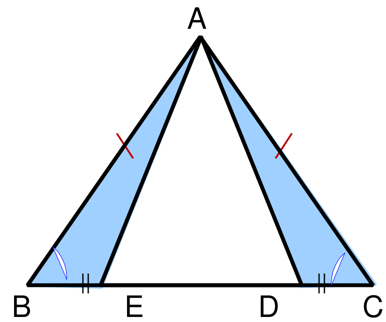
MATH 310 PRACTICE FINAL EXAM – solutions to selected problems(Part G)

The solutions given are not the only valid methods of proof.

G. CONGRUENCE Added features of diagrams are shown in colors.

1. Triangle ABC below is equilateral. Congruent line segments are indicated. Identify one pair of congruent triangles and explain carefully why they are congruent.

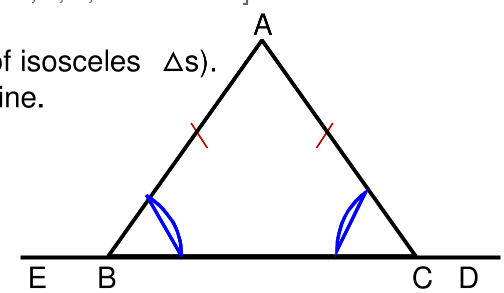


Note there are several pairs of congruent triangles in the figure sketched.
Probably the easiest pair to prove congruent consists of △ABE and △ACD.

△ABE ≅ △ACD**.** Here's why:

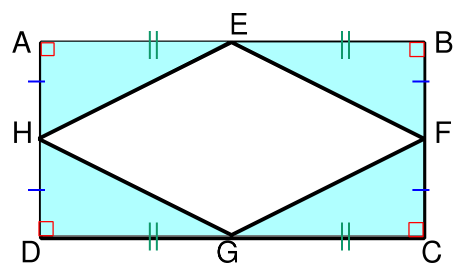
Because △ABC is equilateral, $\overline{AB}$ ≅ $\overline{AC}$ (marked \\ )**.**
∠B ≅ ∠C because △ABC is also isosceles ($\overline{AB}$ ≅ $\overline{AC}$)**.**
That $\overline{BE}$ ≅ $\overline{CD}$, is given as part of the hypothesis**.**
By "SAS", △ABE ≅ △ACD. ∎

2. Prove that ∠EBA is congruent to ∠DCA [given $\overline{AB}$ ≅ $\overline{AC}$ in △ABC, and E,B,C,D collinear] .



Since $\overline{AB}$ ≅ $\overline{AC}$ (given), ∠ABC ≅ ∠ACB (previously proven property of isosceles △s).
∠EBA & ∠ABC are supplementary, since E,B,C and D lie on a straight line.
∠DCA & ∠ACB are likewise supplementary
Since ∠EBA and ∠DBA are supplementary to congruent angles,
their measures must be the same (details below). ∎
(ie. m (∠EBA) = 180° − m(∠ABC) = 180° − m(∠ACB) = m (∠ACD))

3. E, F, G, H are the midpoints of the sides of Rectangle ABCD. Prove that Quadrilateral EFGH is a rhombus.



Since ABCD is a rectangle, all four angles, ∠A, ∠B, ∠C, ∠D, are right, and thus are congruent.

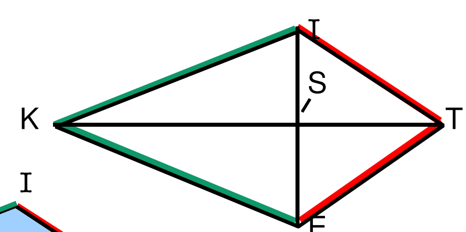
$\overline{AB}$ ≅ $\overline{CD}$ and $\overline{AD}$ ≅ $\overline{BC}$ because opposite sides of any parallelogram are congruent.

Since E is the midpoint of $\overline{AB}$, $\overline{EB}$ ≅ $\overline{AE}$.
Similarly, G is midpoint of $\overline{CD}$, so $\overline{DG}$ ≅ $\overline{CG}$— halves of $\overline{CD}$.
Furthermore, they are all congruent to each other, since they are all halves of two congruent sides.
That is, $\overline{EB}$ ≅ $\overline{AE}$ ≅ $\overline{DG}$ ≅ $\overline{CG}$.

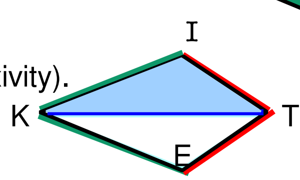
By the same reasoning, $\overline{AH}$ ≅ $\overline{DH}$ ≅ $\overline{BF}$ ≅ $\overline{CF}$

Thus by "SAS", △AEH ≅ △BEF ≅ △CGF ≅ △DGH
Since CPCTC, $\overline{EH}$ ≅ $\overline{GF}$ ≅ $\overline{GH}$ ≅ $\overline{EF}$.

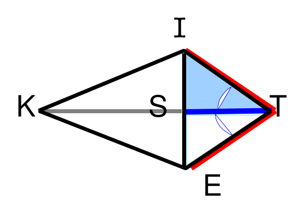
Thus, by definition, EFGH is a rhombus. ∎



4. Given: Quadrilateral KITE with KI ≅ KE and IT ≅ ET
Prove KT ⊥ IE.



$\overline{KI}$ ≅ $\overline{KE}$ and $\overline{TI}$ ≅ $\overline{TE}$ (given), and $\overline{KT}$ ≅ $\overline{KT}$ (reflexivity).
So, by "SSS", △KIT ≅ △KET .
Since CPCTC, ∠ITS ≅ ∠ETS
That, plus $\overline{TI}$ ≅ $\overline{TE}$ and $\overline{TS}$ ≅ $\overline{TS}$, guarantees, by "SAS", that △ITS ≅ △ETS .



Since CPCTC, ∠TSI ≅ ∠TSE ; further, the sum of the measures of ∠TSI and ∠TSE is 180°, so each must measure 90°.
Since $\overline{KT}$ and $\overline{IE}$ meet at right angles, they are perpendicular. ■ (That's all folks!)