Solutions Section K: Solids, Surface Area, and Volume

1. Both the radius and height of a cylinder are the same as the radius of a sphere.
a) Which of the two, if either, has the greater volume? Explain your answer. No credit for guessing.
b) Which of the two solids, if either, has the greater surface area (include the two bases in the case of the cylinder)? Explain your answer, no credit for guessing.

Solution part a): Let r denote the radius of the sphere. The volume V_S of the sphere is given by $V_S = 4\pi r^3/3$. The volume V_C of a cylinder is given by $V_C = \pi r^2 h$, where h is the height of the cylinder. Since h = r by assumption,

$$V_C = \pi r^2 h = \pi r^3 < 4\pi r^3/3 = V_S$$

The volume of the sphere is greater than the volume of the cylinder.

Solution part b): The surface area A_S of the sphere is given by $A_S = 4\pi r^2$. The surface area A_C of a cylinder, including the two circular bases, is given by $A_C = 2\pi rh + 2\pi r^2$. Since h = r in this case,

$$A_C = 2\pi rh + 2\pi r^2 = 2\pi r^2 + 2\pi r^2 = 4\pi r^2 = A_S$$

The surface area of the sphere is the same as the surface area of the cylinder.

2. Find the surface area (including both bases) of a cylinder whose volume is 18π m^3 and whose base has area 9π m^2.

Let r be the radius of the cylinder and h its height. Then the circular base of the cylinder has area $\pi r^2 = 9\pi$ m^2. Then $r^2 = 9$ m^2, and therefore,

$$r = 3 \text{ m}$$

The volume of the cylinder is the area of its base times its height. Since $9\pi \times 2 = 18\pi$, the height h = 2 m. You can also find the height this way: Since the volume of the cylinder is both equal to $\pi r^2 h$ and equal to 18π m^3, we get,

$$\pi r^2 h = 18\pi \text{ m}^3$$
$$r^2 h = 18 \text{ m}^3$$

Since $r^2 = 9$ m^2,

$$9h \text{ m}^2 = 18 \text{ m}^3$$

Therefore,

$$h = 2 \text{ m}$$

Now that the radius and height of the cylinder are known, its surface area A_C in square meters can be calculated as follows:

$$A_C = 2\pi rh + 2\pi r^2 = 2\pi(3)2 + 2\pi 3^2 = 12\pi + 18\pi = 30\pi$$

The surface area of the cylinder is 30π m^2.

3. If the volume of a cube is 8 cm^3, what is its surface area?

Since the volume of the cube is 8 cm^3, its side must be 2 cm long. The cube has 6 faces, each a square with side 2 cm. The area of one of these square faces is 4 cm^2. Therefore the surface area of the cube is 6×4 cm^2 = 24 cm^2.

4. Find the surface area of a square pyramid if the area of the base is 36 cm^2 and the height is 4 cm.

Since the area of the base of the pyramid is 36 cm^2, one side is 6 cm. The distance from the center of the square base to the midpoint of a side is therefore 3 cm. To calculate the altitude h in centimeters of one of the triangular sides, use the Pythagorean theorem.

$$h^2 = 4^2 + 3^2$$
$$h^2 = 25$$
$$h = 5$$

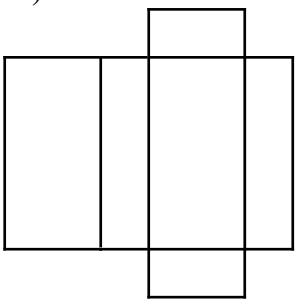
The altitude of each triangular face is 5 cm. Since the base of a triangular face is 6 cm, its area is 15 cm^2. Therefore the surface area A of the pyramid (including the square base of the pyramid) is given by,

$$A = 36 \text{ cm}^2 + 4 \times 15 \text{ cm}^2 = 96 \text{ cm}^2$$

The surface area of the pyramid is 96 cm^2.

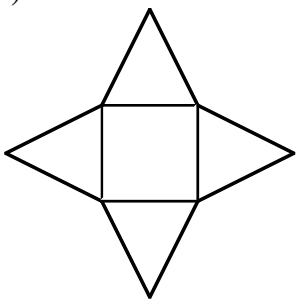
5. Name the polyhedron formed by each net.

a)



a) Right Rectangular Prism

b)



b) Right Square Pyramid

6. Find the volume of a right circular cone whose slant height is 50 cm and whose height is 0.4 m.

The volume V of a cone is given by

$$\mathbf{V = (1/3)Bh,}$$

where B is the area of the base and h is the height of the cone. Since the base of a right circular cone is a circle, $B = \pi r^2$, where r is the radius of the circular base. To find r^2 in square centimeters, use the Pythagorean theorem and the fact that the slant height 0.4m = 40 cm:

$$\mathbf{r^2 + 40^2 = 50^2}$$

$$\mathbf{r^2 = 50^2 - 40^2 = 2500 - 1600 = 900}$$

(Note that although the radius r is not actually needed, r = 30 cm). The volume in cubic centimeters is,

$$\mathbf{V = (1/3)Bh = (1/3)900\pi\ 50 = 300\pi 50 = 15{,}000\pi}$$

The volume of the cone is $15{,}000\pi$ cm^3.

7. (a) If the volume of a sphere is $\frac{500\pi}{3}$ m^3, what is the radius?

(b) Find the volume of a cylinder whose height is 2m and whose base has a radius of 3 m.

(c) Find the height of a square pyramid whose base has side 5 cm and whose volume is 75 cubic centimeters.

Solution part a): The volume V_S of a sphere is given by

$$\mathbf{V_S = 4\pi r^3/3}$$

So,

$$\mathbf{4\pi r^3/3 = \frac{500\pi}{3},}$$

where r is given in meters. Multiplying both sides by 3 and dividing by 4π gives,

$$\mathbf{r^3 = 500/4 = 125}$$

$$\mathbf{r = 5}$$

Therefore the radius of the sphere is 5 meters.

Solution part b): The volume of a cylinder is given by,

$$V_C = \pi r^2 h,$$

where r is the radius of the base and h is the height of the cylinder. Therefore the volume in cubic meters is given by,

$$V_C = \pi 3^2 \times 2 = 36\pi$$

The volume of the cylinder is 36 cubic meters.

Solution part c): The volume V_P of a pyramind is given by,

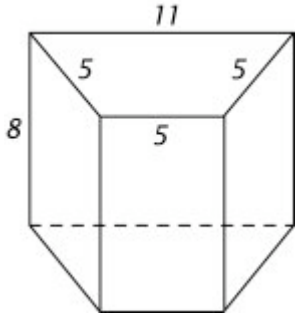
$$V = (1/3)Bh,$$

where B is the area of the base and h is the height of the pyramid. In this case, B = $5^2 cm^2 = 25\ cm^2$. So, the height in centimeters can be determinded from,

$$75 = (1/3)25\ h$$

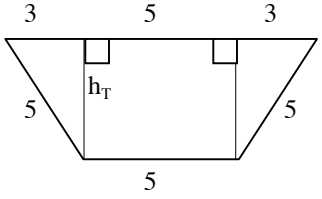
Solving for h gives h = 9. The height of the pyramind is 9 cm.

8. The bases of the right prism shown below are isosceles trapezoids.



Find its volume.

The volume of a prism, V_{Prism} is given by $V_{Prism} = Bh_{Prism}$, where B is the area of the base (in this case the two bases are trapezoids), and h_{Prism} is the height of the prism (in this case 8 units).



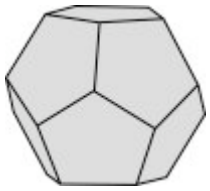
The height h_T of the trapezoid is a leg of a right triangle whose hypothenuse is 5 units and whose other side is 3 units. From the Pythagorean theorem it follows that h_T is 4 units.

The area of a trapezoidal base B is the sum of the lengths of its bases times one-half of h_T. Therefore,

$$\mathbf{B = (1/2)4(5 + 11) = 32}$$

So, $V_{Prism} = 32 \times 8 = 256$. The volume of the trapezoidal prism is 256 cubic units.

9. This solid is a regular dodecahedron.



a) How many faces does it have? Hint: How many faces border the face on top? On the bottom?

b) Explain why the expression $\frac{12 \times 5}{3}$ can be used to find its number of vertices.

c) How many edges does a dodecahedron have?

Solution part a): The five edges of the top face border 5 faces as do the edges of the bottom face. Counting the top and bottom faces along with these 10 faces gives a total of 12 faces for the dodecahedron.

Solution part b): Each pentagonal face has five vertices and there are 12 faces, which gives 12 × 5 in the numerator. But this over counts the number of vertices by a factor of 3 because each vertex is shared by 3 faces. Dividing gives (12 × 5)/3.

Solution part c): Each pentagonal face has five edges and there are 12 faces, which gives a total of 12 × 5 edges. But this over counts the edges by a factor of 2 because each edge is shared by 2 faces, so the total number of edges is (12 × 5)/2 = 30.