

Mathematics 150A Semester Review Problems

1. Evaluate

(a) $\lim_{x \rightarrow 1} \frac{x^2 - 3x + 2}{x - 1}$

(d) $\lim_{x \rightarrow 0^-} \frac{x - |x|}{x}$

(b) $\lim_{x \rightarrow 0} \frac{x - 1}{x}$

(e) $\lim_{x \rightarrow \infty} \frac{2x^2 - 5x + 6}{8 + x + 3x^2}$

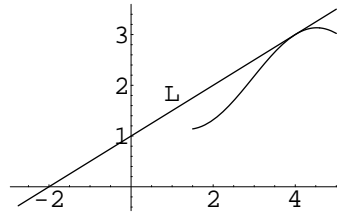
(c) $\lim_{x \rightarrow 0} \frac{x}{\tan 4x}$

(f) $\lim_{x \rightarrow \infty} \frac{\sin x}{x}$

2. Let $f(x) = \begin{cases} ax & \text{for } x < 3 \\ bx^2 + x + 9 & \text{for } x \geq 3 \end{cases}$

Find a and b so that f is continuous and has a continuous derivative at 3.

3. Suppose line L is tangent to the curve $y = f(x)$ at the point $(4, 3)$, as shown. Find $f(4)$ and $f'(4)$.

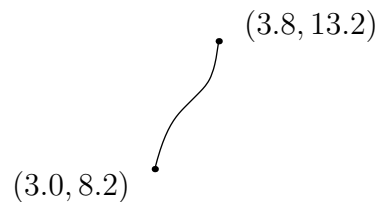


4. If $f(x) = x|x|$ for all x , find $f'(0)$ by directly using the definition of derivative.

5. Let f be a differentiable function satisfying the following table of values:

x	3.0	3.2	3.4	3.6	3.8
$f(x)$	8.2	9.5	10.5	11.0	13.2

If the graph of f is as shown, estimate $f'(3.2)$ and $f'(3.5)$. (In each case, there can be more than one answer that is acceptable.)



6. For each of the following, find $f'(x)$:

(a) $f(x) = \frac{1}{9}x^5 + x^{2/3} - \frac{1}{x^6}$

(d) $f(x) = \sqrt{a^2 + x^2}$

(b) $f(x) = x \sin^2 x$

(e) $f(x) = \frac{x}{(1 + 2x)^9}$

(c) $f(x) = \frac{x^2 + 1}{x - 5}$

(f) $f(x) = \tan^2(x^2 + 1)$

7. Find the equation of the line tangent to the curve $x^3 + 3x^2y^2 + y^3 = 3$ at the point $(1, -1)$.

8. Suppose that $f(T)$ represents the daily cost to heat a certain house in dollars when the mean outside temperature is T degrees fahrenheit, $40 \leq T \leq 60$. Give a physical interpretation, with units specified, of the statement $f'(50) = -0.15$.

9. A revolving search light, which is 800 yards from the shore, makes 2 revolutions (4π radians) per minute. How fast is the light traveling along the straight beach when it is 1000 yards from the lighthouse?

10. The area of a right triangle with hypotenuse of length one unit is given by $A(\theta) = \frac{1}{2} \sin \theta \cos \theta$, where θ is one of the acute angles. If θ is measured to be $\frac{\pi}{6} \pm 0.02$ radians (i.e., within 0.02 of $\frac{\pi}{6}$), use a differential or a linear approximation to estimate the maximum error made if $A(\pi/6)$ is used to approximate the actual area.

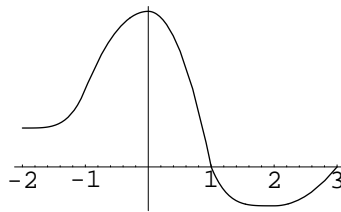
11. Sketch the following curves carefully. Label asymptotes, extrema (local and global), points of inflection, intercepts and intervals of concavity and monotonicity (increase and decrease).

(a) $y = \frac{1}{6}x^6 - x^5$ with domain $[-2, 6]$

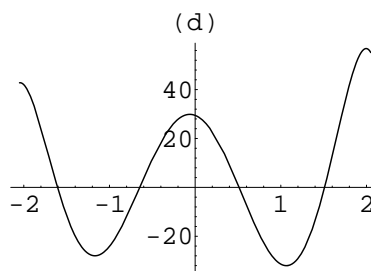
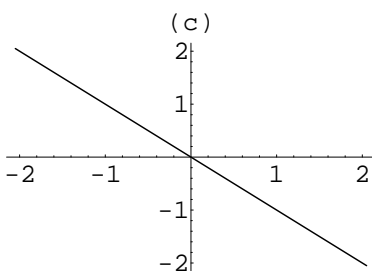
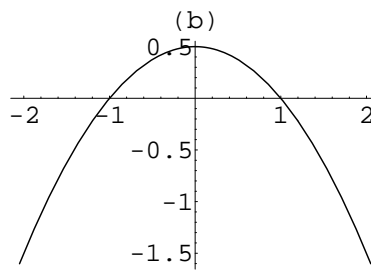
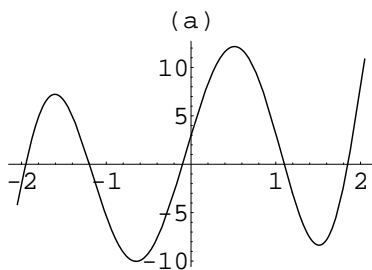
(b) $y = \frac{x^2}{x^2 - 4}$

12. Let f be a function that exists and is continuous for all real numbers except $x = 2$, and whose first two derivatives are $f'(x) = \frac{x^3}{x - 2}$ and $f''(x) = \frac{2x^3 - 6x^2}{(x - 2)^2}$. On what intervals is f increasing and on what intervals is f decreasing? Find the domain values (if there are any) of the points where f has a local maximum. Do the same for any local minimum. On what intervals is f concave up and on what intervals is f concave down? Give the domain values for any inflection points. If $f(1) = f(3) = 0$, use the information found above to sketch a possible graph $y = f(x)$.

13. The graph of $y = f'(x)$ (not $y = f(x)$) is given to the right.



- (a) On what interval(s) is f increasing? decreasing? Why?
- (b) On what interval(s) is f concave up? concave down? Why?
- (c) At what x is $f(x)$ greatest? At what x is $f'(x)$ greatest? At what x is $f''(x)$ greatest?
- (d) Assuming $f(0) = 0$, sketch a possible graph of $y = f(x)$.
14. Here are four graphs. Two of them are plots of the derivatives of the functions plotted in the other two. Match the plots of the functions with the plots of their derivatives.

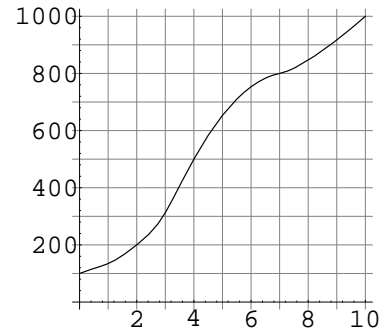


15. Sketch a possible graph of a function f that has all the following properties:
 $f'(x) > 0$ for $x < 0$ and $x > 3$; $f'(x) < 0$ for $0 < x < 2$ and $2 < x < 3$;
 $f''(x) > 0$ for $x < 0$, $0 < x < 1$, and $2 < x < 4$; $f''(x) < 0$ for $1 < x < 2$ and $x > 4$;
 $\lim_{x \rightarrow -\infty} f(x) = 1$; $\lim_{x \rightarrow \infty} f(x) = \infty$; $\lim_{x \rightarrow 2^-} f(x) = -\infty$; $\lim_{x \rightarrow 2^+} f(x) = \infty$.

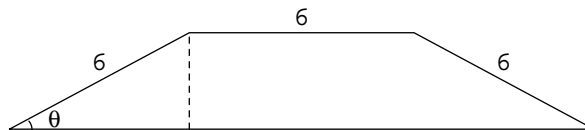
16. If $v(t) = t + 1 + \cos 2t$ is the velocity of a particle in terms of time, what is the time t in $[0, \pi]$ when the velocity is the least? What is the maximum velocity for this interval of time?

17. At what value of x is the graph of $y = -x^3 + 3x^2 + 9x - 27$ rising most rapidly?
18. An epidemic was identified 10 months ago. The progress of this epidemic is indicated by the graph below, which gives y as a function of x , where y is the total number of diagnosed cases as of time x measured in months. The rate of increase of the epidemic is measured by the number of new cases per unit time.

- (a) What is the total number of new cases during the 10 months, and what is the average rate of increase per month?
- (b) Estimate the initial rate of increase.
- (c) At what time was the rate of increase in the number of cases greatest?



19. Find the shortest distance from the point $(2, 1/2)$ to the parabola $y = x^2$.
20. A sheep pen in the shape of a trapezoid is to be formed using three pieces of rigid fencing (6 feet each) against a wall. Find the maximum possible area of the sheep pen.



21. Use Newton's method to approximate all solutions of $x^4 - 2x^3 = 2$, correct to within three places past the decimal point. Start by sketching a graph to find good initial approximations.

22. Let $G(x) = \int_0^x \sqrt{t^4 + 1} dt$.

- (a) Find $G(0)$.
 (b) Find $G'(x)$.
 (c) Find $G''(x)$.
 (d) Where is G increasing? Decreasing?
 (e) Where is G concave up? Concave down?
 (f) Sketch the graph of G .

23. Evaluate

(a) $\int \left(x + \frac{1}{x}\right)^2 dx$

(d) $\int_0^t \frac{d}{dx} (\cos(x^2 + 1)) dx$

(b) $\int \cos^5(3x) \sin(3x) dx$

(e) $\int_0^3 |x^2 - 4x + 3| dx$

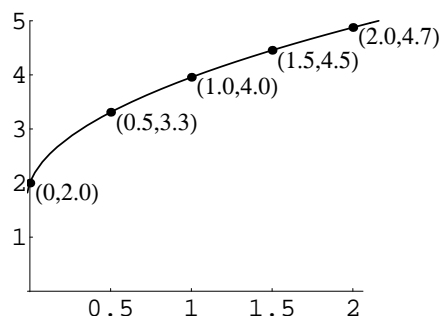
(c) $\int \frac{1}{\sqrt{x}} (\sqrt{a} + \sqrt{x})^{5/2} dx$

(f) $\int_1^2 \frac{du}{(3 - 5u)^2}$

24. A ball is thrown vertically from a point 25 feet above the ground. What should the initial velocity be so that the ball will reach a maximum height of 50 feet? (Assume that the acceleration due to gravity is 32 ft/sec^2 directed downward.)

25. Compute the maximum and the minimum Riemann sums for $f(x) = 2x^2 - x + 1$ over the interval $[-1, 1]$ if the interval is divided into two subintervals of equal length.

26. Consider the graph shown. Write the following Riemann sums for the integral $\int_0^2 f(x) dx$. Use four intervals of equal length.



(a) Right hand sum.

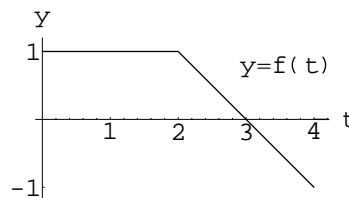
(b) Left hand sum.

(c) Use your answers from parts (a) and (b) to give your best estimate for $\int_0^2 f(x) dx$.

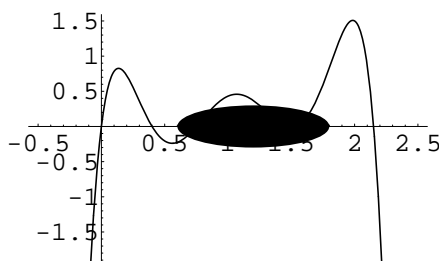
27. Find the area above the x -axis that lies under the curve $y = 15 - 2x - x^2$.

28. Draw the graph of the area function

$A(x) = \int_0^x f(t) dt$ for $0 \leq x \leq 4$, where $f(t)$ is as shown.



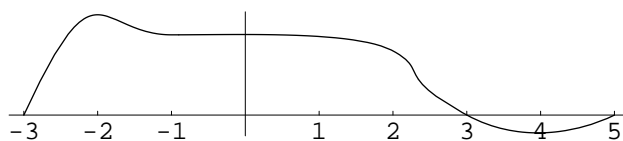
29. Suppose the graph of a continuously differentiable function f is given in the figure to the right. As you can see, a section of the graph is concealed. Decide whether each of the following definite integrals is positive, equal to 0, or negative. Explain your reasoning.



(a) $\int_0^2 f(x) dx$

(b) $\int_0^2 f'(x) dx$

30. The graph below is of the function $y = f(x)$.



Put the following in increasing order:

$$A = \int_{-3}^3 f(x) dx \quad B = \int_0^3 f(x) dx \quad C = \int_3^5 f(x) dx \quad D = \int_0^5 f(x) dx$$

31. When you are fresh, you find that you can harvest 150 bushels of corn per hour. But as the day wears on, your efficiency is somewhat decreased. In fact, after t hours from the beginning of the day, you find that you are harvesting at a rate of $150(1 + 0.5t)^{-3}$ bushels per hour. Write down the integral that measures how many bushels of corn you harvest after arriving in the field fresh and working for 5 consecutive hours.
32. Criticize the following argument: Since $\frac{d}{dx} \left(\frac{1}{1-x} \right) = \frac{1}{(1-x)^2}$, it follows by the Fundamental Theorem of Calculus that

$$\int_{-2}^2 \frac{dx}{(1-x)^2} = \int_{-2}^2 \frac{d}{dx} \left(\frac{1}{1-x} \right) dx = \left[\frac{1}{1-x} \right]_{-2}^2 = \frac{1}{-1} - \frac{1}{3} = -\frac{4}{3}.$$

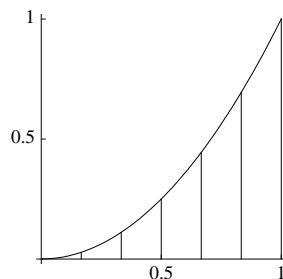
33. Find the area of the region enclosed by the pair of curves

(a) $y = x^2$ and $y = 8\sqrt{x}$

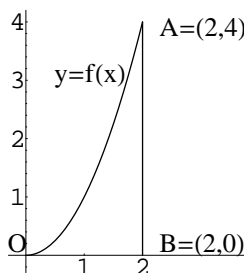
(b) $x = 2y^2$ and $x = y + 3$

(c) $y = x^3 - x$ and $y = x - x^2$

34. The base of a solid is the region bounded by the curve $y = x^2$, the x -axis, and the line $x = 1$. The cross sections of the solid perpendicular to the base and parallel to the y -axis are squares. Find the volume of the solid.



35. Which of the expressions give the volume of the solid generated by revolving the region OAB about the line AB ?



(a) $\int_0^2 2\pi x f(x) dx$

(d) $\int_0^4 \pi [f^{-1}(y)]^2 dy$

(b) $\int_0^2 2\pi(2-x)f(x) dx$

(e) $16\pi - \int_0^4 2\pi f^{-1}(y) dy$

(c) $\int_0^2 \pi [f(x)]^2 dx$

(f) $\int_0^4 \pi [2 - f^{-1}(y)]^2 dy$

36. Let R be the region between $y = 1 - x^2$ and the x -axis. Find the volume of the solid generated by rotation of R about the line $x = 2$.

37. Find the average value of the function $f(x) = \sqrt{x}$ on the interval $[1, 4]$.

38. A water tank in the shape of a cylinder of radius 6 feet and length 15 feet is lying on its side. It is half filled with water, which weighs about 62.5 pounds per cubic foot. How much work is required to pump it dry, if the pump is 9 feet above the top edge of the tank? Give the answer in terms of an integral of a single variable. (Do not integrate.)