Drawing graphs with vertices and edges in convex position and large polygons in Minkowski sums

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LIF Marseille

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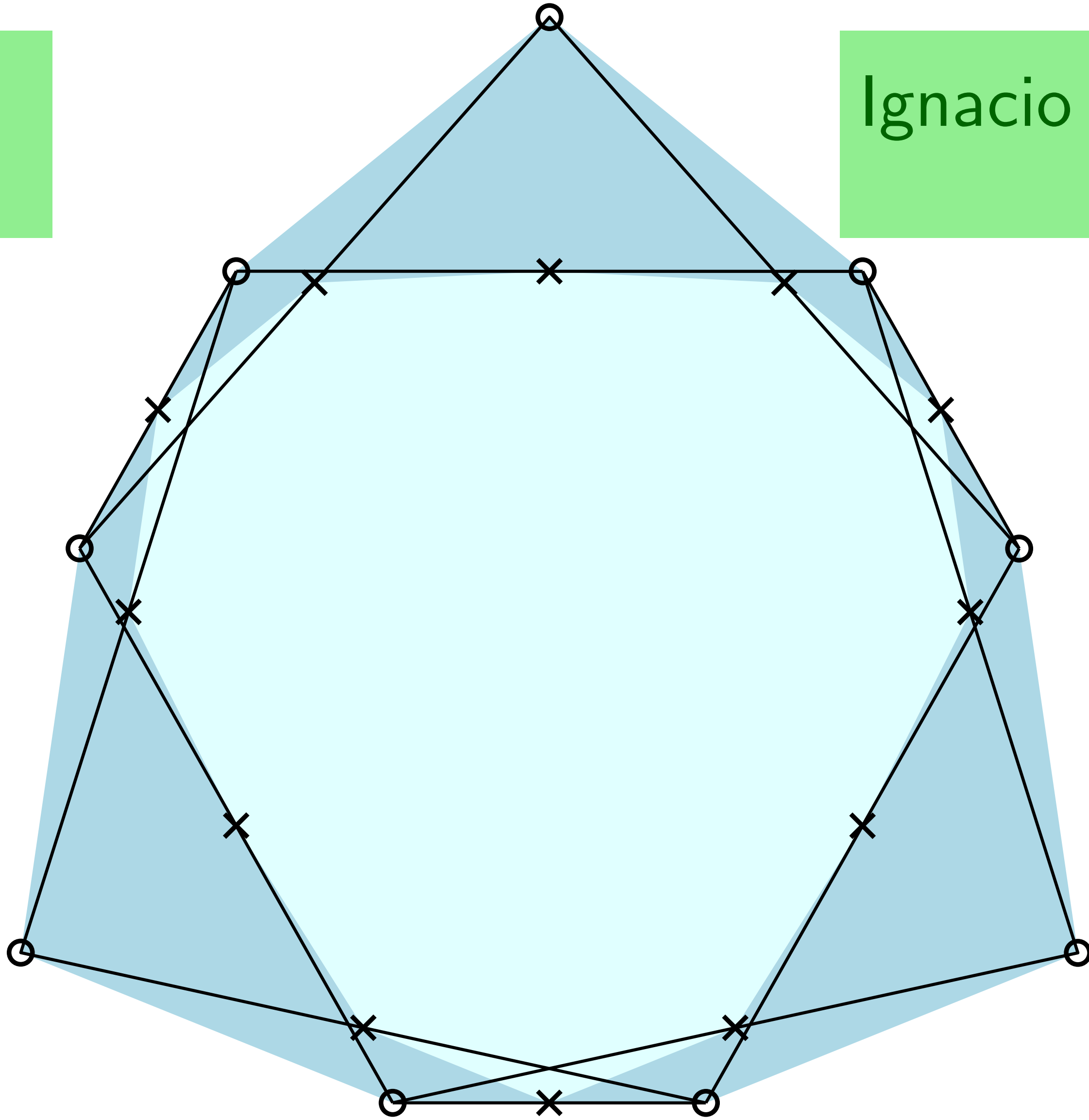
LIP ENS Lyon



Graph Drawing, September 29, 2015

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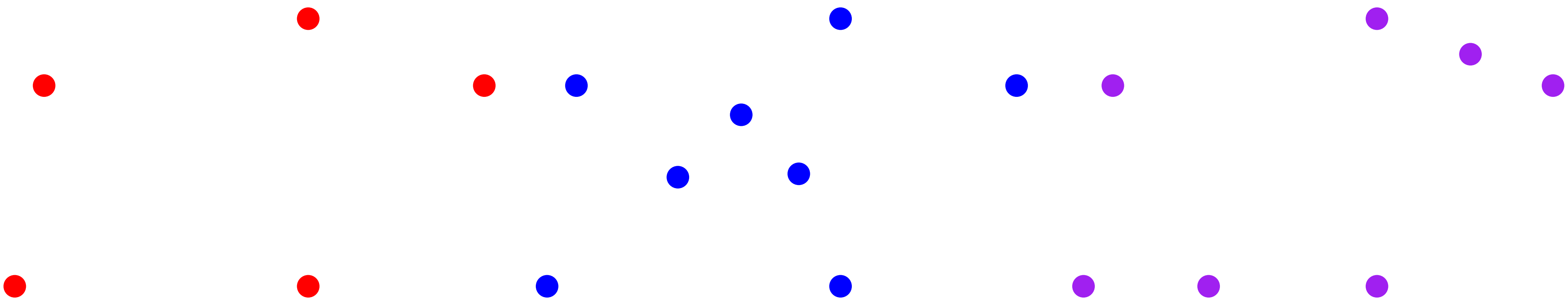

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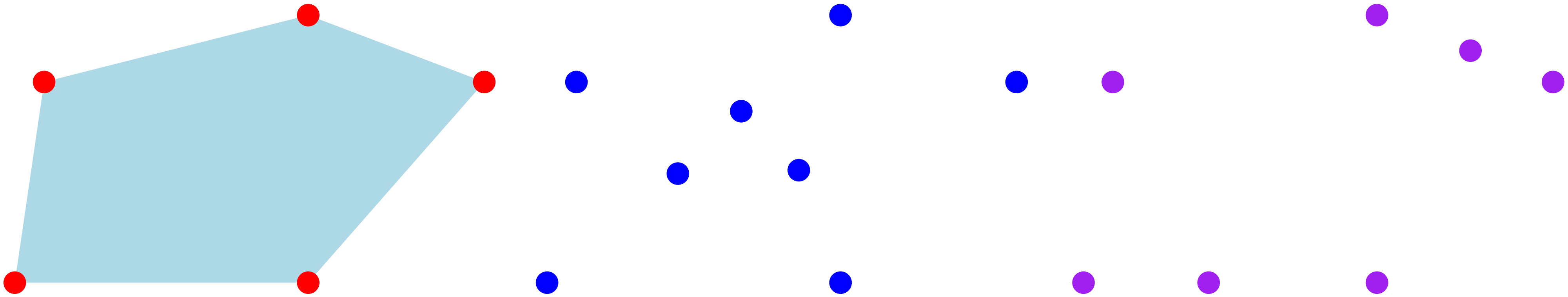

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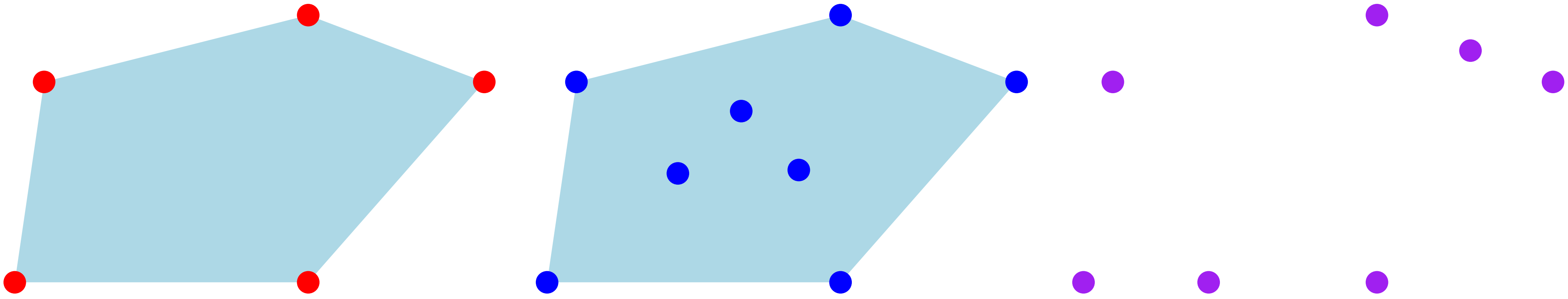

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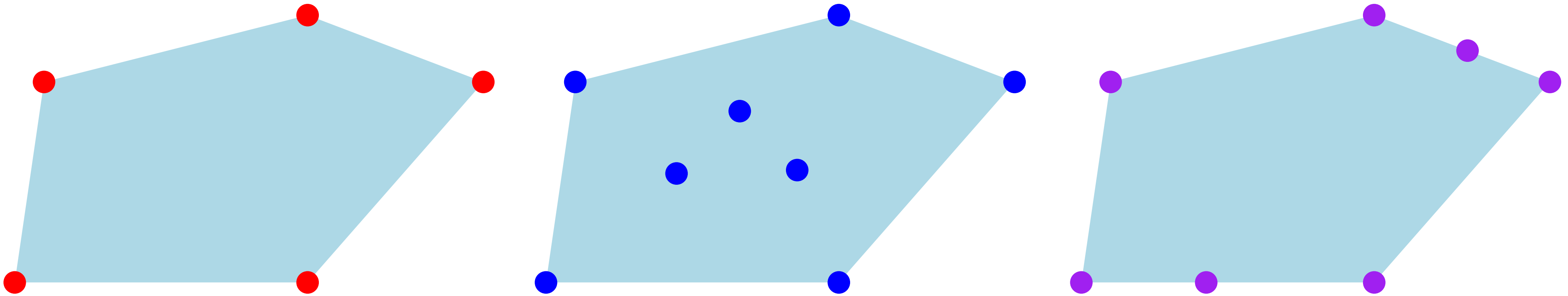

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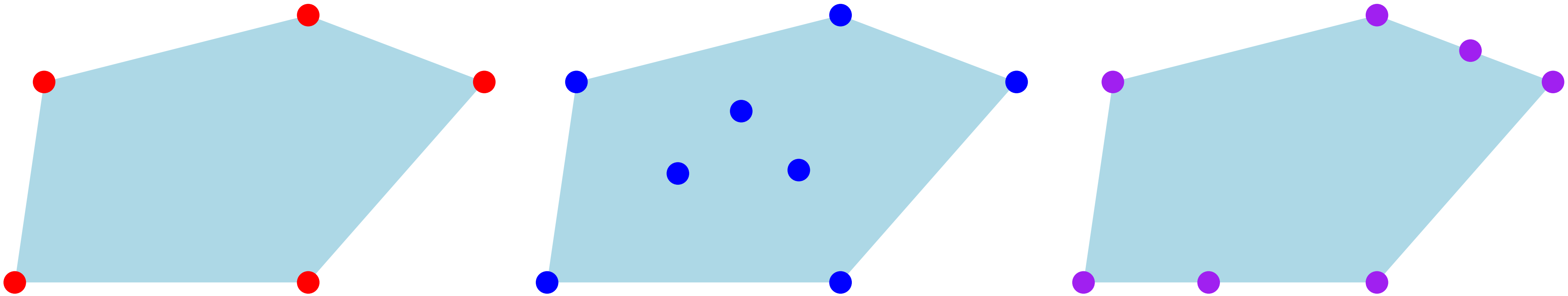

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- edges straight-line segments
- vertices and midpoints of edges on different points

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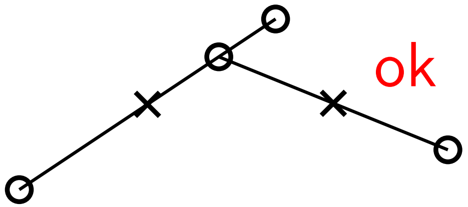

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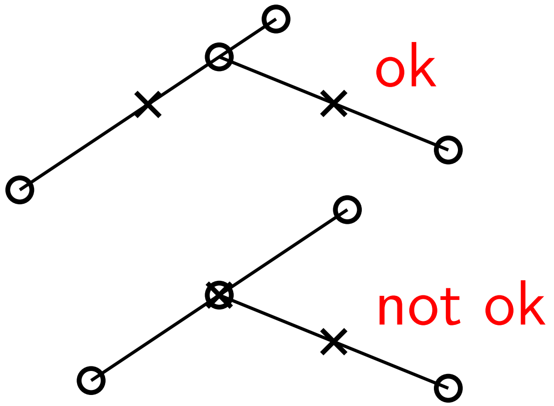

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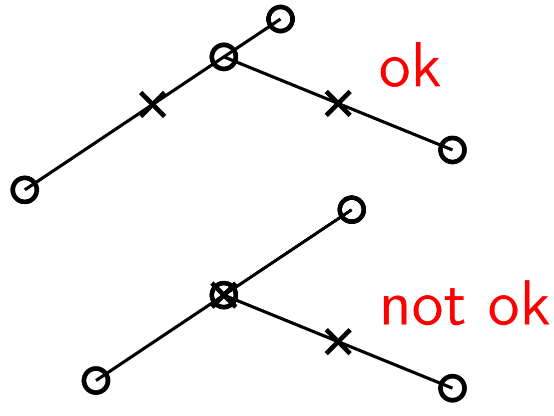

for $i, j \in \{s, w, a\}$ define $\mathcal{G}_i^j$ as class of graphs **drawable** s.th.

midpoints position $\begin{cases} \text{strictly convex} & \text{if } j = s \\ \text{weakly convex} & \text{if } j = w \\ \text{arbitrary} & \text{if } j = a. \end{cases}$

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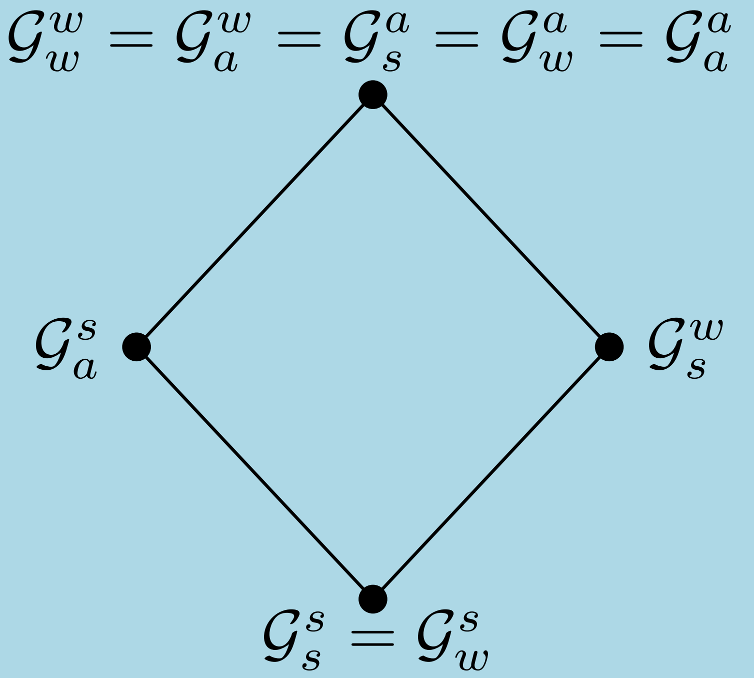

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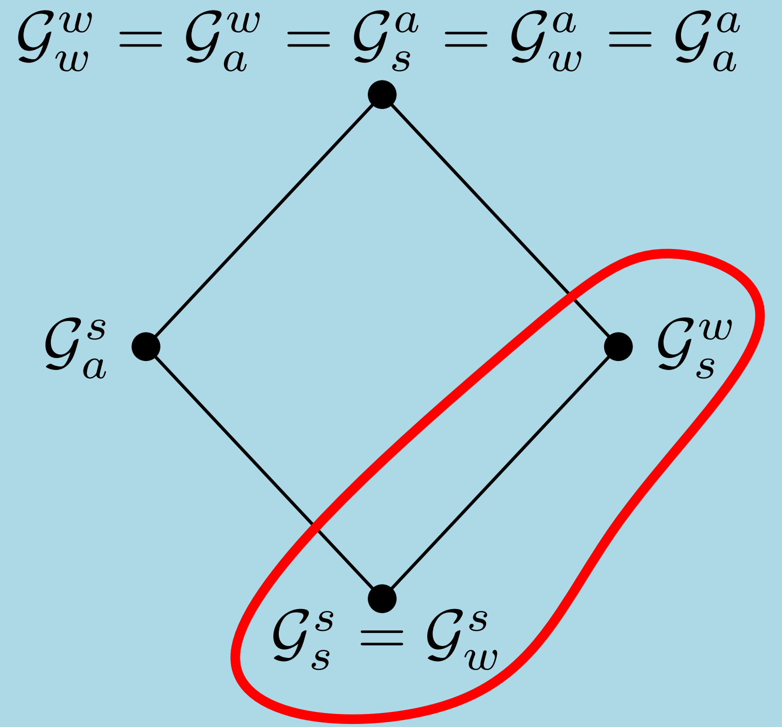

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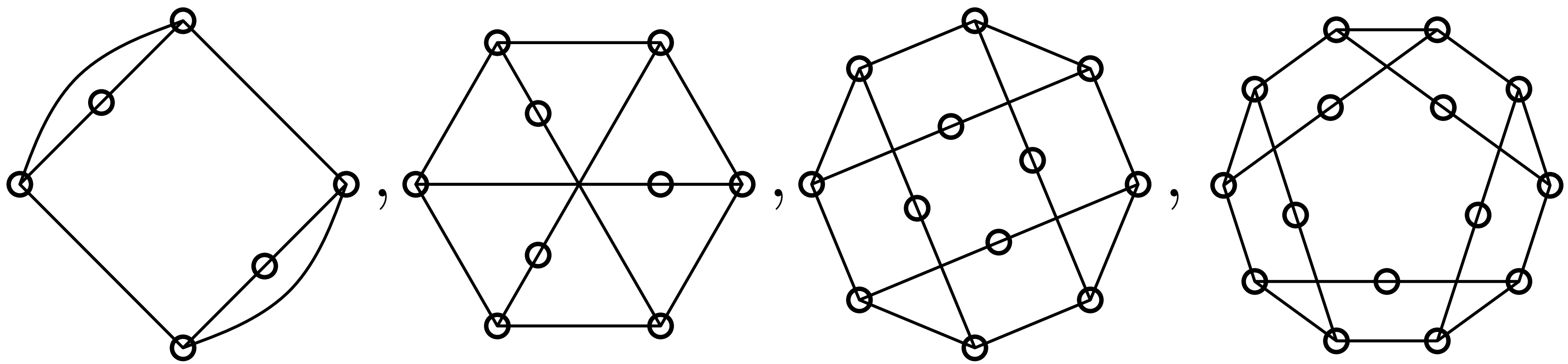

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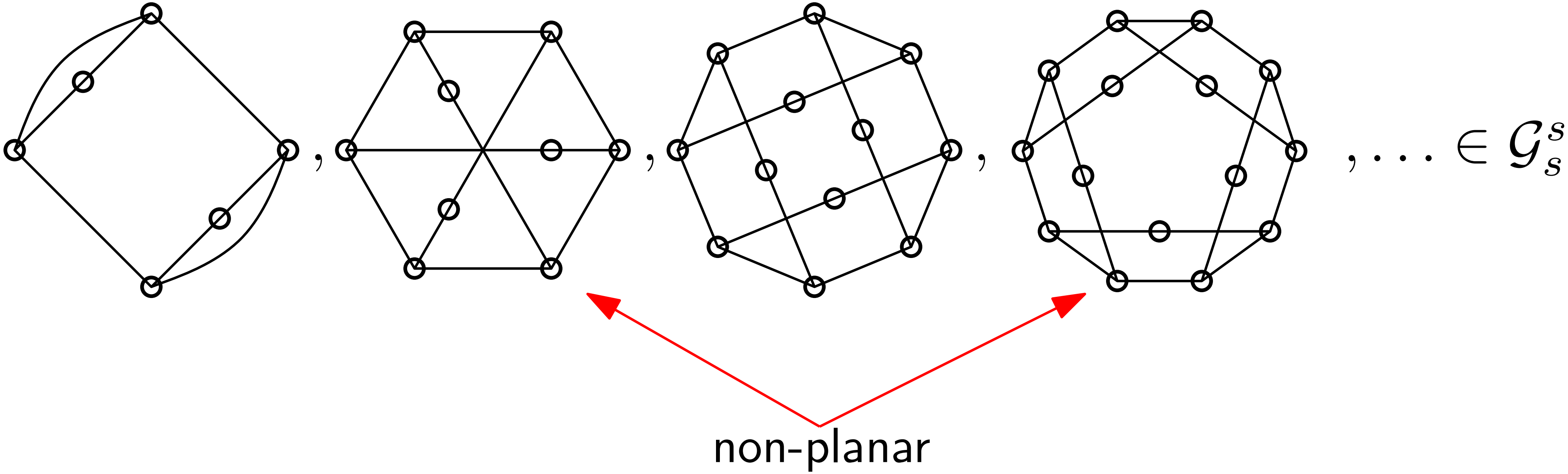

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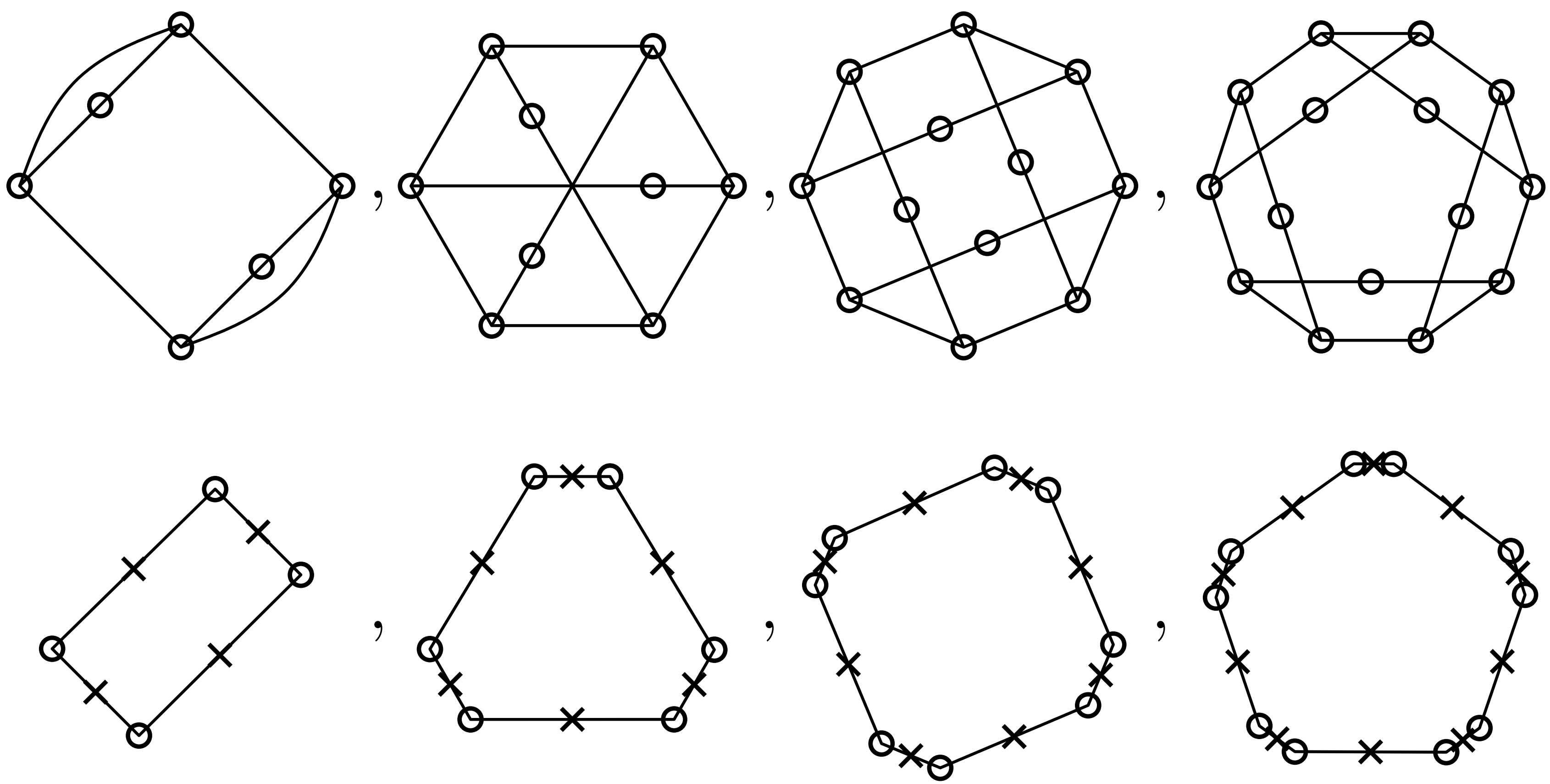

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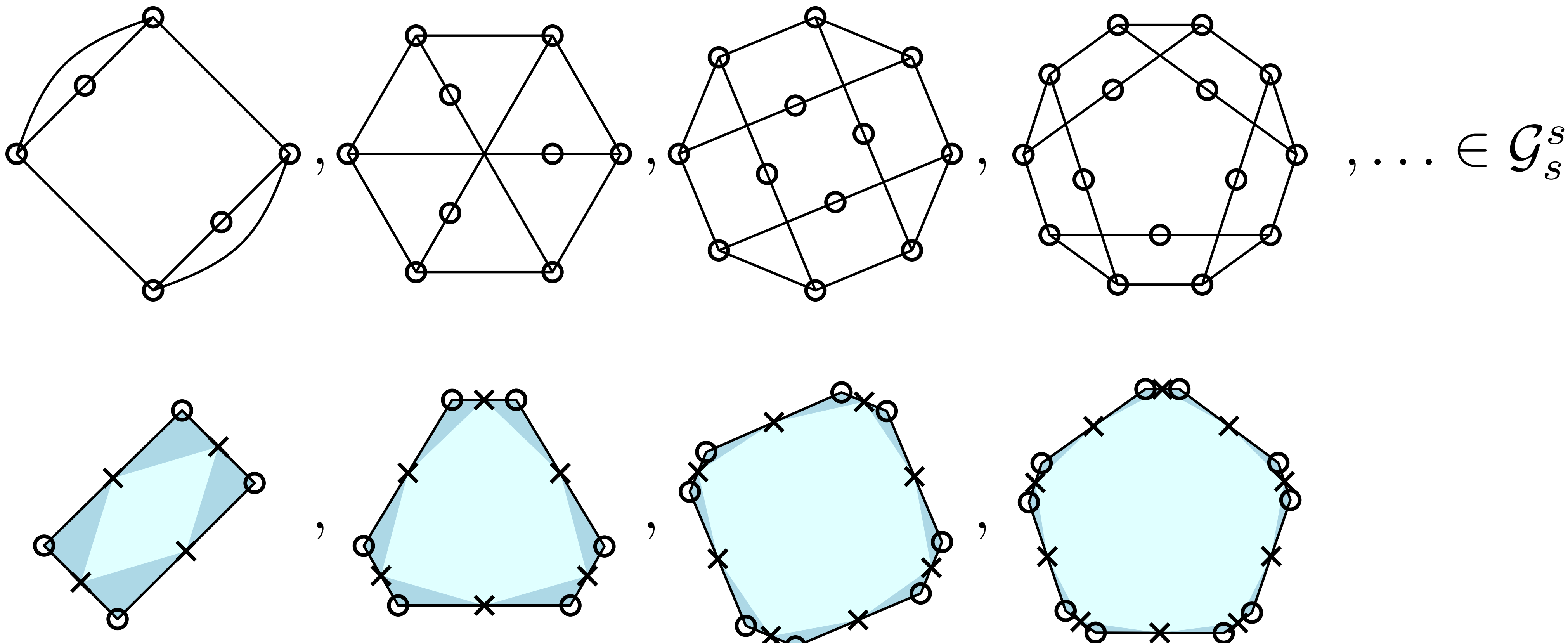

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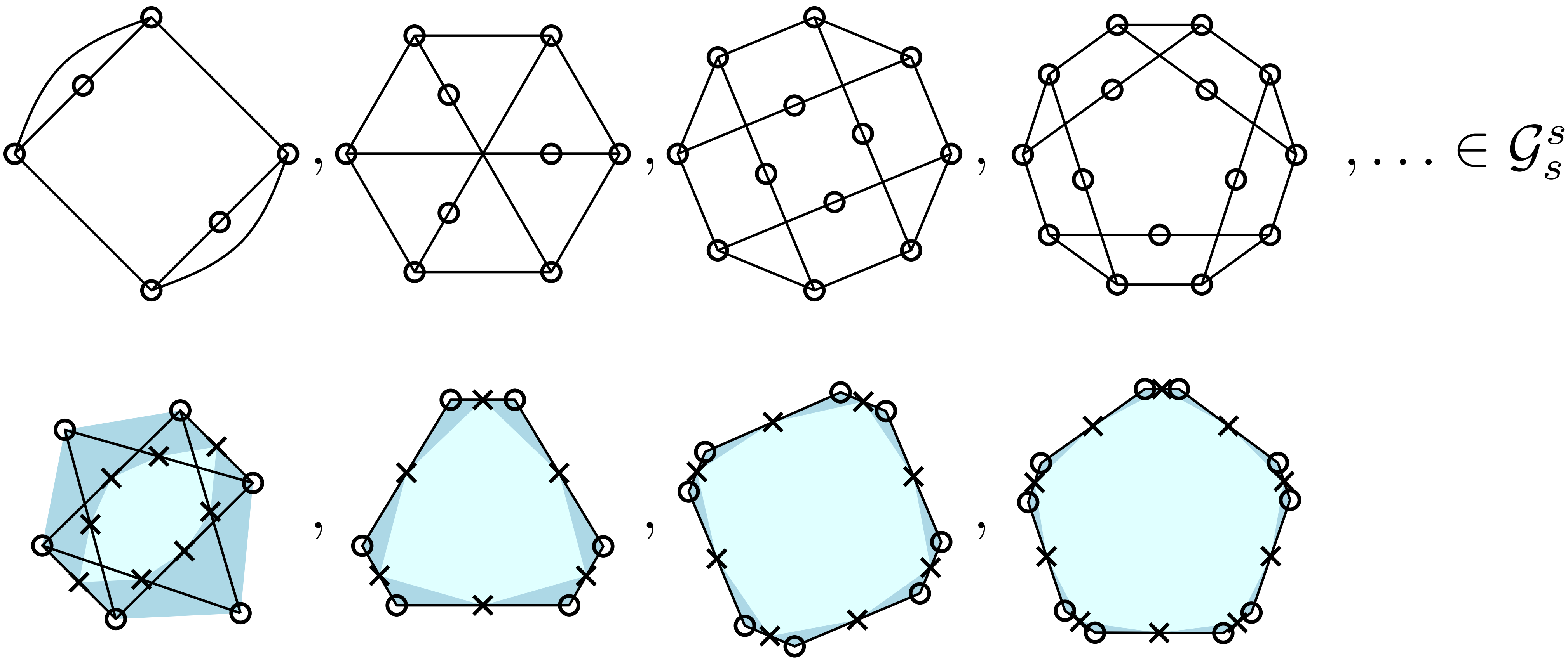

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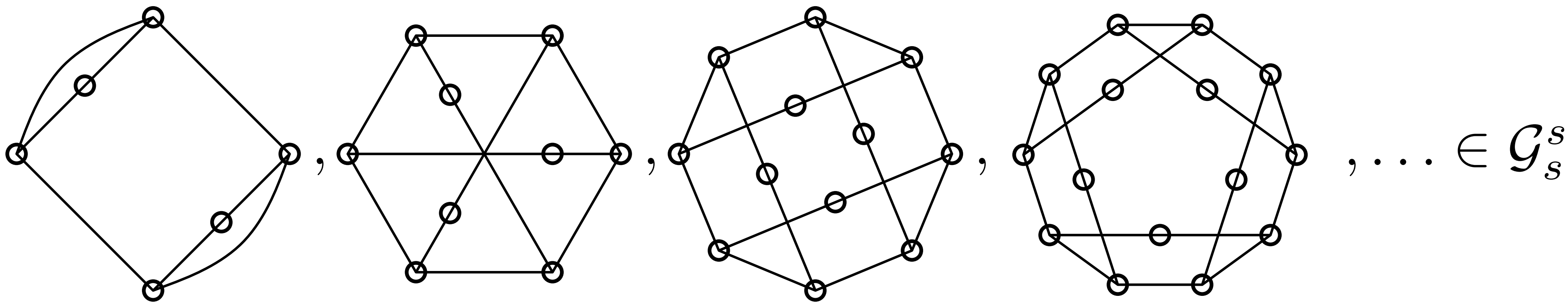

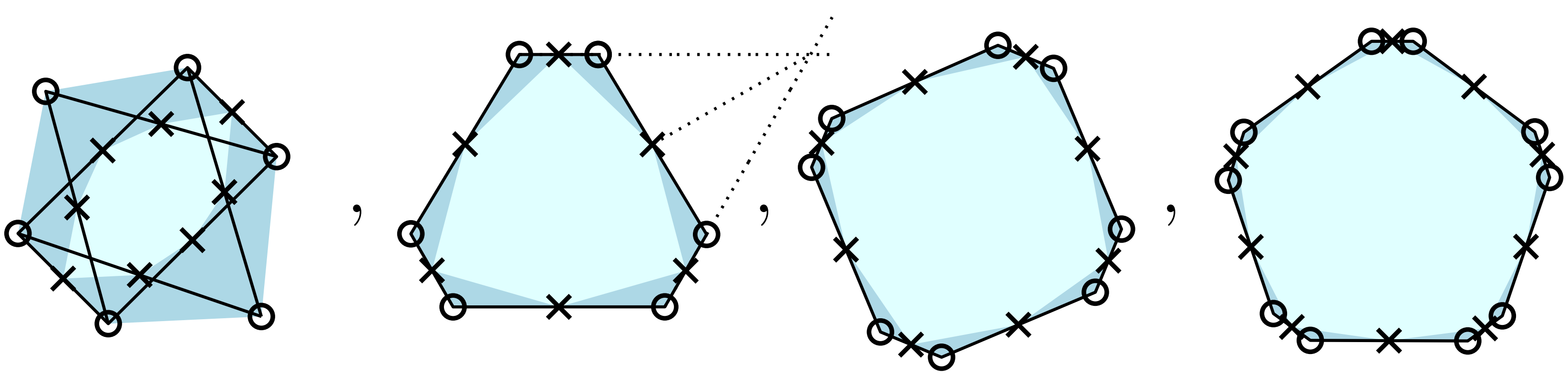

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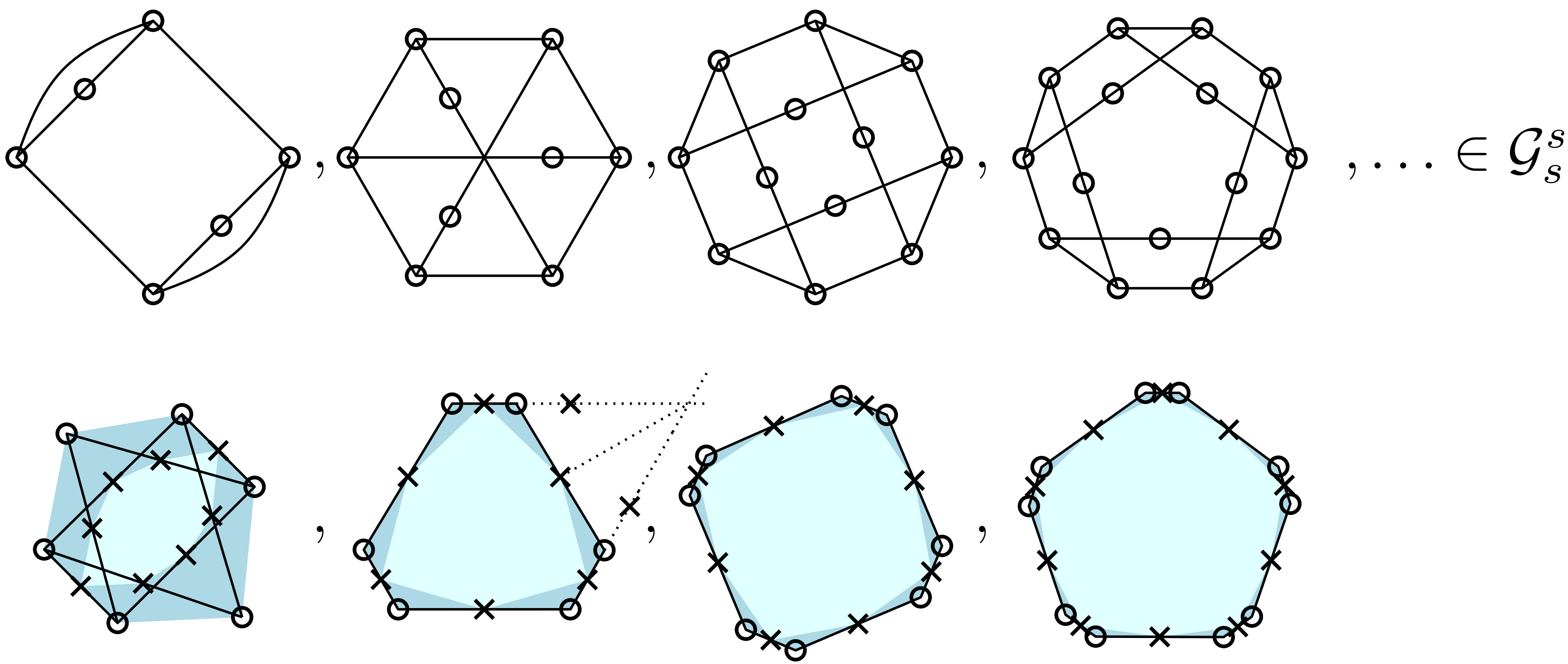

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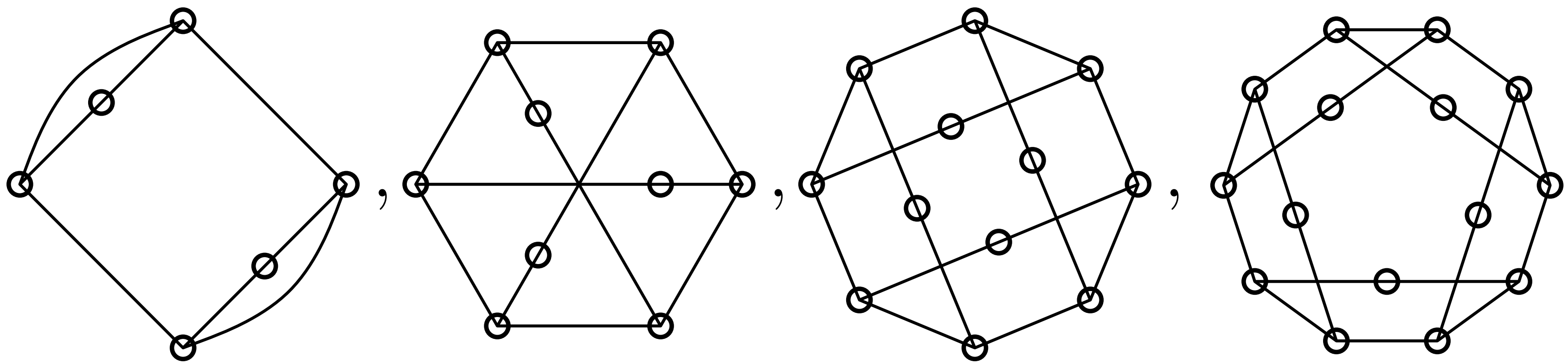

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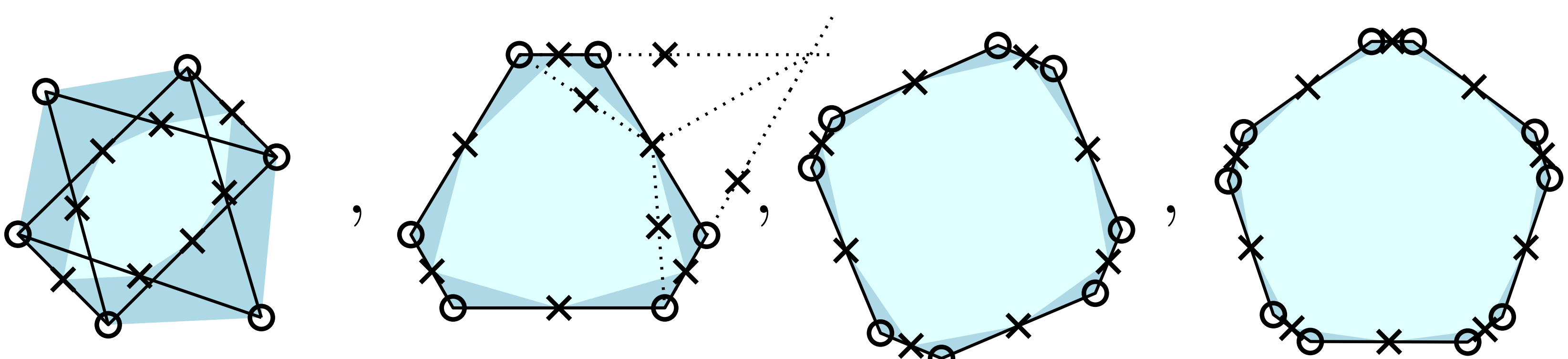

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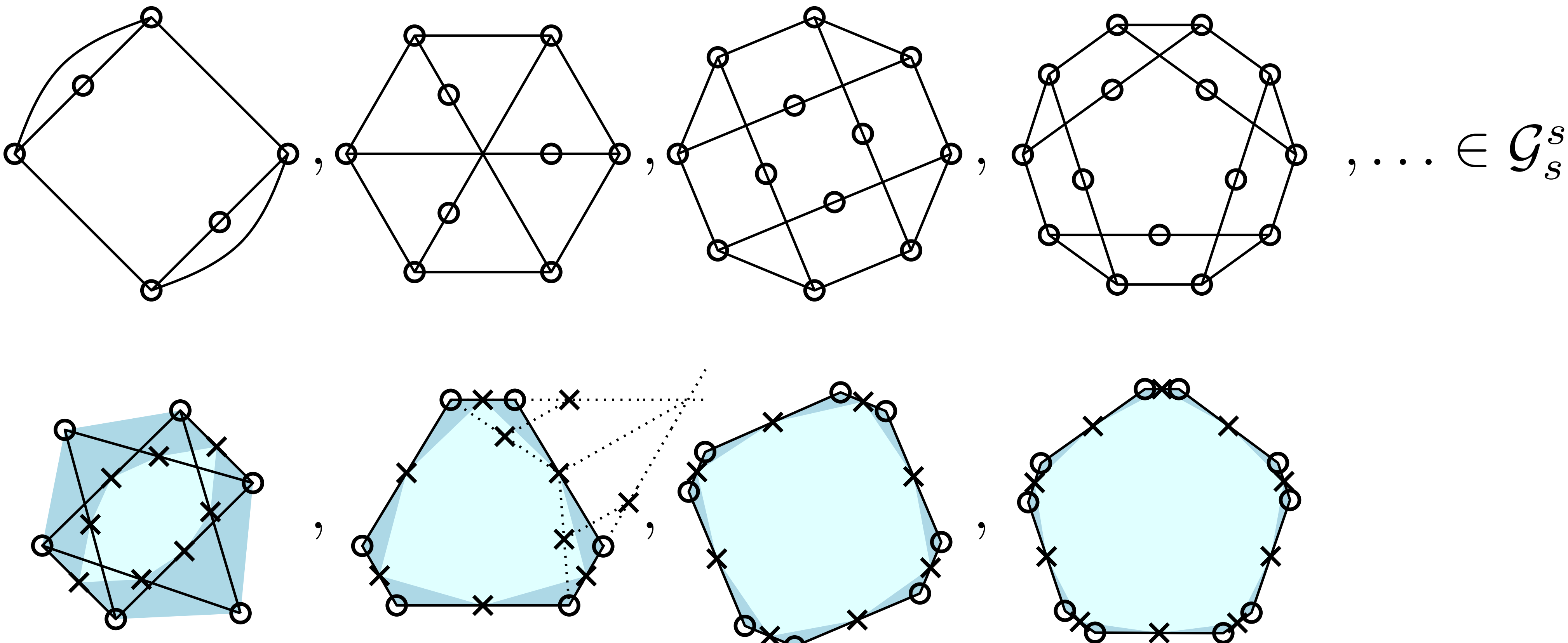

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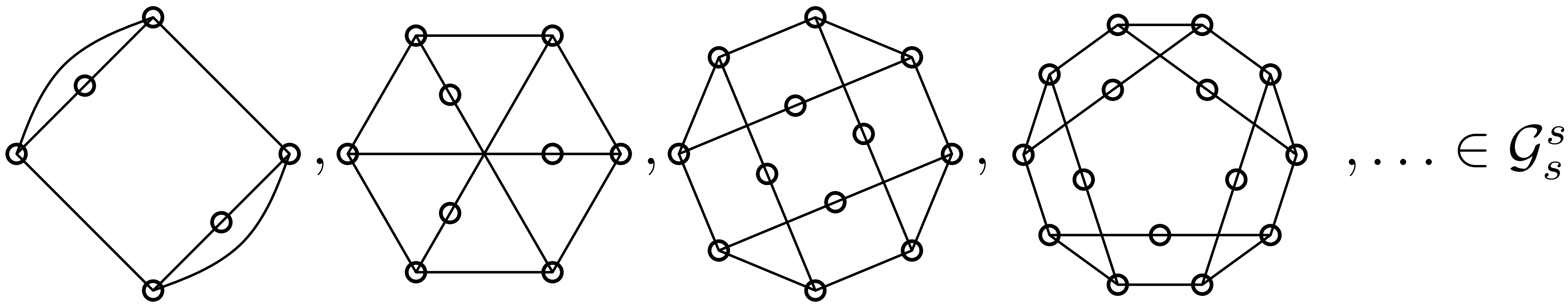

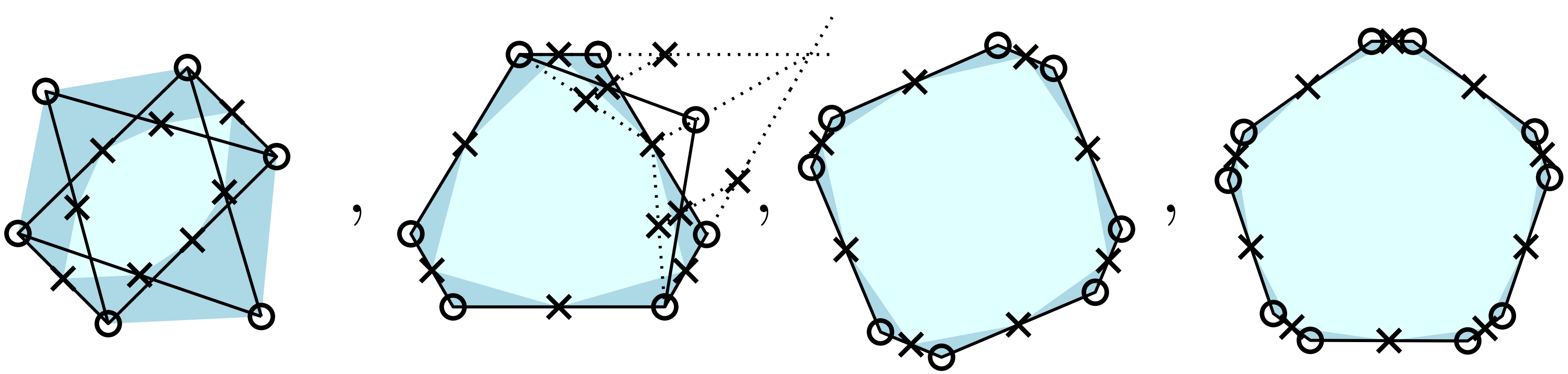

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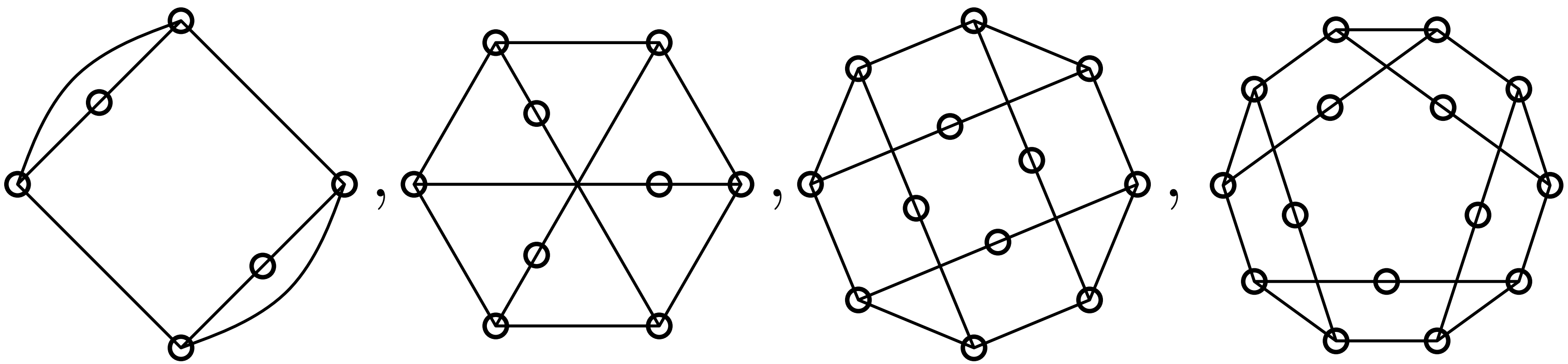

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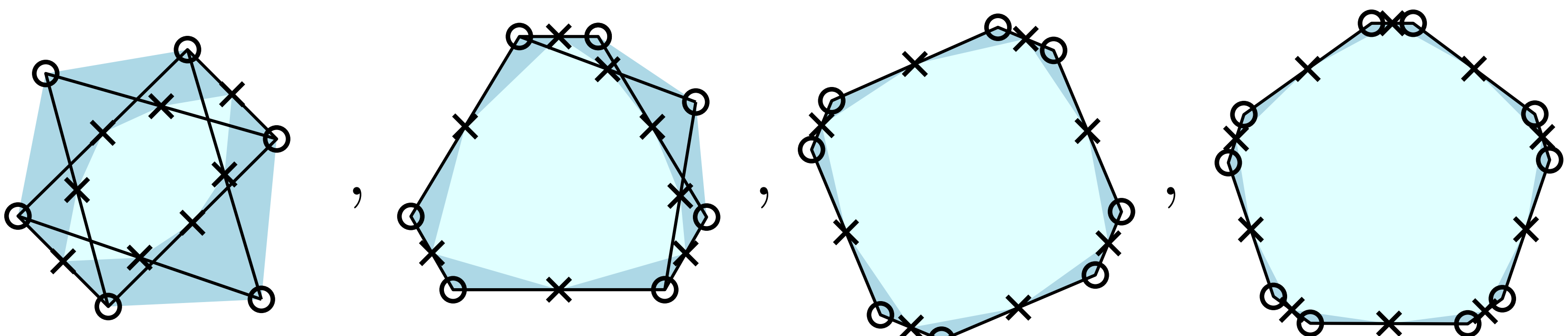

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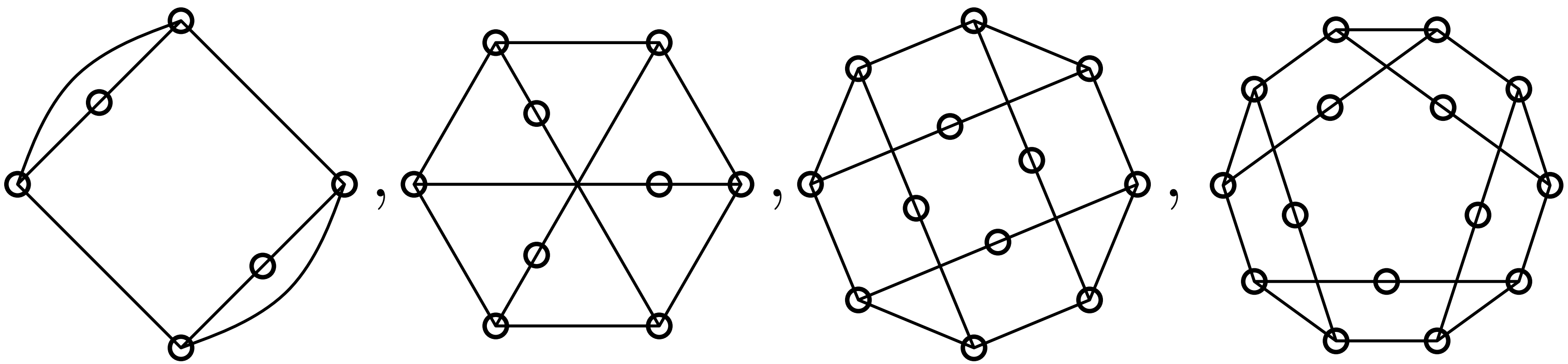

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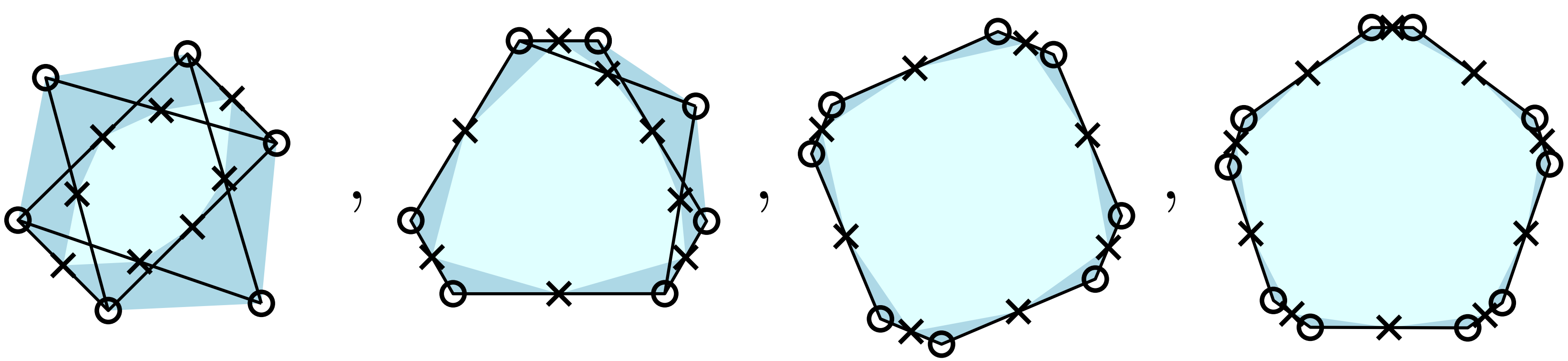

so, this works...and I wont finish these drawings...

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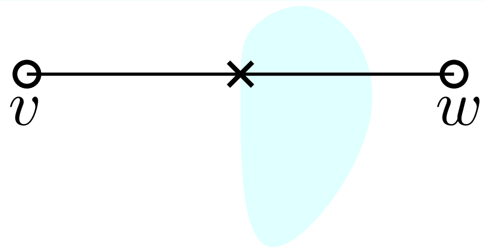

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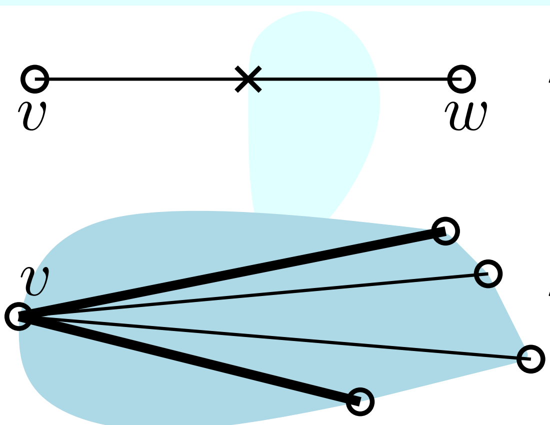

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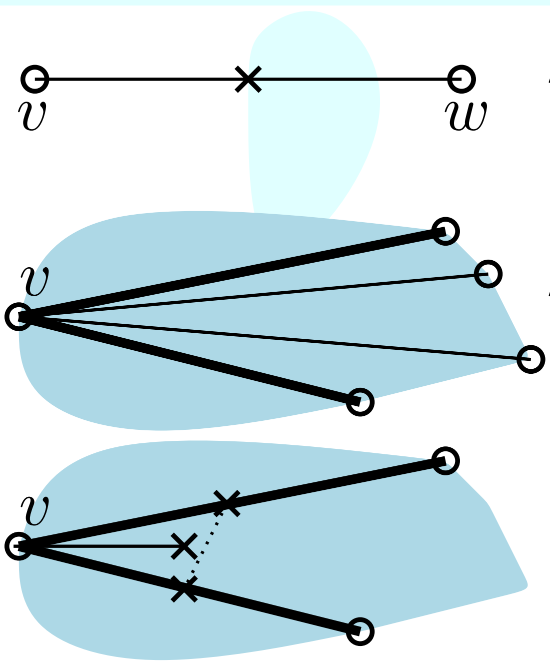

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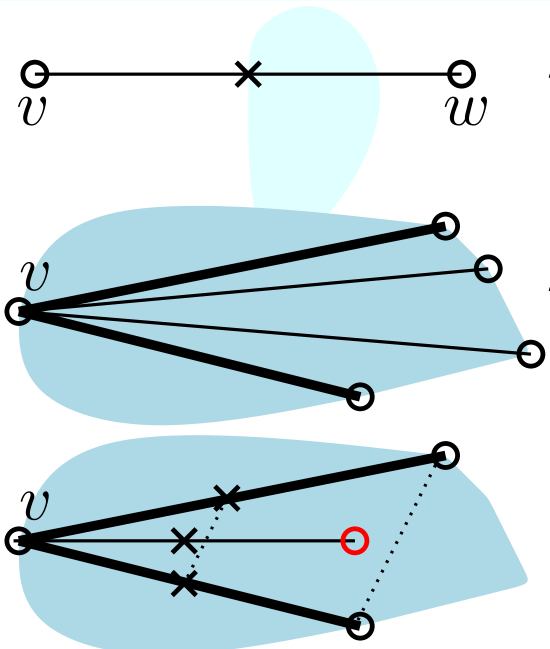

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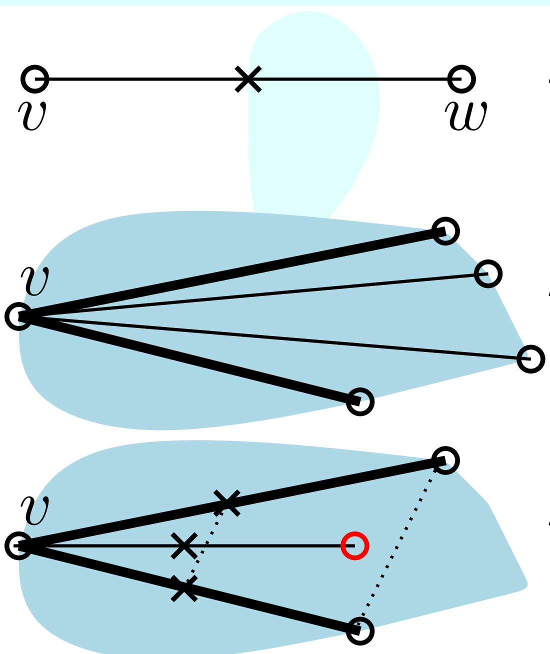

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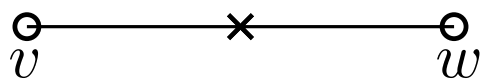

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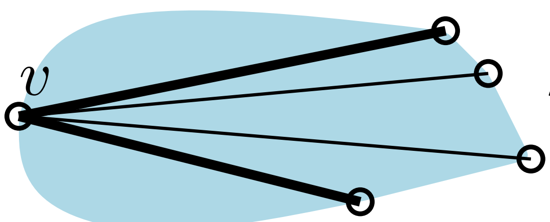

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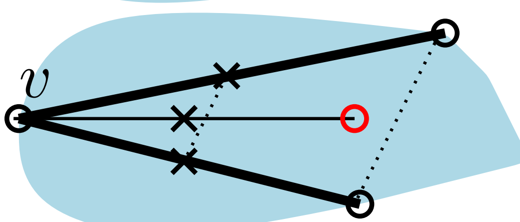

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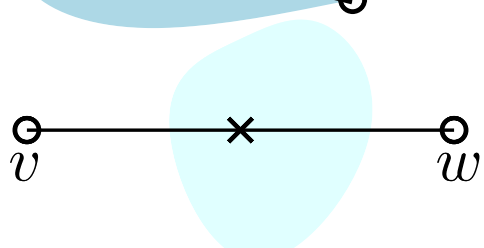

every edge is seen at least once

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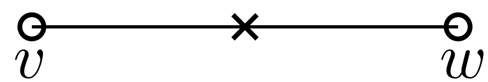

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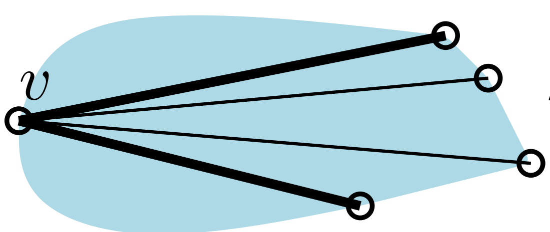

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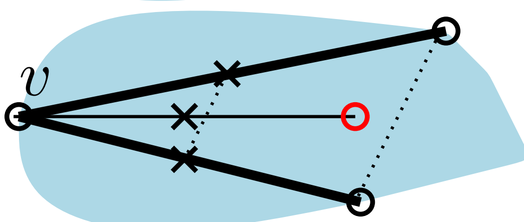

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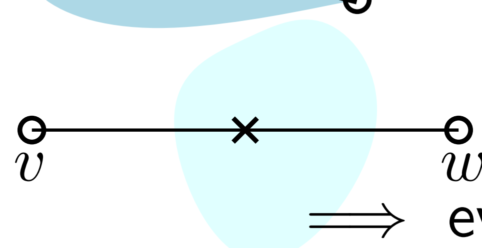

every edge is seen at least once

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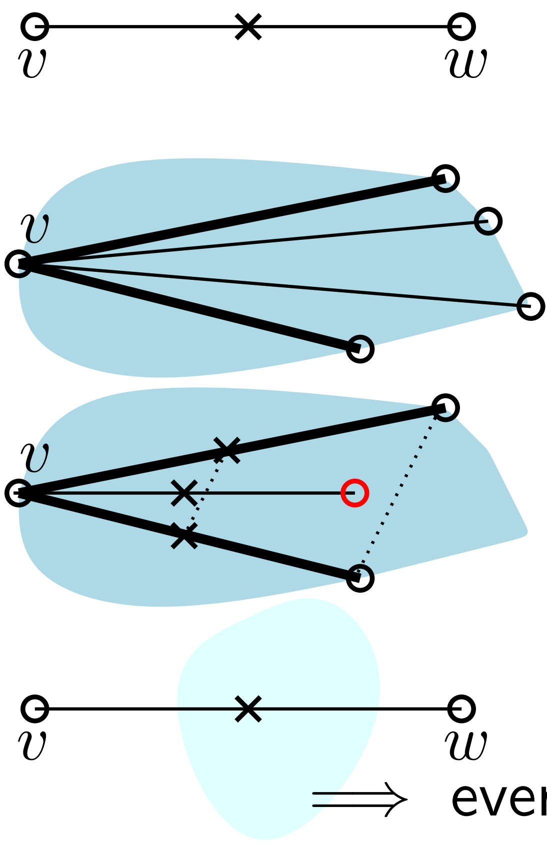

v **sees** vw and w does not

v has at most 2 **exterior** edges

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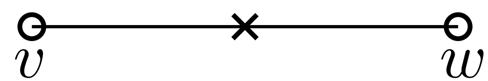

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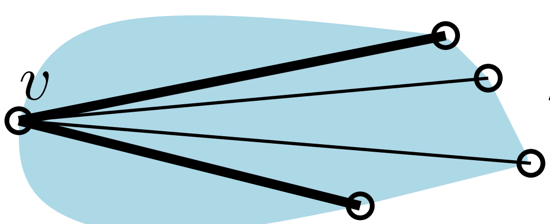

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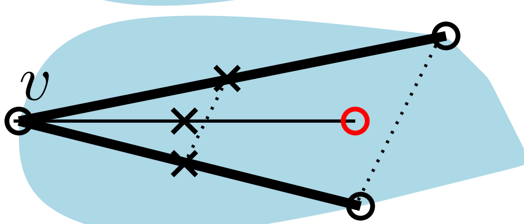

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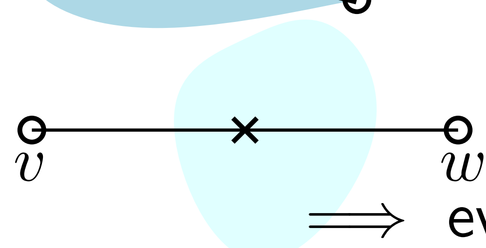

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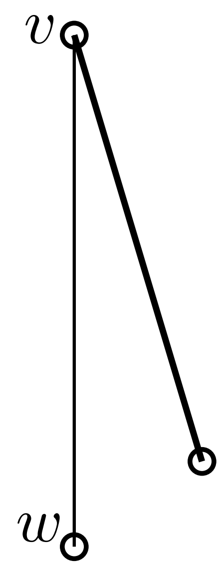

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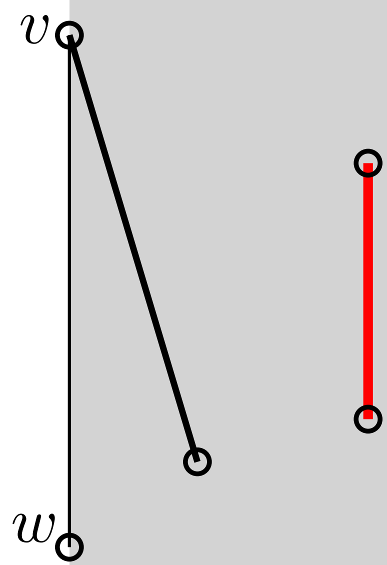

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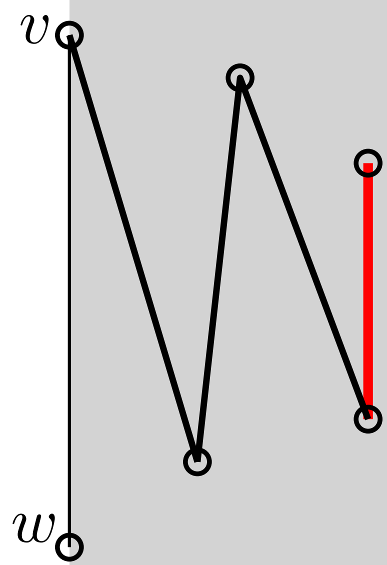

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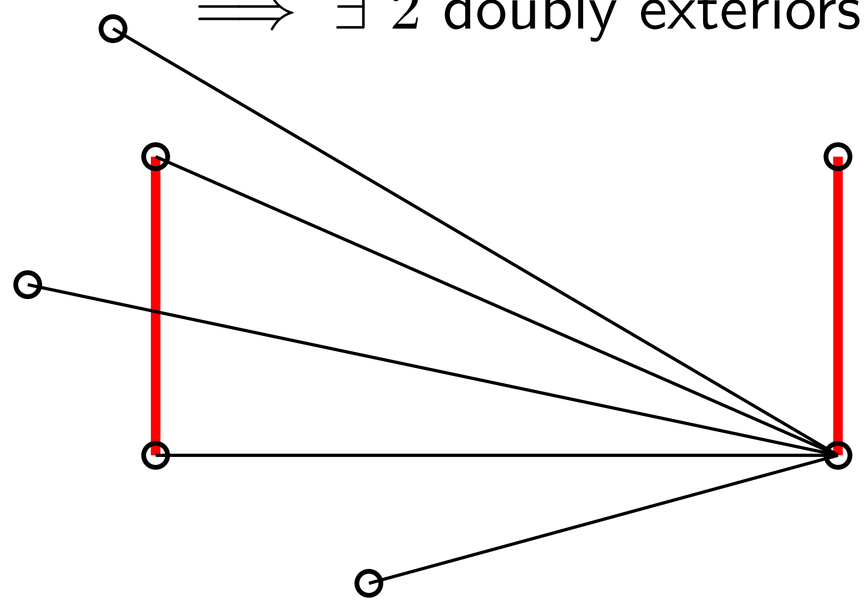

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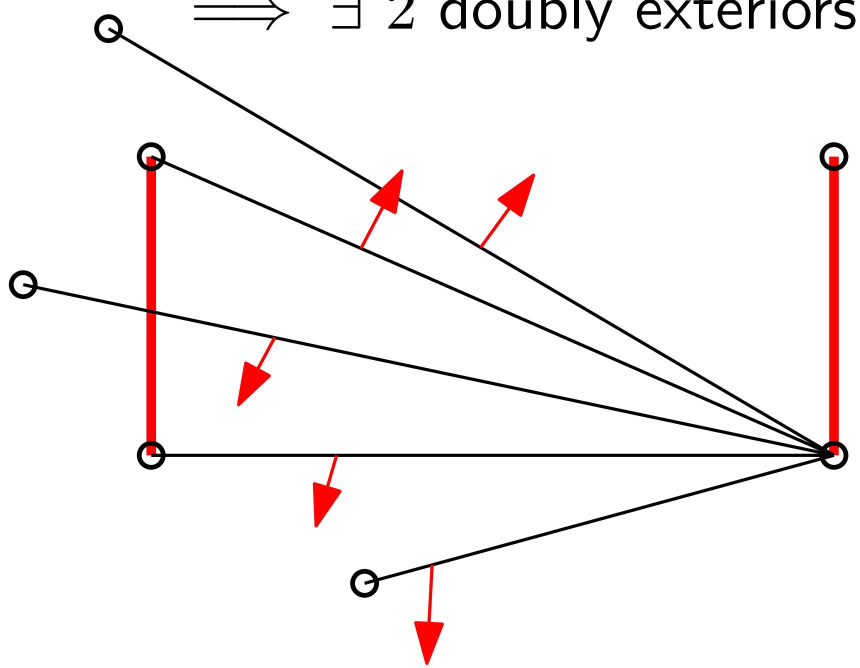

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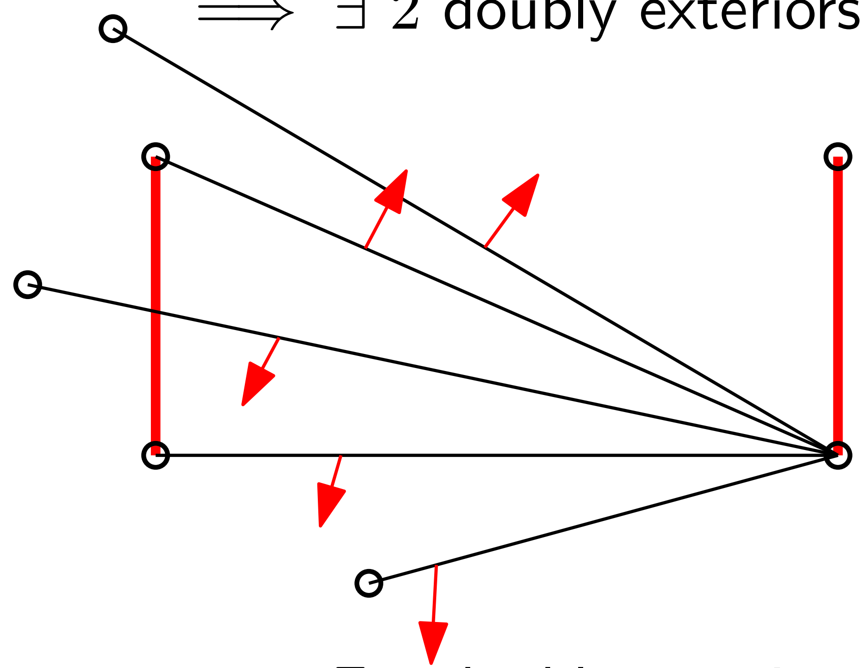

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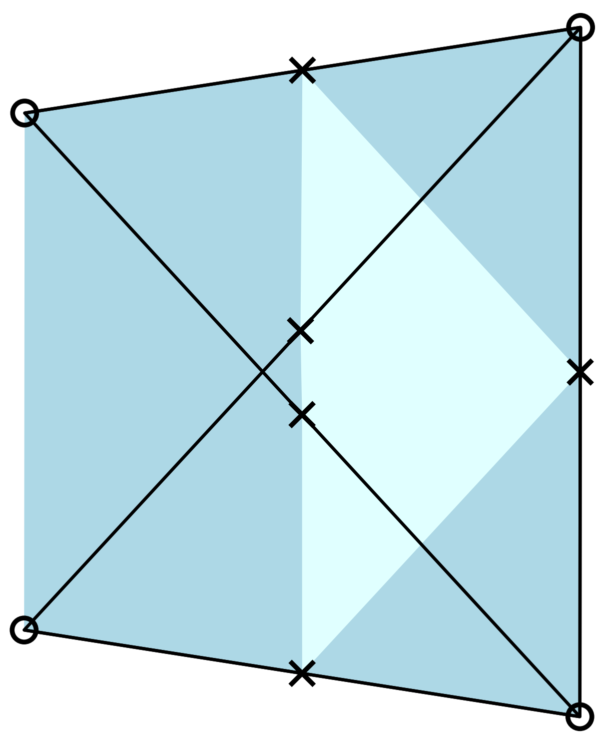

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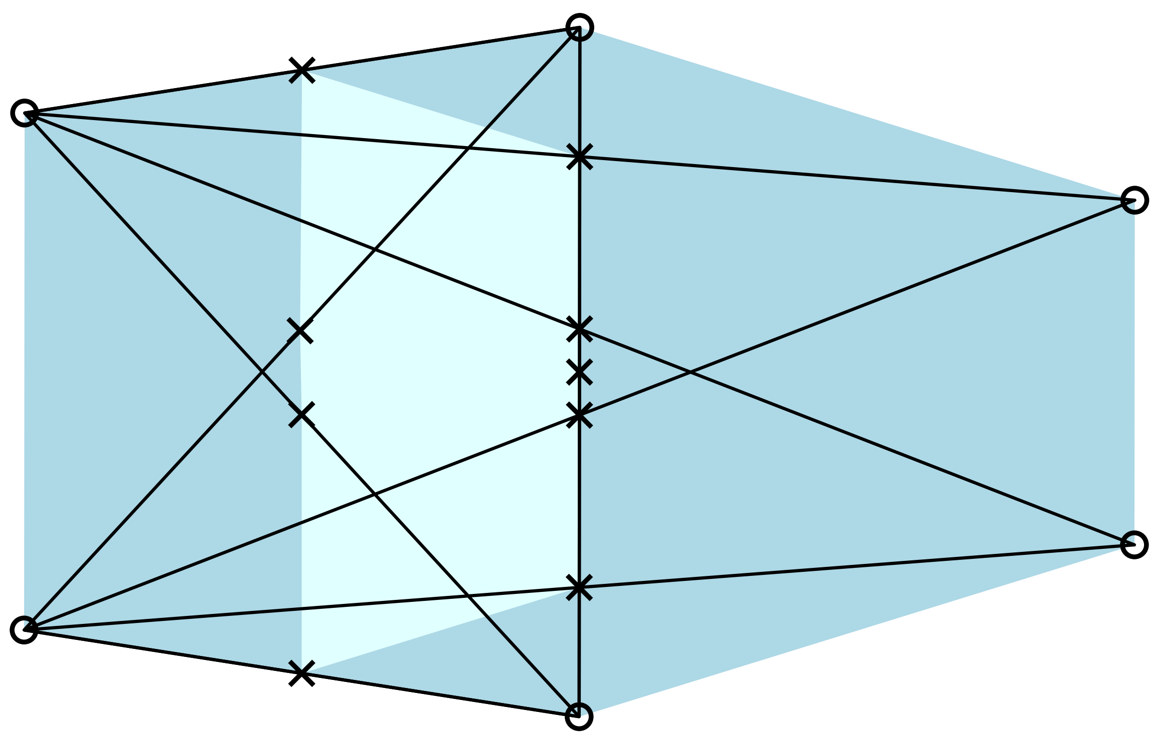

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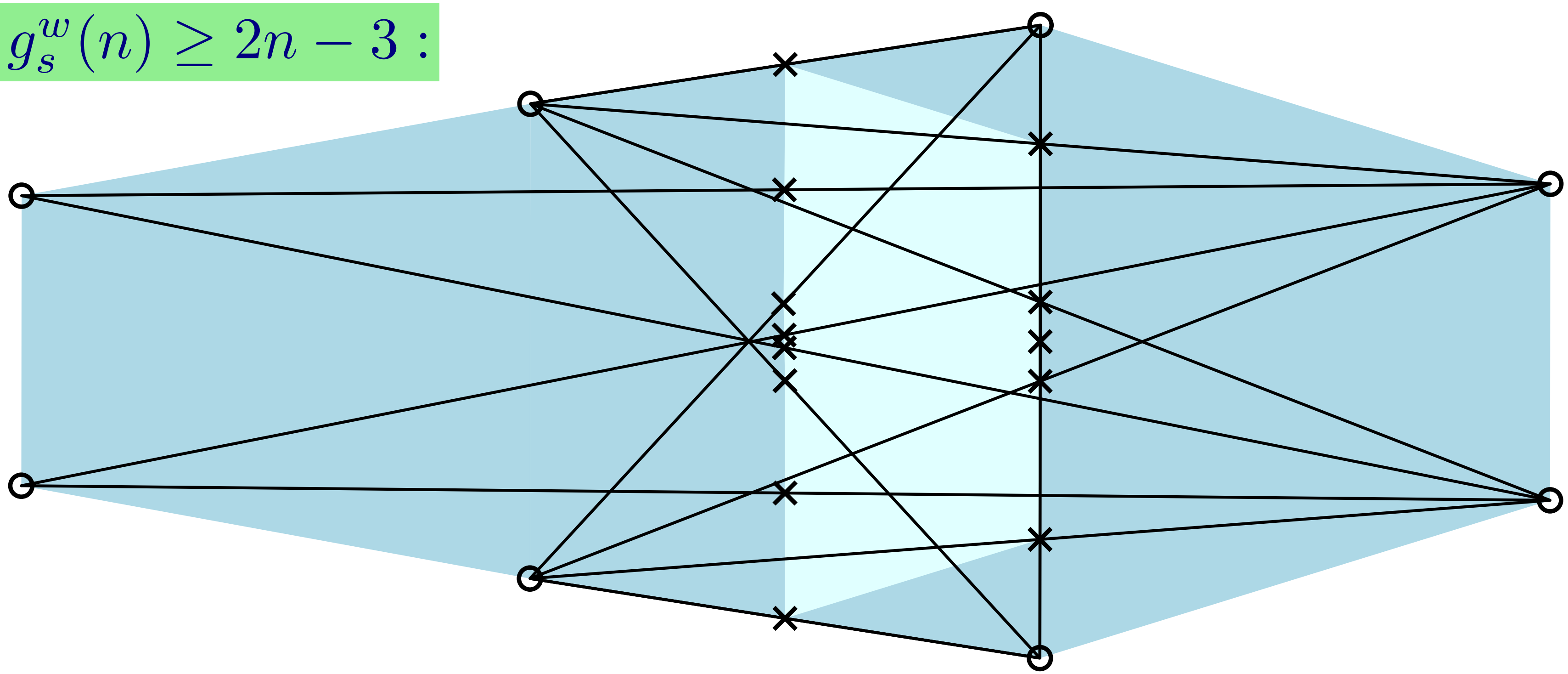

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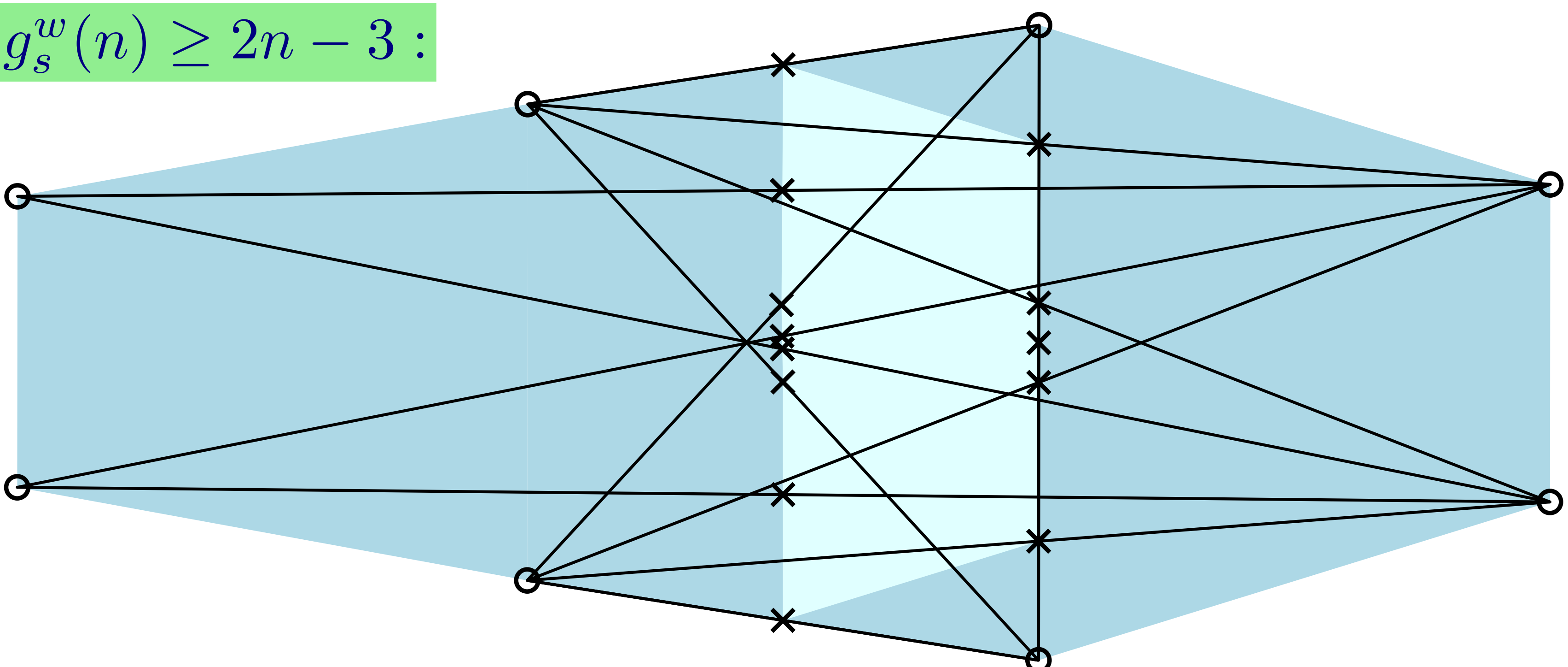

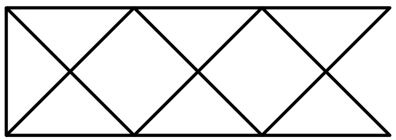

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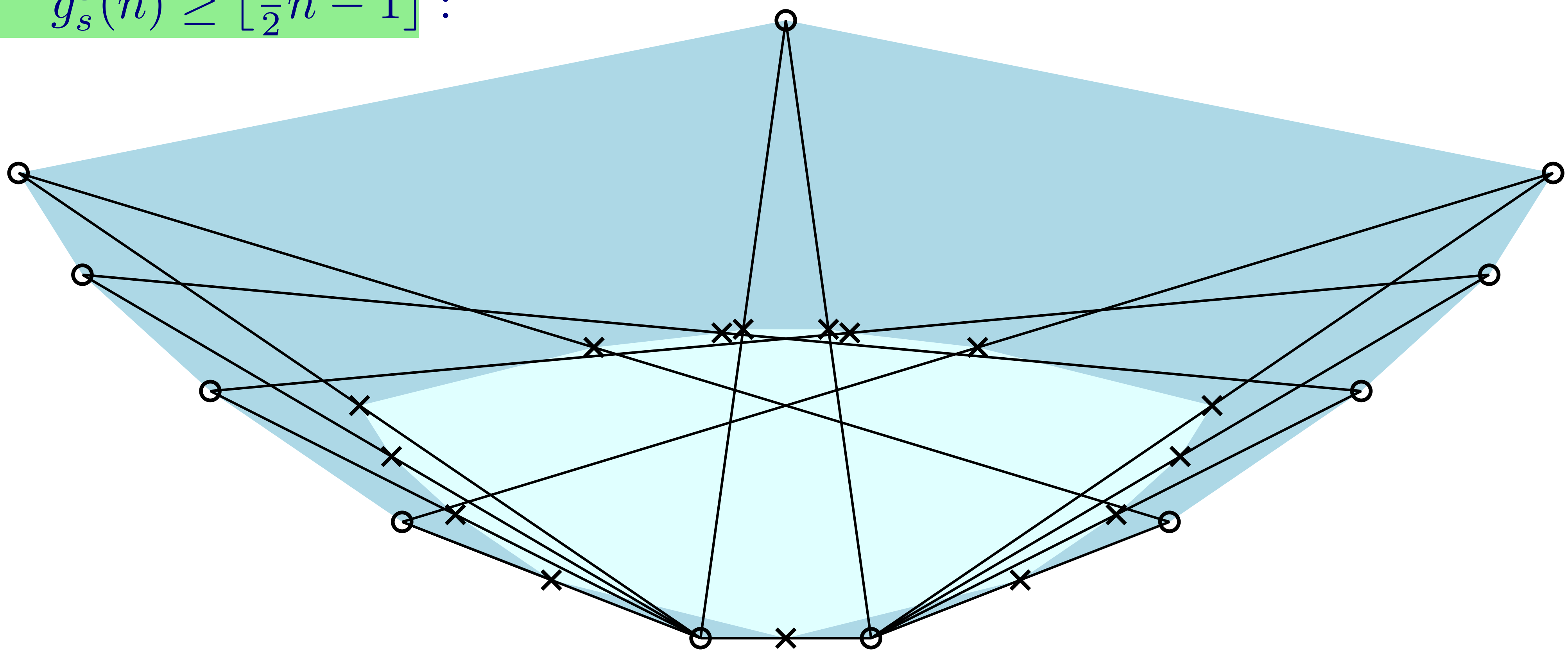

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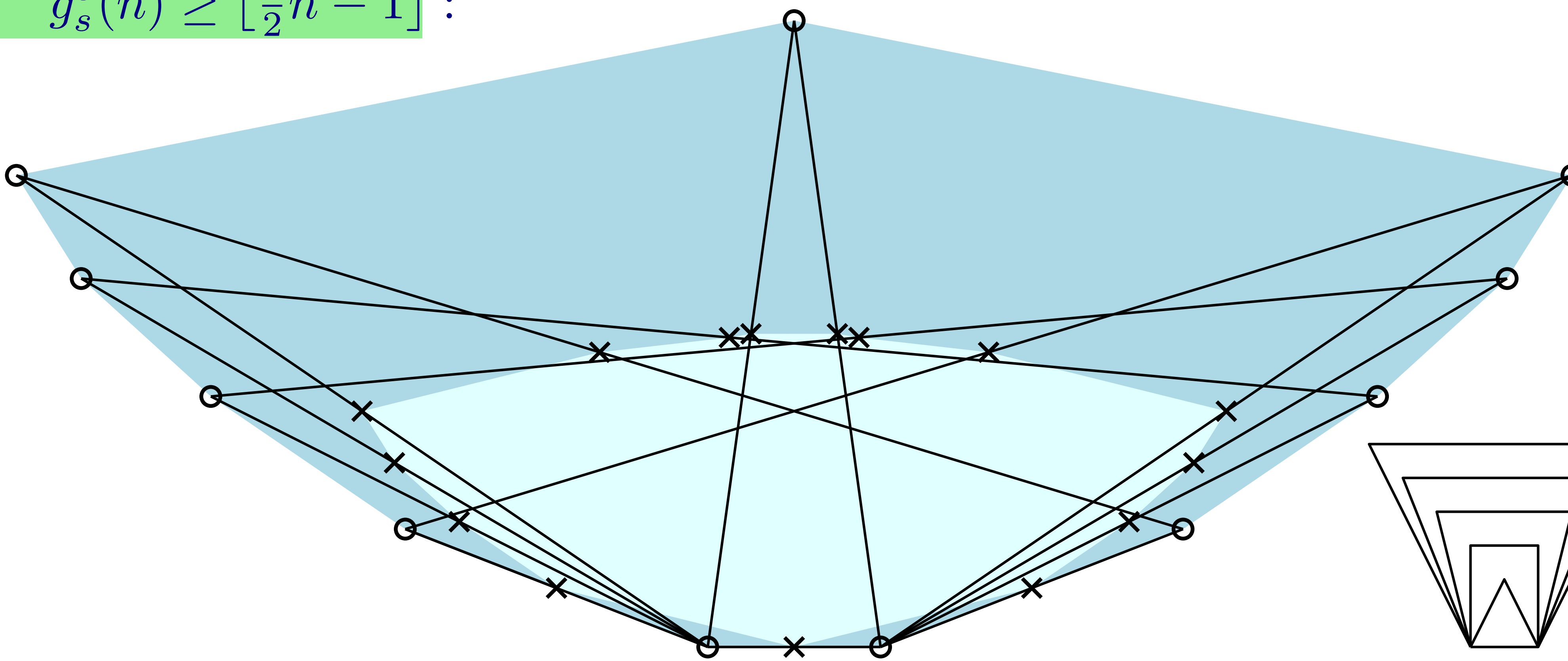

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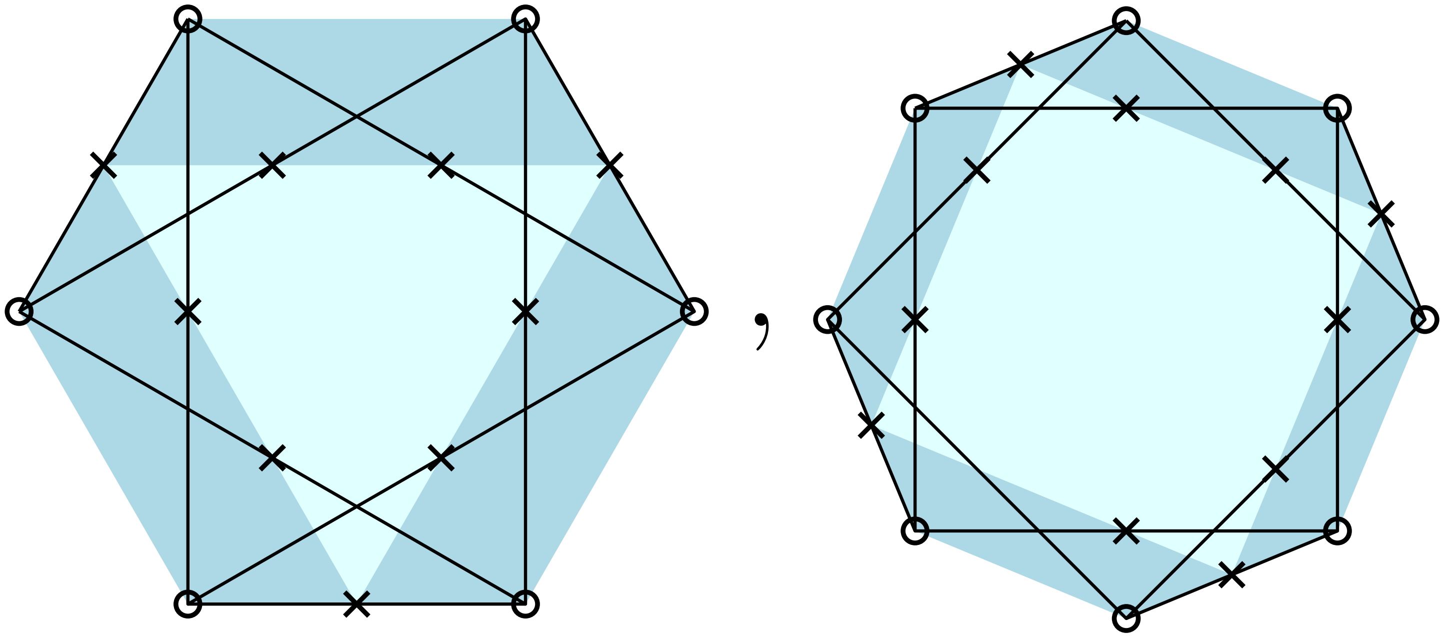

$, \ldots C_k \square K_2$

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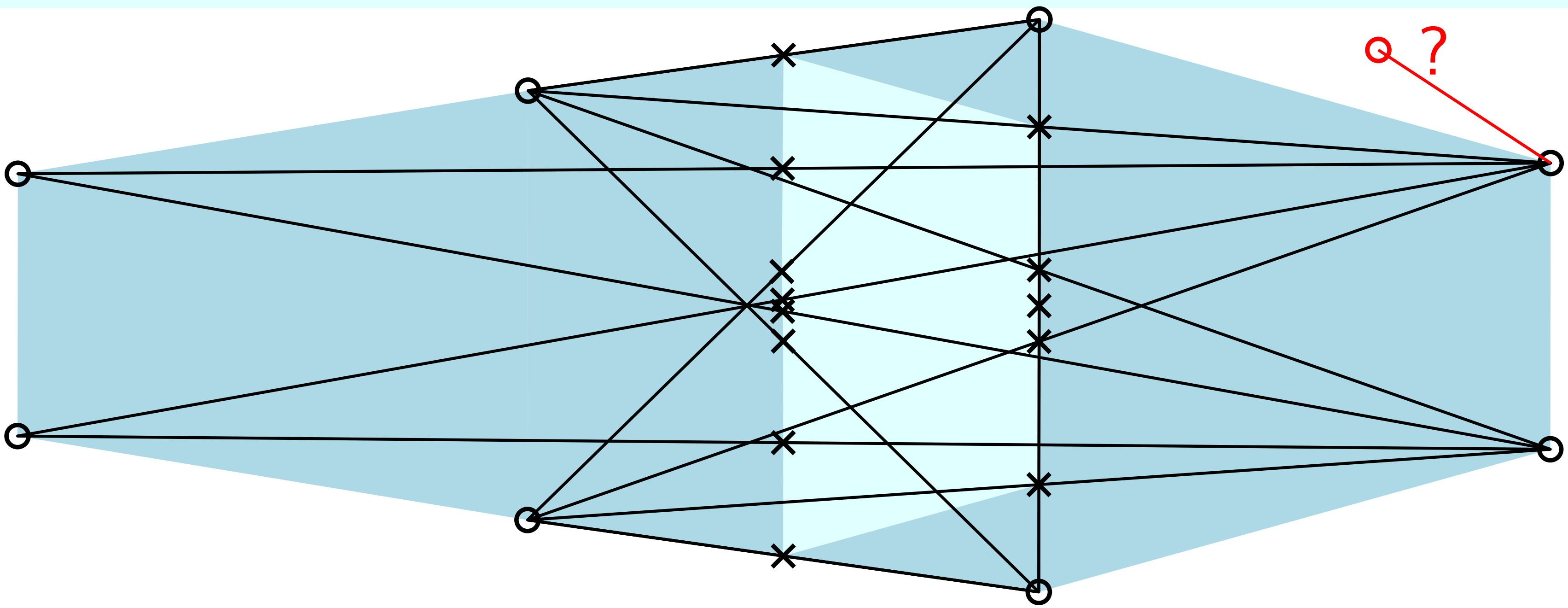

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Conjecture [G-M,K]: Every graph has a (multiple) subdivision in $\mathcal{G}_s^s$.

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fighting for constants

$\widetilde{g}_i^j(n) := \max n' + m$, such that $G \in \mathcal{G}_i^j$ with $|E(G)| = m$, $|V(G)| = n$ and n' of its vertices can be added to the set of midpoints, such that the

$$\text{resulting set is in } \begin{cases} \text{strictly convex} & \text{if } j = s \\ \text{weakly convex} & \text{if } j = w \\ \text{arbitrary} & \text{if } j = a. \end{cases} \text{ position.}$$

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Corollary: largest number of convexly independent points in $A + A$ for n-vertex convex set $A \subseteq \mathbb{R}^2$ lies within $\lfloor \frac{3}{2}n \rfloor$ and $2n - 2$.

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Large convexly independent sets in Minkowski sums

$A, B \subseteq \mathbb{R}^2$ or $\mathbb{R}^3$, $|A| = m$, $|B| = n$.

$A = B$	A convex	B convex	large convex in minkowski sum $\mathbb{R}^2$
○	○	○	$O(m^{\frac{2}{3}}n^{\frac{2}{3}} + m + n)$
○	×	○	$\Omega(m^{\frac{2}{3}}n^{\frac{2}{3}} + m + n)$
×	○	○	$\Omega(n^{\frac{4}{3}} + n)$
○	×	×	$O((m + n)\log(m + n))$
×	×	×	$\frac{2}{3}n \leq \cdot \leq 2n - 2$

Eisenbrand, Pach, Rothvoß, Sopher

Bílka, Buchin, Fulek, Kiyomi, Tanigawa, Tóth

Swanepoel, Valtr

Tiwary

Large convexly independent sets in Minkowski sums

$A, B \subseteq \mathbb{R}^2$ or $\mathbb{R}^3$, $|A| = m$, $|B| = n$.

$A = B$	A convex	B convex	large convex in minkowski sum $\mathbb{R}^2$
○	○	○	$O(m^{\frac{2}{3}} n^{\frac{2}{3}} + m + n)$
○	×	○	$\Omega(m^{\frac{2}{3}} n^{\frac{2}{3}} + m + n)$
×	○	○	$\Omega(n^{\frac{4}{3}} + n)$
○	×	×	$O((m + n) \log(m + n))$
×	×	×	$\frac{2}{3} n \leq \cdot \leq 2n - 2$

Eisenbrand, Pach, Rothvoß, Sopher
Bílka, Buchin, Fulek, Kiyomi, Tanigawa, Tóth
Swanepoel, Valtr
Tiwary

linear expected

Large convexly independent sets in Minkowski sums

$A, B \subseteq \mathbb{R}^2$ or $\mathbb{R}^3$, $|A| = m$, $|B| = n$.

$A = B$	A convex	B convex	large convex in minkowski sum $\mathbb{R}^2$	$\mathbb{R}^3$
○	○	○	$O(m^{\frac{2}{3}}n^{\frac{2}{3}} + m + n)$	$\cdot \leq mn$
○	×	○	$\Omega(m^{\frac{2}{3}}n^{\frac{2}{3}} + m + n)$	$\cdot \leq mn$
×	○	○	$\Omega(n^{\frac{4}{3}} + n)$	$\frac{1}{3}n^2 \leq \cdot \leq \frac{3}{8}n^2 + O(n^{\frac{3}{2}})$
○	×	×	$O((m+n)\log(m+n))$	$mn \leq \cdot$
×	×	×	$\frac{2}{3}n \leq \cdot \leq 2n - 2$	$\frac{1}{4}n^2 \leq \cdot$

Eisenbrand, Pach, Rothvoß, Sopher
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Fukuda, Weibel
Halman, Onn, Rothblum

linear expected

Large convexly independent sets in Minkowski sums

$A, B \subseteq \mathbb{R}^2$ or $\mathbb{R}^3$, $|A| = m$, $|B| = n$.

$A = B$	A convex	B convex	large convex in minkowski sum $\mathbb{R}^2$	$\mathbb{R}^3$
∘	∘	∘	$O(m^{\frac{2}{3}} n^{\frac{2}{3}} + m + n)$	$\cdot \leq mn$
∘	×	∘	$\Omega(m^{\frac{2}{3}} n^{\frac{2}{3}} + m + n)$	$\cdot \leq mn$
×	∘	∘	$\Omega(n^{\frac{4}{3}} + n)$	$\frac{1}{3} n^2 \leq \cdot \leq \frac{3}{8} n^2 + O(n^{\frac{3}{2}})$
∘	×	×	$O((m+n)\log(m+n))$	$mn \leq \cdot$
×	×	×	$\frac{2}{3} n \leq \cdot \leq 2n - 2$	$\frac{1}{4} n^2 \leq \cdot$

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$\Leftrightarrow \exists c > 1 : K_{c,c,c,c} \in 3D - \mathcal{G}_a^s$

linear expected