?
- ✔ How many more to make ten?
- ✔ What would happen if we could only use numbers 1–9, and not 0?
- ✔ How is this game like a game you have played before?

Please return the form below with your comments to school tomorrow.

A HOME GAME FOR TWO



Jack and Billie have made up a number game. They call it "Tit-tat-toe; three-in-a-row." It takes two to play the game. They take turns in writing some number not larger than 9. The one that finishes a row of three numbers that make 10 wins one point. When all the spaces are filled, they draw new lines and begin another game.

Sometimes they agree to write numbers that will make some other number instead of 10.

_____ and I played the game last night. We thought this game was like the _____ game that we played before.

Comments:_____

_____

_____

Student_____ Parent_____

Math Riddles: Helping Children Connect Words and Numbers

Lesson A:

Teacher. Today, class, we will practice counting groups of coins and finding different ways of making a certain amount of money. From the tray on your table, take one dime, two nickels, and four pennies. . . .

Lesson B:

Teacher (holding a lunch bag). I have something in the bag. *(Shakes the bag)*

Children. Money!

Teacher. Yes, I have some coins in the bag, but you'll have to figure out how much I have and exactly which coins I have. A girl named Jessi wrote the clues. Here's the first clue: "I have seven coins." Take coins from the tray on your table and show which ones I might have in the bag. Then I'll give another clue.

Lesson A is a good, hands-on mathematics activity. Lesson B is based on the solving and writing of mathematics riddles. Both offer potentially valuable learning experiences, but the second lesson is more likely to excite and engage children. *Principles and Standards for School Mathematics* (NCTM 2000) states, "In effective teaching, worthwhile mathematical tasks are used to introduce important mathematical ideas and to engage and challenge students intellectually. Well-chosen tasks can pique students' curiosity and draw them into mathematics" (p. 18).

As an elementary-level teacher of mathematics, I am constantly challenged to find ways to help students connect words, numbers, and concrete experiences. About ten years ago, I began trying to meet some of the needs of diverse groups of children by incorporating student-written riddles and puzzles in my classroom mathematics program.

Early on, I discovered that writing good riddles was not as easy as it first appeared, and that the activity sometimes left students frustrated and confused. Still, I was impressed by the effort that students put into the process and the excitement that resulted when they had the opportunity to solve riddles that their classmates had created.

Over the years, by trial and error, I have developed procedures that enable a diverse range of students to write interesting, solvable riddles that reinforce a variety of mathematics skills and concepts.

The process of creating new riddles provides an impetus for students to express themselves fully and clearly because they want other children to solve—or at least attempt to solve—their riddles. Also, children working together tend to give editorial feedback in a constructive way, with peer comments ranging from remarks about penmanship and spelling to more substantive discussions of riddle clues and solutions. *Principles and Standards for School Mathematics* (NCTM 2000) notes, "Teachers also need to provide students with assistance in writing about mathematical concepts. . . . Students will also need opportunities to check the clarity of their work with peers" (pp. 198–99).

The process of writing and solving riddles provides a rich context for learning new vocabulary

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Photograph by Carl M. Sherrill; all rights reserved

and phrases, from "It's a trapezoid" to "Three more black beans than white beans" to "There are equal numbers of dimes and pennies, and half as many nickels as dimes." Using this type of language in conjunction with concrete materials helps reinforce meaning. Also, because the vocabulary required for a particular type of riddle is somewhat limited, children with special needs in reading, as well as those who are English learners, find most of the activities accessible. *Principles and Standards* states, "Representing numbers with various physical materials should be a major part of mathematics instruction. . . . Having a concrete referent helps students develop understandings that are clearer and more easily shared" (pp. 33, 197; see **fig. 1**).

Guiding Student Riddle-Writers

Before children attempt to write original riddles of a given type, they need experience solving riddles that others have written. The teacher can create sample riddles or provide riddles that students in another class have written. For most groups, it is a good idea to select some riddles that will be easy for most children and other riddles that will provide a challenge for highly capable students.

To ensure that most of the riddles will fall within a reasonable range of difficulty, the teacher needs to limit the quantities used. For example, when I have second-grade students write riddles involving four kinds of beans, I ask that they use "from five to twenty beans, using two or three types of beans." A child might start with five white beans, five red beans, and two pintos—or twelve beans in all. With third and fourth graders, I might allow "from fifteen to thirty-five beans of three or four kinds."

You may want to provide an answer key for riddles that you have written or those from other sources, so that students can check their own work. Make sure that students record the solution to each riddle that they write for their classmates. One of the simplest methods is to have the student write or draw the solution at the bottom of the page, then fold it several times to cover the answer. The riddle

Second graders Derek and Jared worked together to create a riddle using three kinds of beans.

Bean Riddle A by Derek and Jared
We have 20 beans in all.
We have 8 red beans
We have 2 more reds than Blacks
We have the same amount of black beans and white beans

Second grader Kayla wrote a good, solvable riddle, but did not use the class chart to proofread her spelling.

Coin Riddle A by Kayla

I have 18 cens.
I have 5 coins.
I have more pennys thne Dims.

Ian's riddle provided an interesting challenge for most of his third-grade classmates.

Coin Riddle A

1. I have the same number of quarters as nickels.
2. I have eight coins.
3. I have $1.00
4. I don't have any pennies.

may also be written directly on an envelope with the solution inside.

After students have created their own riddles, they need a system for exchanging the riddles with their classmates. Options include the following:

- Place all riddles on a table or other common area. Students select riddles to solve and take them back to their own work space. (This is simple but can result in a lot of movement and noise.)
- Have groups work on a small subset of the riddles, perhaps placing them in a tray in the center of a table or group of desks.
- Have students work on copies that you have made, writing directly on the copies.
- Make the riddles part of a learning center or independent work station.

To allow children to work at varying rates, consider assigning some of the activities in a menu format, with required items and choices. This can work especially well in academically diverse classes.

A Sample Lesson: Coin Riddles

In the process of solving coin riddles, students refer to actual coins as concrete representations of their thinking. They extend their command of the vocabulary of comparison and practice finding the values of collections of coins; see **figs. 2** and **3.**

The following materials will be needed at various times during the coin-riddle activities:

- Coins (real or play): About four quarters and six dimes, six nickels, and six pennies for each pair of students
- Trays, plates, or shallow dishes
- Copies for each student of the worksheet "Coin Riddles" (see **fig. 4**)
- Overhead transparency or chart of "Creating Coin Riddles" (see **fig. 5**)
- 8 1/2-by-11 lined paper, one sheet per student
- Transparent overhead coins (optional)

Make a transparency of "Creating Coin Riddles," or copy it on large chart paper or on the board. In two paper bags or envelopes, place coins to indicate the solutions for the riddle examples. Label these "Jessi," containing one dime, two nickels, and four pennies; and "Michael," with two dimes, one nickel, and five pennies.

Coin riddles

Kayla's Riddle
I have 18¢.
I have 5 coins.
I have more pennies than dimes.

Nick's Riddle
I have 4 coins.
My coins are worth 30¢.
I have no pennies.

Aaron's Riddle
I have 7 coins worth 29¢.
I have twice as many pennies as dimes.

Danny's Riddle
I have 6 coins.
My coins are worth 65¢.
I have more dimes than nickels.
I don't have any pennies.

Alejandra's Riddle
I have 6 coins.
My coins are worth 62¢.
I have the same number of each coin.
I have three kinds of coins.

Mary Ann's Riddle
I have 8 coins.
My coins are worth 45¢.
I have the same number of nickels, dimes, and quarters.

Brady's Riddle
I have 35¢.
I don't have any dimes.
I have 7 coins.
They are not all nickels.

Jennifer's Riddle
I have 9 coins worth 96¢.
There are more dimes than quarters.
There are more quarters than pennies.
There is the same number of dimes and nickels.

Alyssa's Riddle
I have 40¢.
I have no quarters.
I have 11 coins.

Julian's Riddle
I have 87¢.
I have 7 coins.
I have more quarters than nickels.
I have 1 more penny than what a penny is worth.

Curtis's Riddle
I have 10 coins.
My coins are worth 61¢.
I have twice as many nickels as dimes.
I have no quarters.

Brooke's Riddle
I have 11 coins.
My coins are worth 99¢.
I have the same number of pennies and dimes.

Coin Riddle Solutions

Most appropriate for students in grades 2 and 3:

Kayla: 1 dime, 1 nickel, 3 pennies
Nick: 2 dimes, 2 nickels
Aaron: 2 dimes, 1 nickel, 4 pennies
Danny: 1 quarter, 3 dimes, 2 nickels
Alejandra: 2 quarters, 2 nickels, 2 pennies
Mary Ann: 1 quarter, 1 dime, 1 nickel, 5 pennies
Brady: 1 quarter, 1 nickel, 5 pennies

Most appropriate for students in grades 3 and 4:

Jennifer: 2 quarters, 3 dimes, 3 nickels, 1 penny
Alyssa: 1 dime, 5 nickels, 5 pennies
Julian: 3 quarters, 2 nickels, 2 pennies
Curtis: 3 dimes, 6 nickels, 1 penny
Brooke: 2 quarters, 4 dimes, 1 nickel, 4 pennies

Creating coin riddles

You need:

Pennies, nickels, dimes, and quarters

Paper and pencil

Directions:

- Write a title and your name.
- Select from ___ to ___ coins worth up to ___.
- Write three or more clues about your coins.
- Be sure to tell *how many* coins you have and *how much* they are worth altogether.
- These words may come in handy:

worth	*cents (¢)*
twice as many	*half as many*
three times	*money*
same number	*equal groups*
more than	*fewer than*
total	*altogether*

- At the bottom of the paper, draw the coins, or show your answer with words and numbers.
- Fold the paper several times to cover the answer.
- Ask another student to solve your riddle. Make changes if necessary.

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Introducing coin riddles

Tell children that they will be solving coin riddles that other students have written, then writing riddles of their own. Distribute the coins.

Hold up the bag or envelope labeled "Jessi." Explain that it contains several coins, which may include pennies, nickels, dimes, and quarters, and that a student named Jessi has written a riddle about the coins. Distribute the trays of coins and read Jessi's riddle:

I have 7 coins.
I have twice as many pennies as nickels.
I have only one dime.
I have no quarters.
I have 24¢.

Ask students to discover possible solutions as you reveal one clue at a time. Pause occasionally to ask questions, such as "Could there be two pennies?" and "Could there be as much as seventy-five cents?" Ask whether these riddles would be solvable if fewer clues were given: "Which clues are necessary and which are merely helpful?"

When you have exposed all the clues, ask stu-

Third grader Reyna wrote a coin riddle in her first language, with the help of a Spanish vocabulary chart.

Adivinaza de Monedas
por Reyna

Tengo 8 monedas.

Mis monedas valen 63¢

Tengo el mismo numero
de pennies y dimes.

Tengo una quarters.

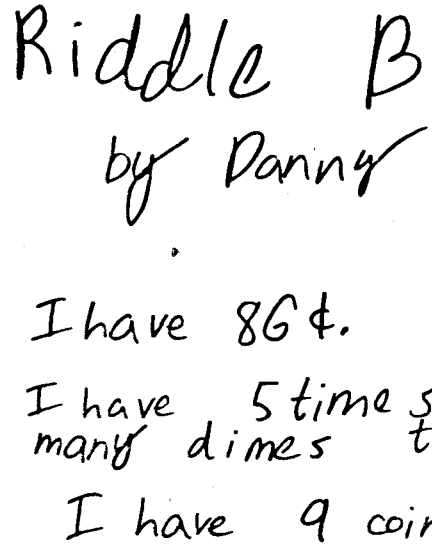
dents to share their solutions and their reasoning. Then open the bag or envelope to reveal "Jessi's" coins. Repeat the process for Michael's riddle:

> I have 30¢.
> I have one nickel.
> I have three more pennies than dimes.
> There are 8 coins in all.

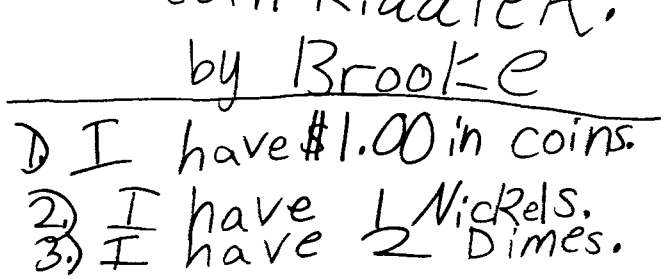
Before distributing the "Coin Riddles" worksheet (see **fig. 4**), demonstrate ways of recording solutions. You may prefer that students draw each collection of coins, write the solutions using words and numbers, or both.

Explain which riddles you would like every child to attempt, and give options for those who finish early. Depending on your time constraints, you may want to continue the activity during another session.

Danny's riddle was unsolvable. He did not count his collection of coins correctly; also, he misunderstood the phrase "five times," intending to indicate "five more dimes than nickels."



Brooke's first coin riddle had many solutions.



Creating coin riddles

Based on how your students did during the first activity, decide on a maximum number of coins and a maximum total value to use in creating new riddles. Write these limits on the transparency "Creating Coin Riddles" (see **fig. 5**).

Display the transparency on the overhead, or use the chart you have prepared. Select a set of coins and draw them schematically, or use clear overhead coins. Following the directions on the transparency, write a title and—with student input—write several clues on the board or on chart paper. Discuss the riddle briefly, asking whether it seems challenging enough and how it might be made clearer or more interesting.

Ask students to write their own riddles, following the format you have modeled (see **fig. 6**). At your discretion, have students work in pairs.

Solving classmate-written riddles

Select a riddle written during the previous session. Write one clue at a time on the board or overhead, having volunteers propose possible solutions as you go along.

Have students share their completed riddles with one another. During this session you may also ask that students edit previously written riddles or write additional riddles (see **figs. 7** and **8**).

A Final Word

Judging from my own classroom experience, and that of a number of my colleagues, having students create and solve mathematics riddles has many benefits. The activities—

- integrate mathematics, reading, and writing;
- address many of the NCTM Standards;
- allow children to engage in algebraic thinking in a hands-on context; and
- are appropriate for diverse ability levels and multilingual classrooms.

Working with mathematics riddles appears to boost students' strategic competence and "the ability to formulate mathematical problems, represent them, and solve them," as Kilpatrick (2001) notes. "Becoming strategically competent involves an avoidance of 'number grabbing' methods . . . in favor of methods that generate problem models" (p. 124).

Just as important, writing and solving mathematics riddles is also a lot of fun—for students and teachers alike!

Bibliography

Burns, Marilyn, and Bonnie Tank. *A Collection of Math Lessons from Grade 1 through 3.* White Plains, N.Y.: Cuisenaire Company of America, 1988.

Kilpatrick, Jeremy, Jane Swafford, and Bradford Findell, eds. *Adding It Up: Helping Children Learn Mathematics.* Washington, D.C.: National Academy Press, 2001.

National Council of Teachers of Mathematics (NCTM). *Principles and Standards for School Mathematics.* Reston, Va.: NCTM, 2000.

Sherrill, Carl M. *RiddleMath: Using Student-Written Riddles to Build Mathematical Power.* Calistoga, Calif.: Morning River Publications, 2001. ▲

Discussion for **Ants, Ants, Ants**

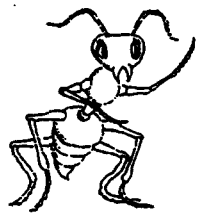
Directions to Children

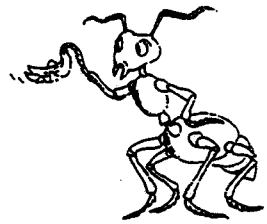
Six ants live in the six houses, one in each. They would love to visit each other, but first they have to build roads between the houses so that each ant has a road that goes directly to the house of each other ant. Although you will see all kinds of interesting paths that can be drawn between any two houses, you should draw in only one, the straightest possible. How many roads are needed to connect each pair of ants?

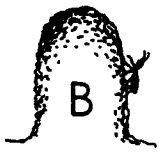
Materials

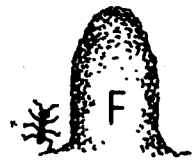
Pencil, the problem sheet, and a ruler.

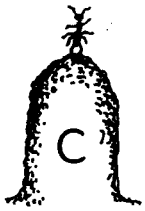
Ants, Ants, Ants

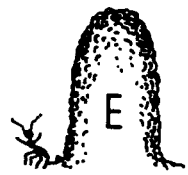












Nine Ways

How do we help floundering students who lack basic math concepts?

Marilyn Burns

Paul, a 4th grader, was struggling to learn multiplication. Paul's teacher was concerned that he typically worked very slowly in math and "didn't get much done." I agreed to see whether I could figure out the nature of Paul's difficulty. Here's how our conversation began:

MARILYN: Can you tell me something you know about multiplication?

PAUL: [*Thinks, then responds*] 6 x 8 is 48.

MARILYN: Do you know how much 6 x 9 is?

PAUL: I don't know that one. I didn't learn it yet.

MARILYN: Can you figure it out some way?

PAUL: [*Sits silently for a moment and then shakes his head.*]

MARILYN: How did you learn 6 x 8?

PAUL: [*Brightens and grins*] It's easy—goin' fishing, got no bait, 6 x 8 is 48.

As I talked with Paul, I found out that multiplication was a mystery to him. Because of his weak foundation of understanding, he was falling behind his classmates, who were multiplying problems like 683 x 4. Before he could begin to tackle such problems, Paul needed to understand the concept of multiplication and how it connects to addition.

Paul wasn't the only student in this class who was floundering. Through talking with teachers and drawing on my own teaching experience, I've realized that in every class a handful of students are at serious risk of failure in mathematics and aren't being adequately served by the instruction offered. What should we do for such students?

Grappling with Interventions

My exchange with Paul reminded me of three issues that are essential to teaching mathematics:

- It's important to help students make connections among mathematical ideas so they do not see these ideas as disconnected facts. (Paul saw each multiplication fact as a separate piece of information to memorize.)
- It's important to build students' new understandings on the foundation of their prior learning. (Paul did not make use of what he knew about addition to figure products.)
- It's important to remember that students' correct answers, without accompanying explanations of how they reason, are not sufficient for judging mathematical understanding. (Paul's initial correct answer about the product of 6 x 8 masked his lack of deeper understanding.)

For many years, my professional focus has been on finding ways to more effectively teach arithmetic, the cornerstone of elementary mathematics. Along with teaching students basic numerical concepts and skills, instruction in number and operations prepares them for algebra. I've developed lessons that help students make sense of number

to Catch Kids Up

PHOTOS BY SUSAN BARCLAY © MATH SOLUTIONS

and operations with attention to three important elements—computation, number sense, and problem solving. My intent has been to avoid the "yours is not to question why, just invert and multiply" approach and to create lessons that are accessible to all students and that teach skills in the context of deeper understanding. Of course, even well-planned lessons will require differentiated instruction, and much of the differentiation needed can happen within regular classroom instruction.

But students like Paul present a greater challenge. Many are already at least a year behind and lack the foundation of mathematical understanding on which to build new learning. They may have multiple misconceptions that hamper progress. They have experienced failure and lack confidence.

Such students not only demand more time and attention, but they also need supplemental instruction that differs from the regular program and is designed specifically for their success. I've recently shifted my professional focus to thinking about the kind of instruction we need to serve students like Paul. My colleagues and I have developed lessons that provide effective interventions for teaching number and operations to those far behind. We've grappled with how to provide instruction that is engaging, offers scaffolded instruction in bite-sized learning experiences, is paced for students' success, provides the practice students need to cement fragile understanding and skills, and bolsters students' mathematical foundations along with their confidence.

In developing intervention instruction, I have reaffirmed my longtime commitment to helping students learn facts and skills—the basics of arithmetic. But I've also reaffirmed that "the basics" of number and operations for all students, including those who struggle, must address all three aspects of numerical proficiency—computation, number sense, and problem solving. Only when the basics include understanding as well as skill proficiency will all students learn what they need for their continued success.

Extra help for struggling learners must be more than additional practice.

Essential Strategies

I have found the following nine strategies to be essential to successful intervention instruction for struggling math learners. Most of these strategies will need to be applied in a supplementary setting, but teachers can use some of them in large-group instruction.

1. Determine and Scaffold the Essential Mathematics Content

Determining the essential mathematics content is like peeling an onion—we

must identify those concepts and skills we want students to learn and discard what is extraneous. Only then can teachers scaffold this content, organizing it into manageable chunks and sequencing these chunks for learning.

For Paul to multiply 683 x 4, for example, he needs a collection of certain skills. He must know the basic multiplication facts. He needs an understanding of place value that allows him to think about 683 as 600 + 80 + 3. He needs to be able to apply the distributive property to figure and then combine partial products. For this particular problem, he needs to be able to multiply 4 by 3 (one of the basic facts); 4 by 80 (or 8 x 10, a multiple of 10); 4 and by 600 (or 6 x 100, a multiple of a power of 10). To master multidigit multiplication, Paul must be able to combine these skills with ease. Thus, lesson planning must ensure that each skill is explicitly taught and practiced.

2. Pace Lessons Carefully

We've all seen the look in students' eyes when they get lost in math class. When it appears, ideally teachers should stop, deal with the confusion, and move on only when all students are ready. Yet curriculum demands keep teachers pressing forward, even when some students lag behind. Students who struggle typically need more time to grapple with new ideas and practice new skills in order to internalize them. Many of these students need to unlearn before they relearn.

3. Build in a Routine of Support

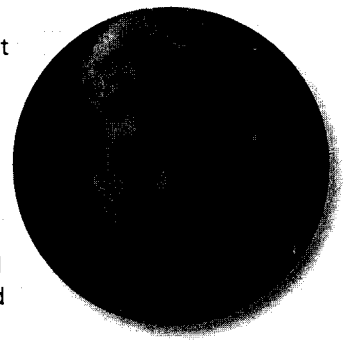
Students are quick to reveal when a lesson hasn't been scaffolded sufficiently or paced slowly enough: As soon as you give an assignment, hands shoot up for help. Avoid this scenario by building in a routine of support to reinforce concepts and skills before students are expected to complete independent work. I have found a four-stage process helpful for supporting students.

In the first stage, the teacher models what students are expected to learn and records the appropriate mathematical representation on the board. For example, to simultaneously give students practice multiplying and experience applying the associative and commutative properties, we present them with problems that involve multiplying three one-digit factors. An appropriate first problem is 2 x 3 x 4. The teacher thinks aloud to demonstrate three ways of working this problem. He or she might say,

My "Aha!" Moment

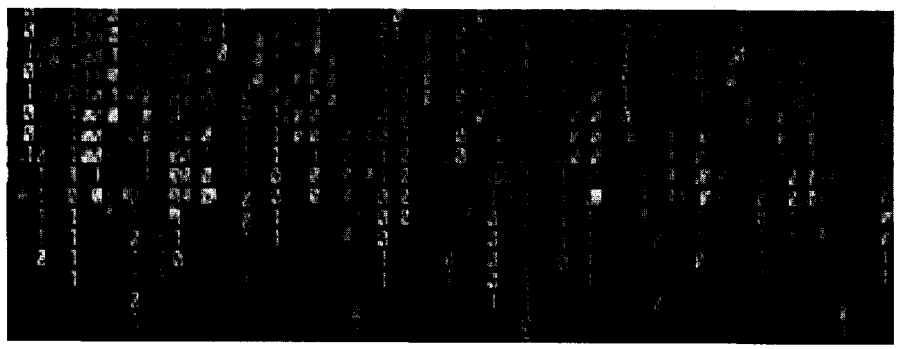
Mary M. Lindquist, Professor of Mathematics Education, Columbus College, Georgia. Winner of the National Council of Teachers of Mathematics Lifetime Achievement Award.

My "aha" moment came long after I had finished a masters in mathematics, taught mathematics in secondary school and college, and completed a doctorate in mathematics education. Although I enjoyed the rigor of learning and applying rules, mathematics was more like a puzzle than an elegant body of knowledge.



Many years of work on a mathematics program for elementary schools led to that moment. I realized that mathematics was more than rules—even the beginnings of mathematics were interesting. Working with elementary students and teachers, I saw that students could make sense of basic mathematical concepts and procedures, and teachers could help them do so. The teachers also posed problems to move students forward, gently let them struggle, and valued their approaches. What a contrast to how I had taught and learned mathematics!

With vivid memories of a number-theory course in which I memorized the proofs to 40 theorems for the final exam, I cautiously began teaching a number-theory course for prospective middle school teachers. My aha moment with these students was a semester long. We investigated number-theory ideas, I made sense of what I had memorized, and my students learned along with me. My teaching was changed forever.



I could start by multiplying 2 x 3 to get 6, and then multiply 6 x 4 to get 24. Or I could first multiply 2 x 4, and then multiply 8 x 3, which gives 24 again. Or I could do 3 x 4, and then 12 x 2. All three ways produce the same product of 24.

As the teacher describes these operations, he or she could write on the board:

2 x 3 x 4	2 x 3 x 4	2 x 3 x 4
6 x 4 = 24	8 x 3 = 24	2 x 12 = 24

It's important to point out that solving a problem in more than one way is a good strategy for checking your answer.

In the second stage, the teacher models again with a similar problem—such as 2 x 4 x 5—but this time elicits responses from students. For example, the teacher might ask, "Which two factors might you multiply first? What is the product of those two factors? What should we multiply next? What is another way to start?" Asking such questions allows the teacher to reinforce correct mathematical vocabulary. As students respond, the teacher again records different ways to solve the problem on the board.

During the third stage, the teacher presents a similar problem—for example, 2 x 3 x 5. After taking a moment to think on their own, students work in pairs to solve the problem in three different ways, recording their work. As students report back to the class, the teacher writes on the board and discusses their problem-solving choices with the group.

In the fourth stage, students work independently, referring to the work recorded on the board if needed. This routine both sets an expectation for student involvement and gives learners the direction and support they need to be successful.

Students are quick to reveal when a lesson hasn't been scaffolded sufficiently or paced slowly enough: As soon as you give an assignment, hands shoot up for help.

4. Foster Student Interaction

We know something best once we've taught it. Teaching entails communicating ideas coherently, which requires the one teaching to formulate, reflect on, and clarify those ideas—all processes that support learning. Giving students opportunities to voice their ideas and explain them to others helps extend and cement their learning.

Thus, to strengthen the math understandings of students who lag behind, make student interaction an integral part of instruction. You might implement the *think-pair-share* strategy, also called *turn and talk.* Students are first asked to collect their thoughts on their own, and then talk with a partner; finally, students share their ideas with the whole group. Maximizing students' opportunities to express their math knowledge verbally is particularly valuable for students who are developing English language skills.

5. Make Connections Explicit

Students who need intervention instruction typically fail to look for relationships or make connections among mathematical ideas on their own. They need help building new learning on what they already know. For example, Paul needed explicit instruction to understand how thinking about 6 x 8 could give him access to the solution for 6 x 9. He needed to connect the meaning of multiplication to what he already knew about addition (that 6 x 8 can be thought of as combining 6 groups of 8). He needed time and practice to cement this understanding for all multiplication problems. He would benefit from investigating six groups of other numbers—6 x 2, 6 x 3, and so on—and looking at the numerical pattern of these products. Teachers need to provide many experiences like these, carefully sequenced and paced, to prepare students like Paul to grasp ideas like how 6 x 9 connects to 6 x 8.

6. Encourage Mental Calculations

Calculating mentally builds students' ability to reason and fosters their number sense. Once students have a foundational understanding of multiplication, it's key for them to learn the basic multiplication facts—but their experience with multiplying mentally should expand beyond these basics. For example, students should investigate patterns that help them mentally multiply any number by a power of 10. I am concerned when I see a student multiply 18 x 10, for example, by reaching for a pencil and writing:

$$\begin{array}{r} 18 \\ \times\ 10 \\ \hline 00 \\ 18 \\ \hline 180 \end{array}$$

Revisiting students' prior work with multiplying three factors can help develop their skills with multiplying mentally. Helping students judge which way is most efficient to multiply three

factors, depending on the numbers at hand, deepens their understanding. For example, to multiply 2 x 9 x 5, students have the following options:

2 x 9 x 5	2 x 9 x 5	2 x 9 x 5
18 x 5 = 90	10 x 9 = 90	2 x 45 = 90

Guiding students to check for factors that produce a product of 10 helps build the tools they need to reason mathematically.

When students calculate mentally, they can estimate before they solve problems so that they can judge whether the answer they arrive at makes sense. For example, to estimate the product of 683 x 4, students could figure out the answer to 700 x 4. You can help students multiply 700 x 4 mentally by building on their prior experience changing three-factor problems to two-factor problems: Now they can change a two-factor problem—700 x 4 — into a three-factor problem that includes a power of 10 — 7 x 100 x 4. Encourage students to multiply by the power of 10 last for easiest computing.

7. Help Students Use Written Calculations to Track Thinking

Students should be able to multiply 700 x 4 in their heads, but they'll need pencil and paper to multiply 683 x 4. As students learn and practice procedures for calculating, their calculating with paper and pencil should be clearly rooted in an understanding of math concepts. Help students see paper and pencil as a tool for keeping track of how they think. For example, to multiply 14 x 6 in their heads, students can first multiply 10 x 6 to get 60, then 4 x 6 to get 24, and then combine the two partial products, 60 and 24. To keep track of the partial products, they might write:

14 x 6
10 x 6 = 60
4 x 6 = 24
60 + 24 = 84

They can also reason and calculate this way for problems that involve multiplying by three-digit numbers, like 683 x 4.

8. Provide Practice

Struggling math students typically need a great deal of practice. It's essential that practice be directly connected to students' immediate learning experiences. Choose practice problems that support the elements of your scaffolded instruction, always promoting understanding as well as skills. I recommend giving assignments through the four-stage support routine, allowing for a gradual release to independent work.

Games can be another effective way to stimulate student practice. For example, a game like *Pathways* (see Figure 1 for a sample game board and instructions) gives students practice with multiplication. Students hone multiplication skills by marking boxes on the board that share a common side and that each contain a product of two designated factors.

9. Build In Vocabulary Instruction

The meanings of words in math—for example, *even*, *odd*, *product*, and *factor*—often differ from their use in common language. Many students needing math intervention have weak mathematical vocabularies. It's key that students develop a firm understanding of mathematical concepts before learning new vocabulary, so that they can anchor terminology in their understanding. We should explicitly teach vocabulary in the context of a learning activity and then use it consistently. A math vocabulary chart can help keep both teacher and students focused on the importance of accurately using math terms.

When Should We Offer Intervention?

There is no one answer to when teachers should provide intervention instruction on a topic a particular student is struggling with. Three

different timing scenarios suggest themselves, each with pluses and caveats.

While the Class Is Studying the Topic
Extra help for struggling learners must be more than additional practice on the topic the class is working on. We must also provide comprehensive instruction geared to repairing the student's shaky foundation of understanding.

■ *The plus*: Intervening at this time may give students the support they need to keep up with the class.

■ *The caveat*: Students may have a serious lack of background that requires reaching back to mathematical concepts taught in previous years. The focus should be on the underlying math, not on class assignments. For example, while others are learning multidigit multiplication, floundering students may need experiences to help them learn basic underlying concepts, such as that 5 x 9 can be interpreted as five groups of nine.

Before the Class Studies the Topic
Suppose the class is studying multiplication but will begin a unit on fractions within a month, first by cutting out individual fraction kits. It would be extremely effective for at-risk students to have the fraction kit experience before the others, and then to experience it again with the class.

■ *The plus*: We prepare students so they can learn *with* their classmates.

■ *The caveat*: With this approach, struggling students are studying two different and unrelated mathematics topics at the same time.

After the Class Has Studied the Topic.
This approach offers learners a repeat experience, such as during summer school, with a math area that initially challenged them.

■ *The plus*: Students get a fresh start in a new situation.

■ *The caveat*: Waiting until after the rest of the class has studied a topic to intervene can compound a student's confusion and failure during regular class instruction.

Many students needing math intervention have weak mathematical vocabularies.

FIGURE 1. Pathways Multiplication Game

72	36	49	88	54
84	77	96	132	56
63	81	48	108	121
66	99	144	64	42

6 7 8 9 **11** 12

Player 1 chooses two numbers from those listed (in the game shown here, 6 and 11) and circles the product of those two numbers on the board with his or her color of marker.

Player 2 changes just one of the numbers to another from the list (for example, changing 6 to 9, so the factors are now 9 and 11) and circles the product with a second color.

Player 1 might now change the 11 to another 9 and circle 81 on the board. Play continues until one player has completed a continuous pathway from one side to the other by circling boxes that share a common side or corner. To support intervention students, have pairs play against pairs.

How My Teaching Has Changed

Developing intervention lessons for at-risk students has not only been an all-consuming professional focus for me in recent years, but has also reinforced my belief that instruction—for all students and especially for at-risk students—must emphasize understanding, sense making, and skills.

Thinking about how to serve students like Paul has contributed to changing my instructional practice. I am now much more intentional about creating and teaching lessons that help intervention students catch up and keep up, particularly scaffolding the mathematical content to introduce concepts and skills through a routine of support. Such careful scaffolding may not be necessary for students who learn mathematics easily, who know to look for connections, and who have mathematical intuition. But it is crucial for students at risk of failure who can't repair their math foundations on their own. EL

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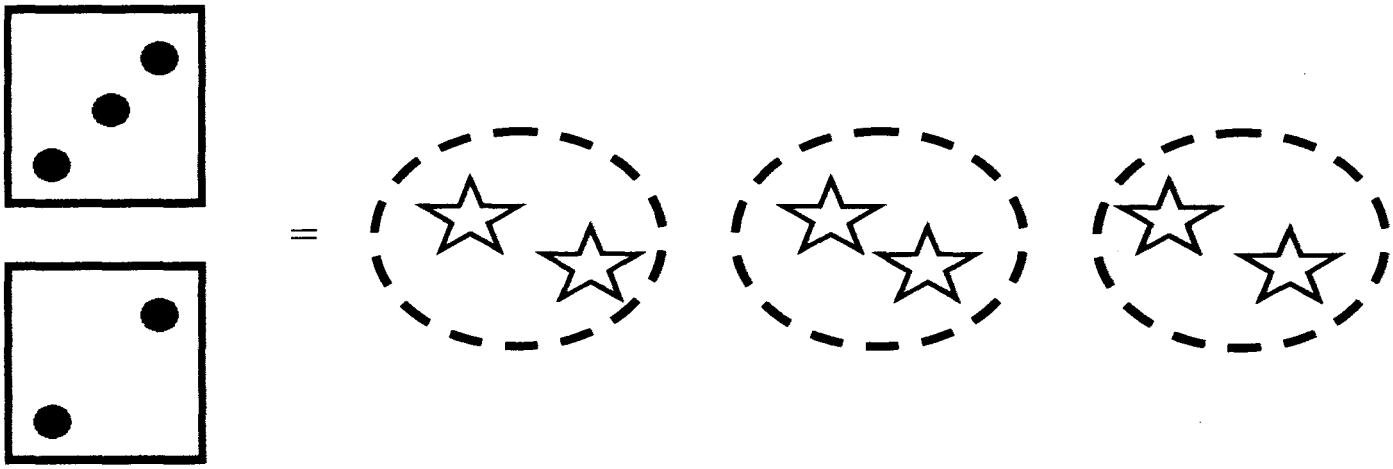
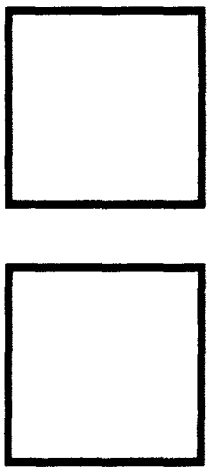
Circles and Stars

Begin by rolling a die. Take the number that comes up and draw that many circles after the equal sign. Have your partner roll the die. Take the number that comes up and draw that many stars in each of your circle. Use the circles and stars to fill in the blanks. Do all seven problems. Add up the total number of stars. The team that has the most stars wins.



3 circles
2 stars in each circle
6 stars in all

3 x 2 = 6



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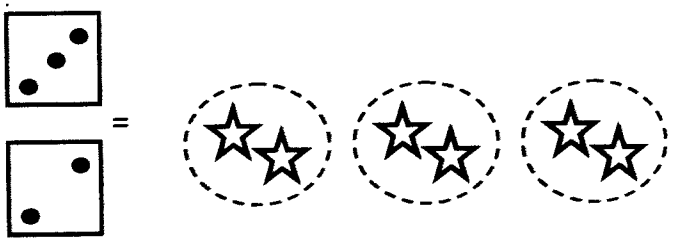

3 circles
2 stars in each circle
6 stars in all
3 x 2 = 6

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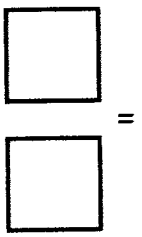
TOTAL STARS



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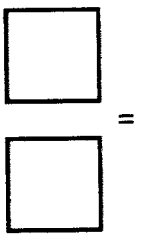

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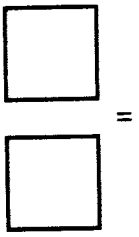

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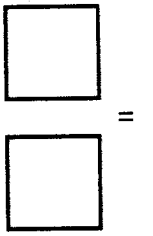

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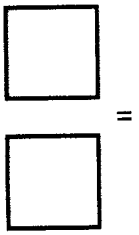

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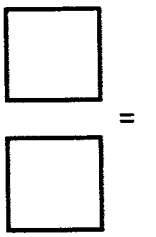

____ circles
____ stars in each circle
____ stars in all

____ x ____ = ____

DICE

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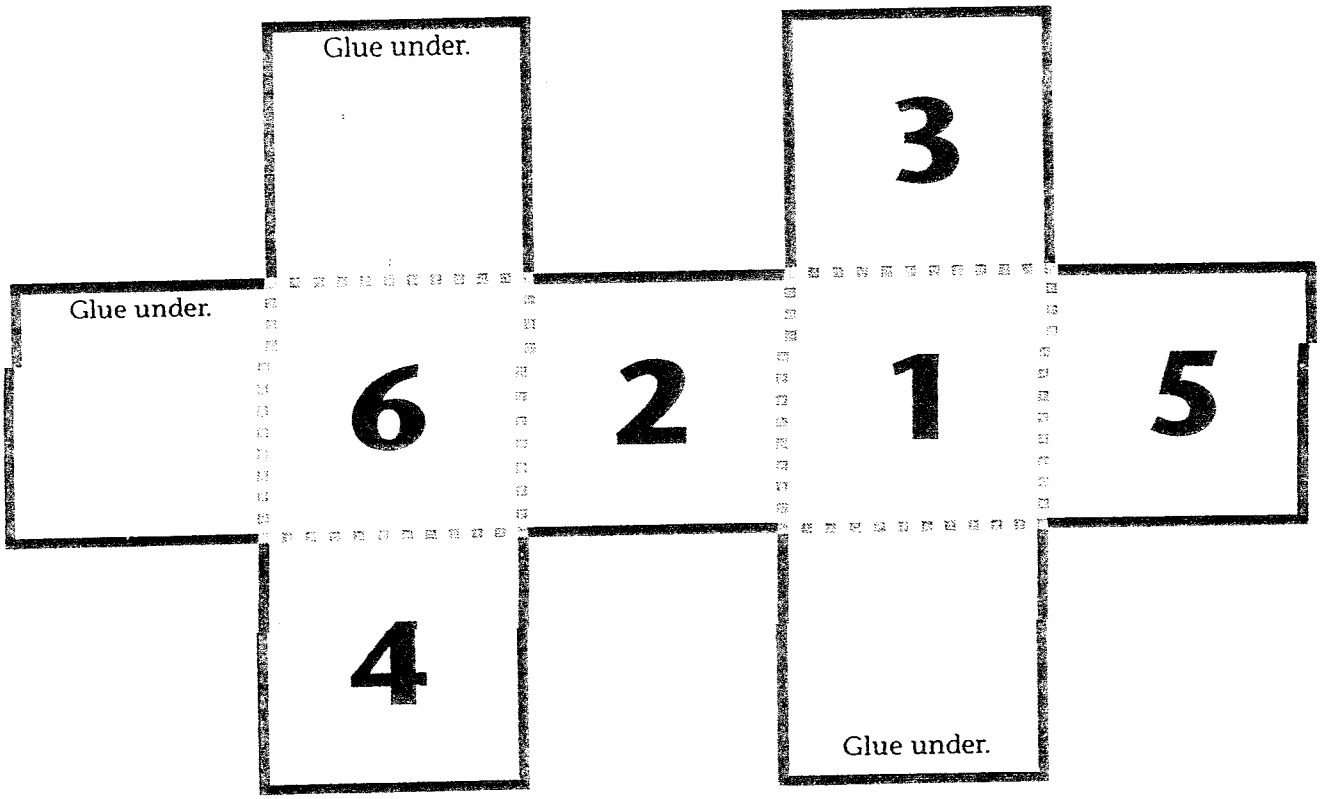

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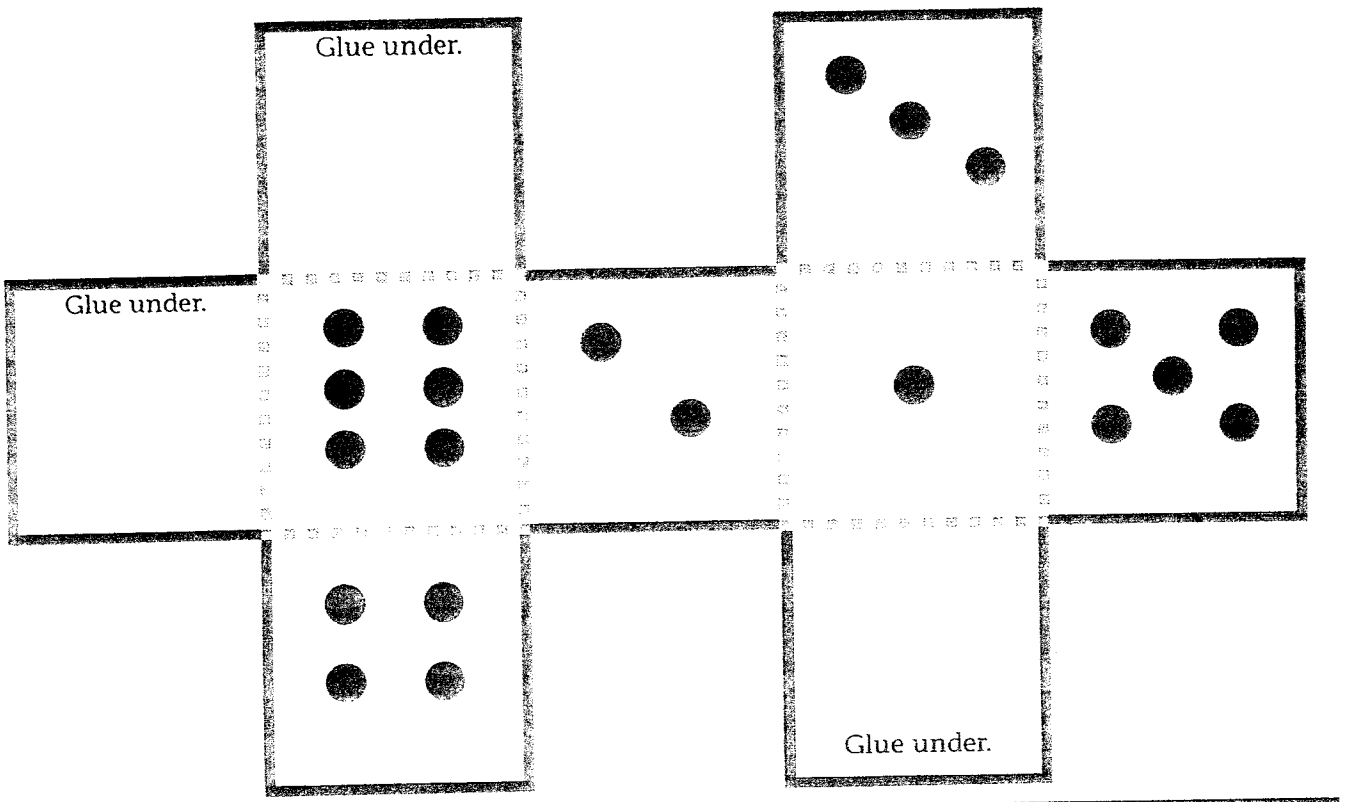
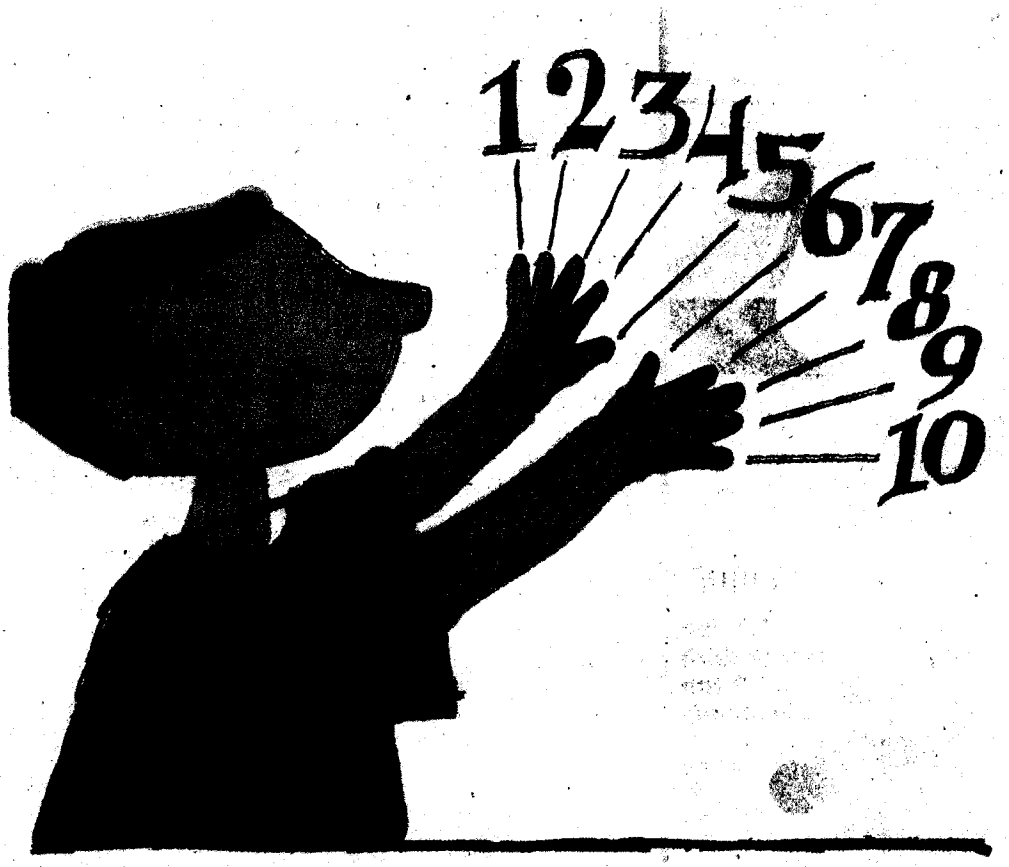


'Fun With Nines'



STEPHEN SEDAM *Los Angeles Times*

By MARILYN GOULD
Special to The Times

"HEY, Michael. Want to play catch?" Poppy poked his mitt with his fist.

"Can't. I have math homework."

"Are you struggling with math again?" Poppy asked. "What's the problem this time?"

"Multiplying nines." Michael shook his head in disgust.

"Hmm," Poppy rubbed his chin. "You really need to memorize the tables. But here's a trick to get you started.

"What's that?" Michael asked.

"A hand calculator. I'll show you. Put your hands in front of you."

Poppy made a face. "Maybe you need to wash them first," he said. "Calculators work better when they're clean."

Michael went to the kitchen sink and scrubbed his hands.

"That's better," said Poppy. "Now put your clean hands in front of you with your thumbs next to each other. Starting with the left hand, your pinkie will be 1, ring finger, 2; middle finger, 3; pointer, 4; and thumb, 5."

Michael wished he had a marker.

"Now the right hand," Poppy continued. "Number 6, thumb; number 7, pointer; number 8, middle finger; number 9, ring finger, and pinkie, 10. There's your hand calculator for multiplying nines."

"OK, I'm plugged in," laughed Michael. "How does it work?"

"Let's start with 6 times 9," Poppy said. "Put the finger 6 down."

"That's the right-hand thumb, right? asked Michael.

"Right. Now how many fingers are there to the left of 6?"

Michael counted the fingers on his left hand. "Five!" he answered.

"Good. And how many fingers are there to the right of 6?"

"Four?"

Poppy smiled. "Right! And that's your answer: 54. 6 x 9 is 54."

"I'm not sure I understand it," Michael said.

"Try 8 x 9. Put finger 8 down."

Poppy watched as Michael lowered the middle finger on his right hand.

"How many fingers are there to the left of 8?" he asked.

"I get it," Michael said excitedly. "There are 7 fingers to the left of my middle finger and 2 on the right. That means 8 x 9 is 72."

"That's it!" Poppy said proudly.

"Thank you," Michael gave Poppy a hug.

"You're welcome, and I love you too," Poppy said. "Now can we play ball?"

This story will be on The Times' website at *latimes.com/kids*. Look for the Kids' Reading Room on Mondays through Fridays in Comics Plus.

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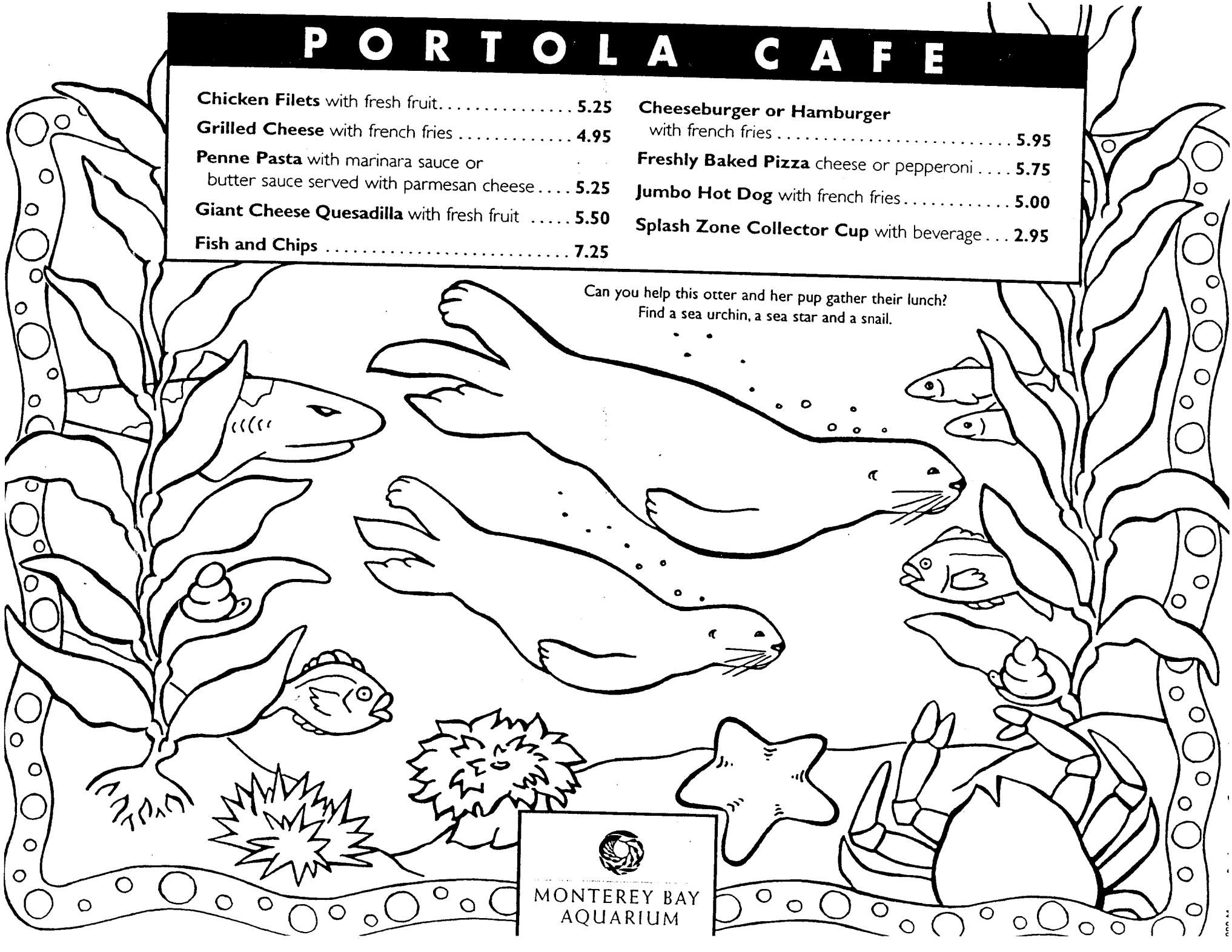
Splash Zone Collector Cup with beverage . . . **2.95**

Can you help this otter and her pup gather their lunch?
Find a sea urchin, a sea star and a snail.



MONTEREY BAY AQUARIUM

Title: Multiplication Rap- When It Comes to 7

Subject area: Math

Grade levels: 3-5

Key CA Content Standards Addressed:
2.2 Memorize to automaticity the multiplication table for numbers between 1 and 10.

ELD stage of students: Intermediate

Key ELD Standards Addressed: Make oneself understood when speaking by using consistent standard English grammatical forms and sounds.

Teacher objectives: Teacher will check students' automaticity (memory) of the multiplication table.

Learner objectives: Students will be able to retain multiplication facts from memory after learning rap and making up their own raps and rhymes.

Materials: Paper, pencil, chalk

Modifications for special needs, including ELL and exceptionalities: Provide student with a multiplication table, allow student to choose a multiplication table for the rap and help student brainstorm rhyming words

Procedure:
(5 minutes) *Initiating activity*: Teacher will list numbers vertically on the board: 7, 14, 21, 28, 35, 42, 49, 56, 63, 70, 77, 84. Ask the students how the numbers are related.

(15 minutes) *Modeling of process and/or guided practice*: Guide students to look for words that rhyme with those numbers. Lead those words into a rap (poem). After class has helped write the rap, have the students perform the rap in front of the class. Here is the one we prepared for the 7's:

> *"When it comes to 7, you know that it is true*
> *When it comes to 7, we know what to do!*
> *7, 14, 21*
> *Now come on, join in, you know what to do!*
> *49,56, 63*
> *We're almost done, so stick with me*
> *70, 77, 84*
> *Look we've made it out the door!"*

(15 minutes) *Independent practice*: Divide students into pairs to make up their own raps to other multiplication tables.

(10 minutes) *Closure*: Allow students to recite their rap in front of the class.

Assessment of student performance: Check for automaticity when students recite rap orally for memory.

Logical follow-up lesson: Have students write their poems for posting around the classroom.

Title: Two Dice and Four Operations: A Mathematical Game for the Classroom

Subject area: Math

Grade levels: 4-5

Key CA Content Standards Addressed:
Number Sense 2.0 Students extend their use and understanding of whole numbers to the addition and subtraction of simple decimals.
Statistics, Data Analysis, and Probability 2.0 Students make predictions for simple probability situations.
Mathematical Reasoning 1.0 Students make decisions about how to approach problems.
Mathematical Reasoning 3.0 Students move beyond a particular problem by generalizing to other situations.

ELD stage of students: Intermediate ELD level

Key ELD Standards Addressed: Participate in social conversations with peers and adults on familiar topics by asking and answering questions and soliciting information

Teacher objectives: Teacher will maintain a learning environment during the game that fosters deep communication among the students.

Learner objectives: Student will be actively involved in mathematical activities such as practicing mental mathematics and reviewing the four basic operations. This game may help develop students' understanding of mathematical concepts like frequency and probability and promote the development of essential mathematical skills like systematic counting, collection and organization of numerical data, analysis of data and inference-making, and formulation of conjectures.

Materials: Two dice, number strips, and counters for each pair of students

Modifications for special needs, including ELL and exceptionalities: Provide a chart of sample plays, model how to play the game

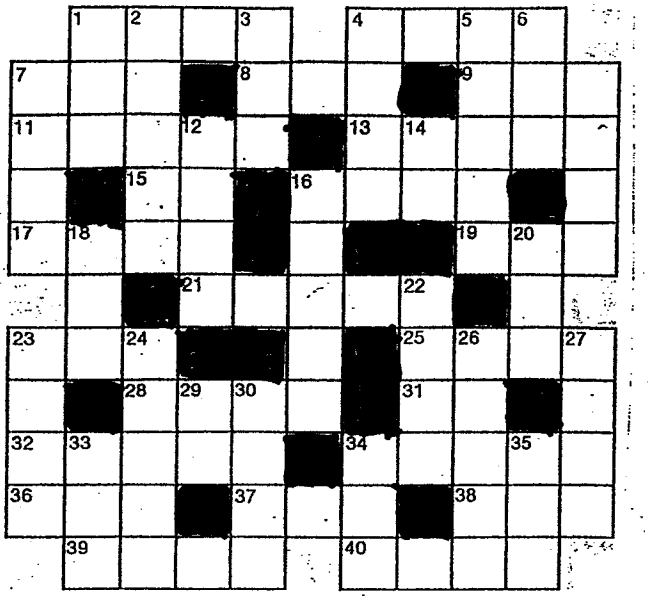
Procedure:
(5 minutes) *Initiating activity*: Introduce the game.
(15 minutes) *Modeling of process and/or guided practice*: Review the rules of the game. Play a demonstration game against the whole class. The game is for two players and requires two standard dice, two plating strips with the natural numbers from 1 to 10, and many counters. The first player rolls the dice and, using the numbers on them and any of the four basic arithmetic operations, tries to create one of the natural numbers from 1 to 10. He/she then covers it with a counter on his/her playing strip. The second player repeats this procedure to do the same with a number on his/her strip. If a player rolls two numbers and all of the possibilities are covered already, he/she loses a turn. The winner is the player who covers all ten numbers first.
(15 minutes) *Independent practice*: Place students in pairs and have them play among themselves. During their playing, the students will try to look for patterns and begin to formulate conjectures about winning strategies.
(10 minutes) *Closure*: Have a discussion about students' observations.

Assessment of student performance: Have students write plays and strategies they used during their game (e.g. rolled 5 and 6 on the dice→6-3=3 or 6+3=9). Collect for assessment.

Logical follow-up lesson: Play the game with two students on each team. During play, have students brainstorm strategies with their partner.

Name: Date:

smartsum



smartsum

Calculate the clues. Enter the answers in the grid above.

ACROSS

1 Average of 5781, 3707, 696, 7916
4 Ten percent of 70250
7 One eighth of 2400
8 Sum of numerals is 8
9 One twelfth of 4944
11 One eighth of 184112
13 Fill 14099, 14551, ?????, 15455
15 Sum of numerals is 4
16 One eighth of 24000
17 One twelfth of 13680
19 One eighth of 3280
21 Sum of numerals is 6
23 Average of 686, 182, 684, 1212
25 Twelve and half percent of 9448
28 Twelve and half percent of 9728
31 Ten percent of 250
32 Average of 15259, 17353, 9866, 5690
34 Average of 22308, 33726, 8727, 15415
36 Ten percent of 1300
37 One twelfth of 1920
38 Average of 135, 19, 133, 273
39 Twelve and half percent of 56000
40 Fill 5354, 5977, ????, 7223

DOWN

1 One eighth of 3224
2 Twelve and half percent of 400272
3 Average of 509, 359, 507, 681
4 Ten percent of 76100
5 Average of 22528, 33506, 8507, 31475
6 One twelfth of 6120
7 Fill 2753, 2977, ????, 3425
10 Sum of numerals is 5
12 Twelve and half percent of 8808
14 Sum of numerals is 5
16 Fill 29706, 30401, ?????, 31791
18 Ten percent of 1090
20 One twelfth of 1416
22 Average of 7370, 5767, 1101, 6242
23 Twelve and half percent of 48088
24 Sum of numerals is 2
26 Average of 15683, 16929, 9442, 17986
27 Twelve and half percent of 8320
29 One eighth of 192
30 One twelfth of 14520
33 Average of 232, 38, 230, 448
34 One twelfth of 2472
35 Sum of numerals is 8

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3. Multiply by 80
4. Add 1
5. Multiply by 250
6. Add the last 4 digits of your phone number
7. Add the last 4 digits of your phone number again.
8. Subtract 250
9. Divide number by 2

Do you recognize the answer?