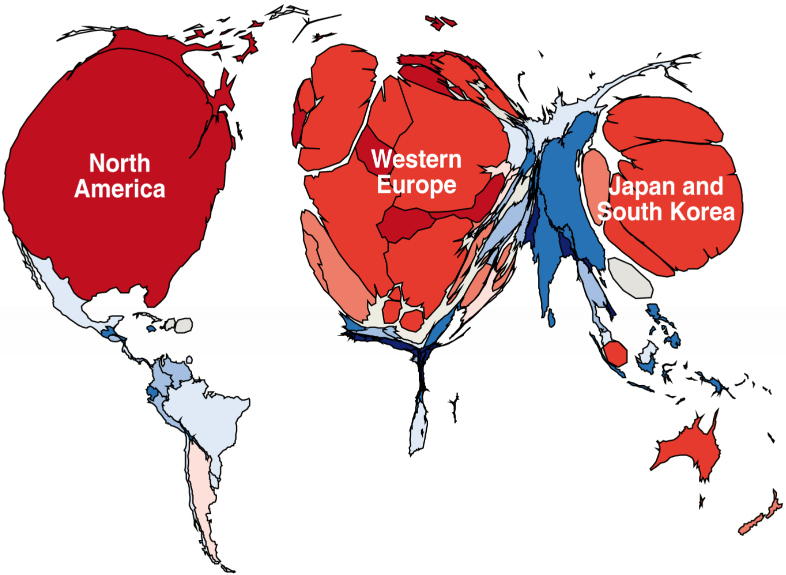
Towards characterizing graphs with a sliceable rectangular dual

Vincent Kusters Bettina Speckmann

ETH Zurich TU Eindhoven

September 26, 2015

Cartograms



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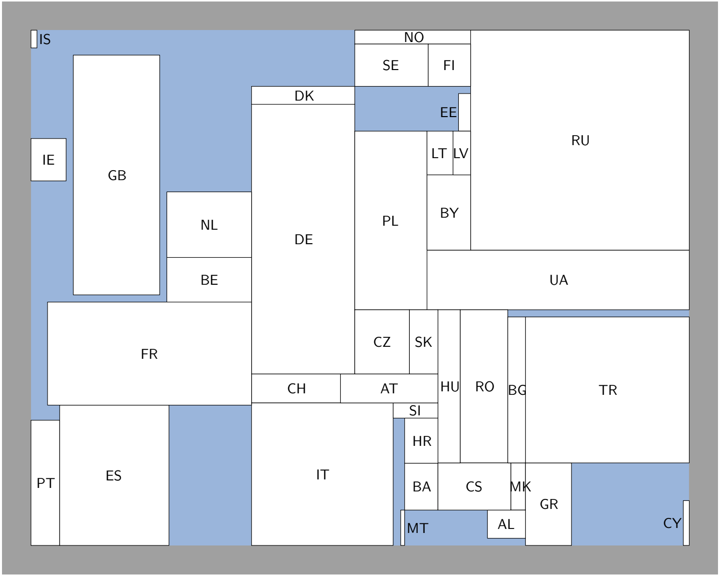

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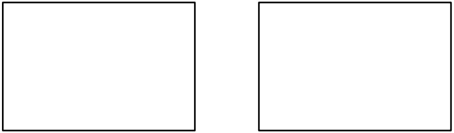
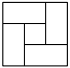
Rectangular partitions

A rectangular partition of a rectangle *R* is a set of non-overlapping rectangles that together form *R*.





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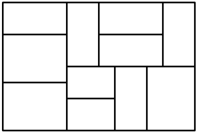

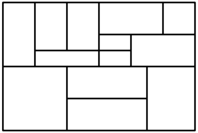

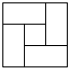

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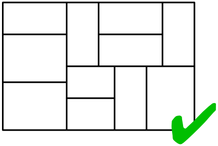

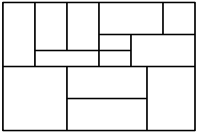

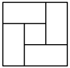

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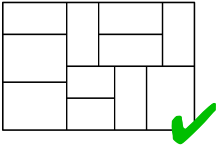
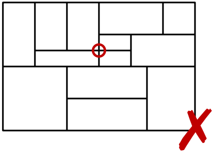
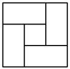


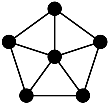


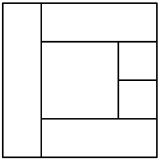


Rectangular duals

A rectangular dual of a planar graph G is a rectangular partition R, such that:

- vertices in G correspond to rectangles in R and
- edges in G correspond to shared borders in R.





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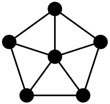
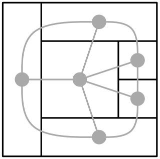
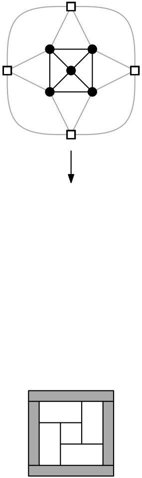
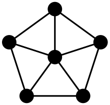


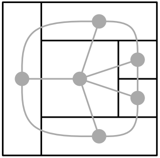


Corner assignments



A corner assignment or extended graph $E(G)$ of G is an extention of G with four vertices. The four vertices form the outer cycle of $E(G)$.





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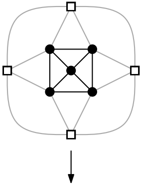

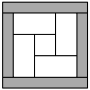

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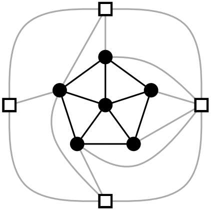

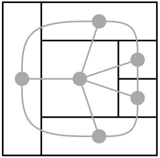

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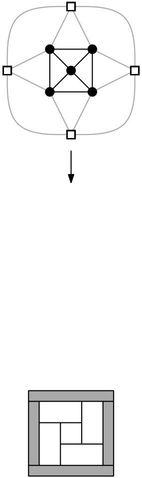

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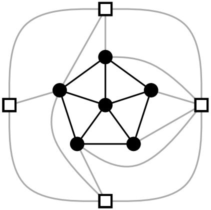
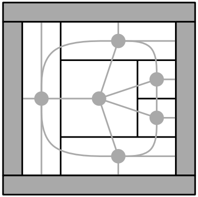
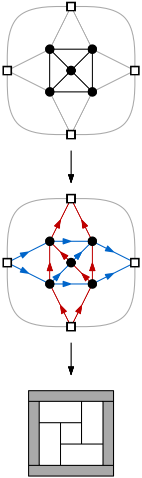
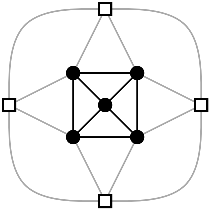


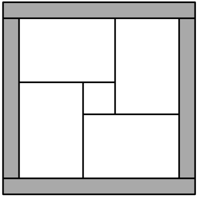


Regular edge labelings



A regular edge labeling of an extended graph $E(G)$ is a partition of the interior edges of $E(G)$ into red and blue directed edges.





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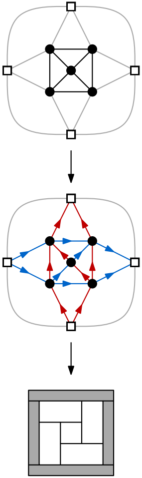

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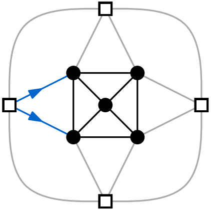

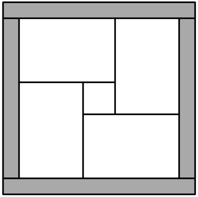

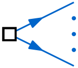

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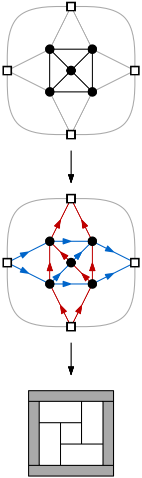

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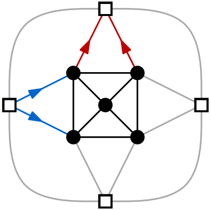

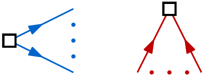

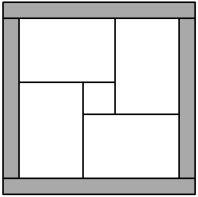

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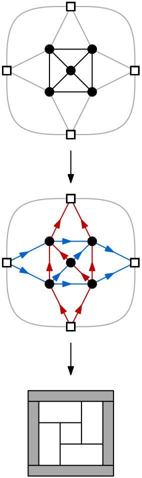

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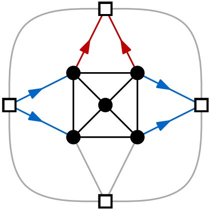

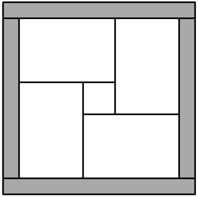

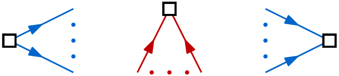

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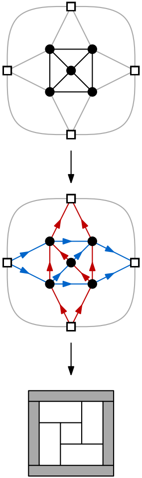

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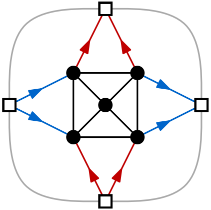

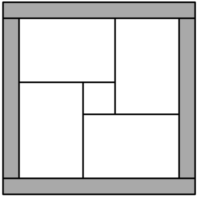

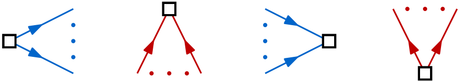

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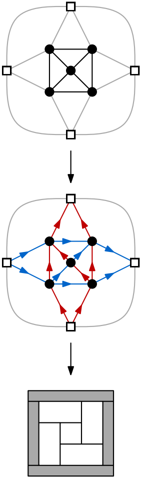

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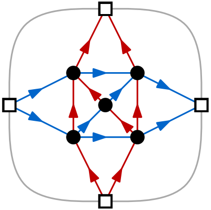

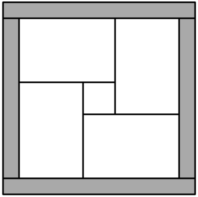

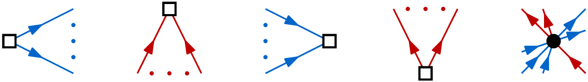

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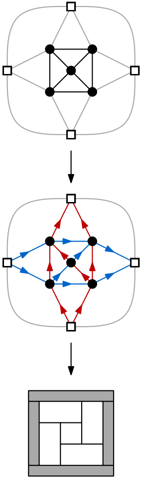

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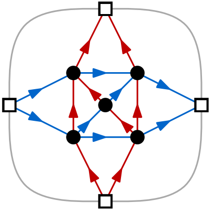
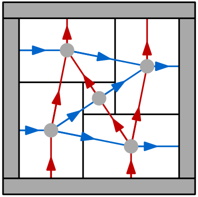
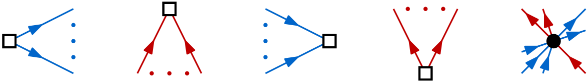
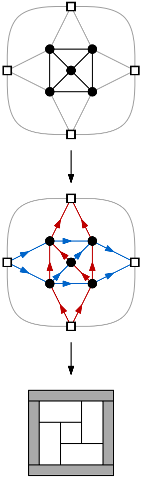


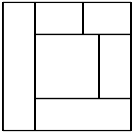


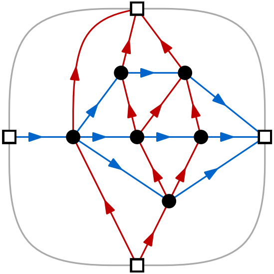


Sliceable rectangular duals



A rectangular dual is sliceable if it can be recursively partitioned along horizontal or vertical lines:





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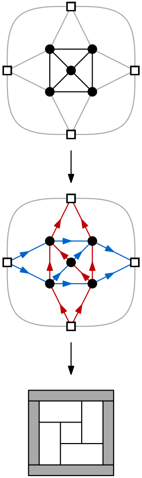

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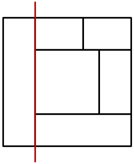

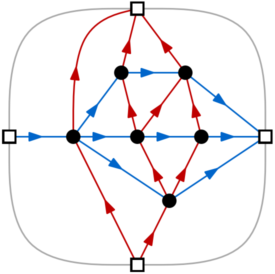

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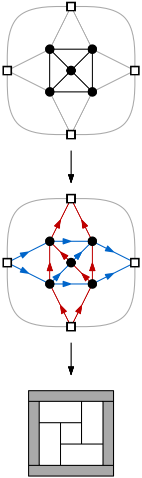

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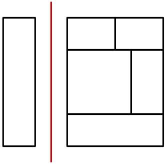

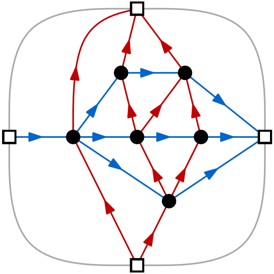

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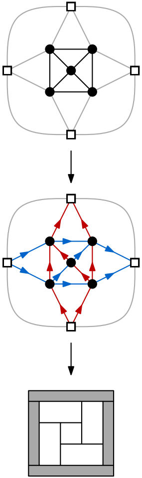

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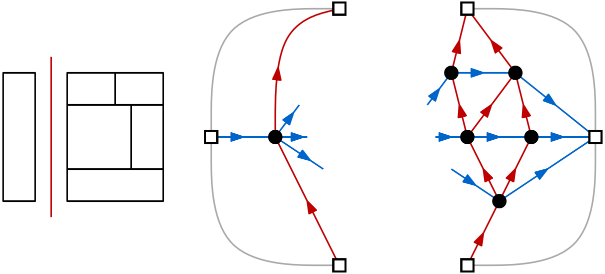

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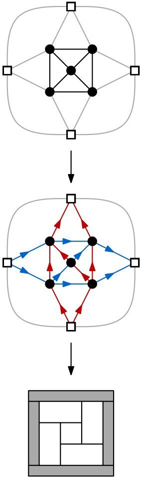

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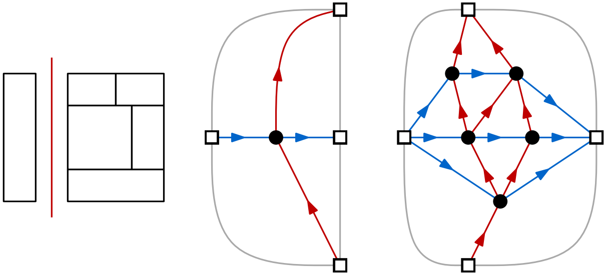

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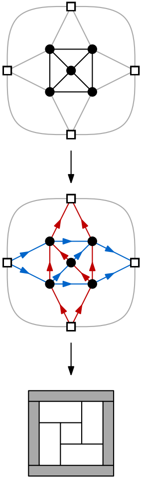

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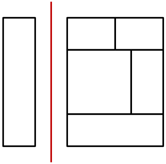
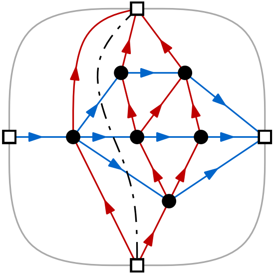
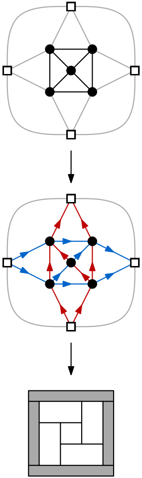




Sliceable rectangular duals



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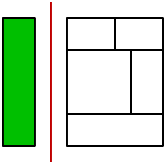

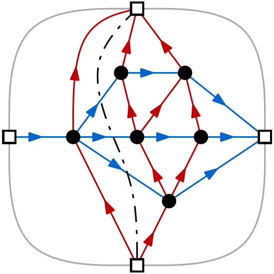

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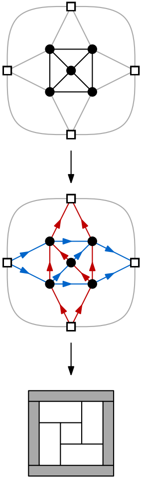

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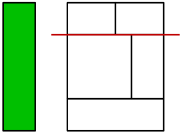

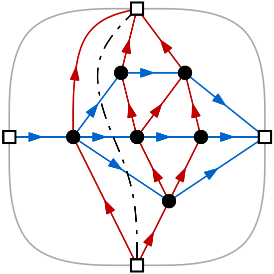

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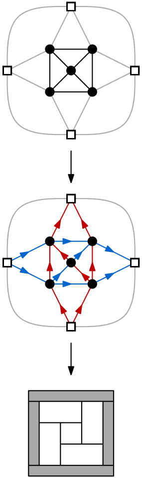

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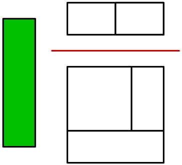

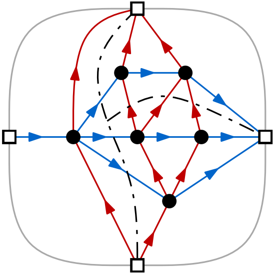

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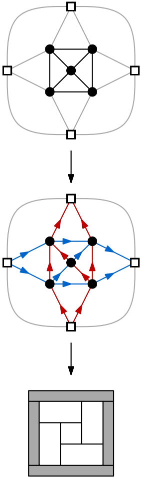

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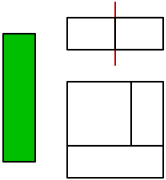

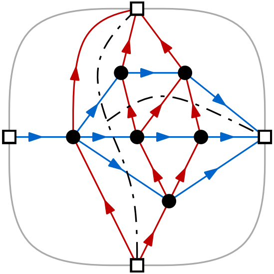

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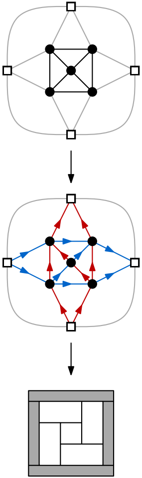

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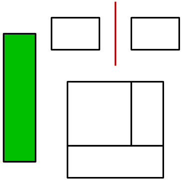

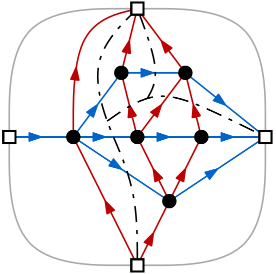

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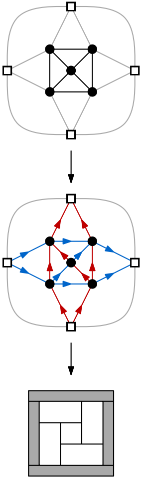

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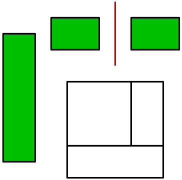

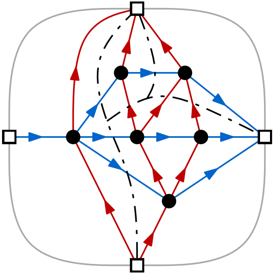

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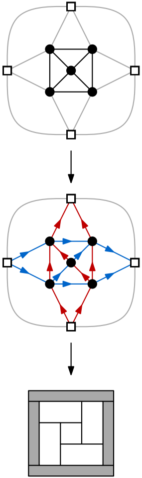

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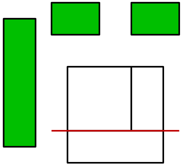

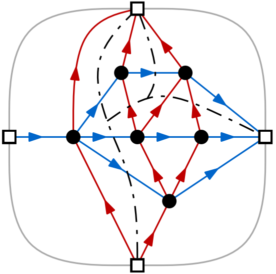

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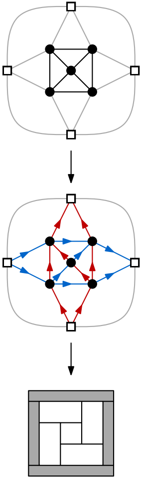

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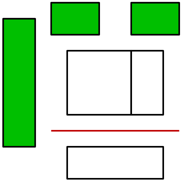
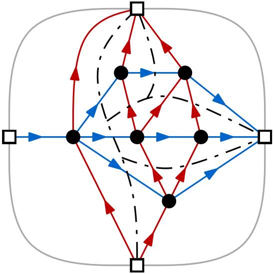
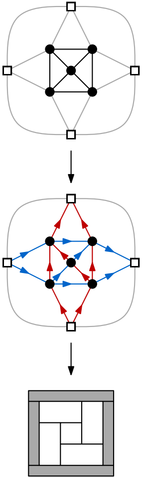




Sliceable rectangular duals



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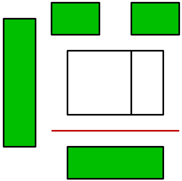

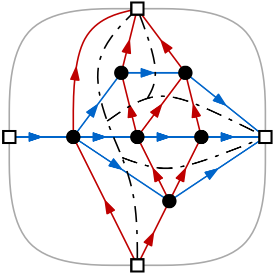

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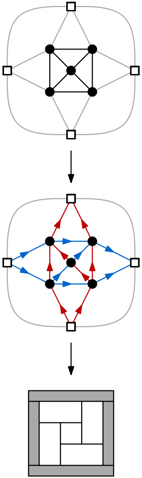

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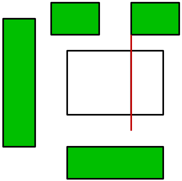

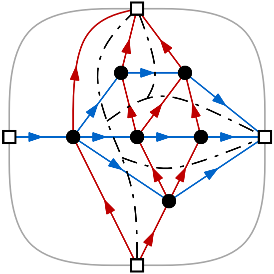

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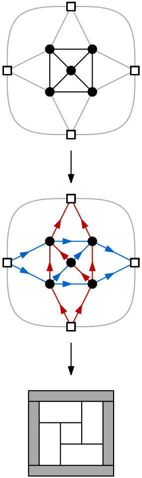

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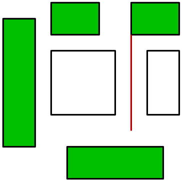

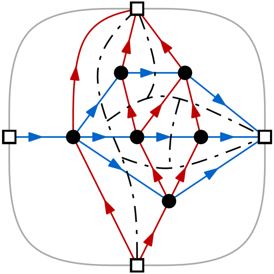

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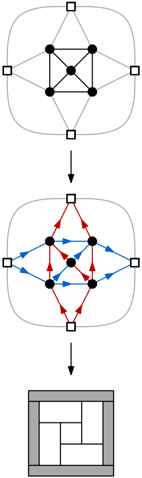

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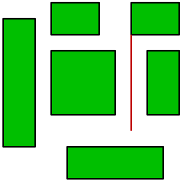

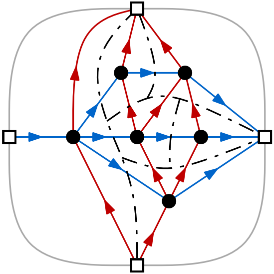

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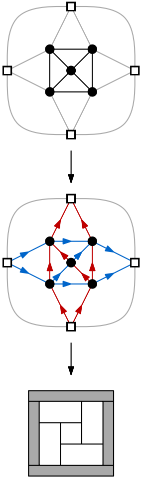

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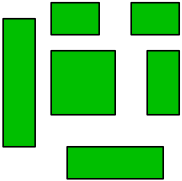
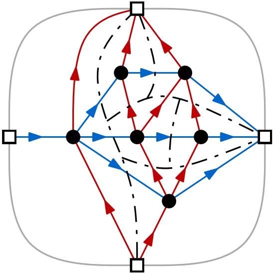
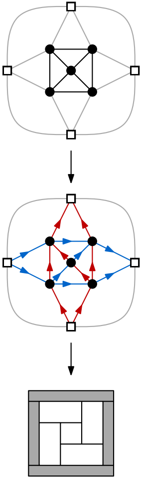


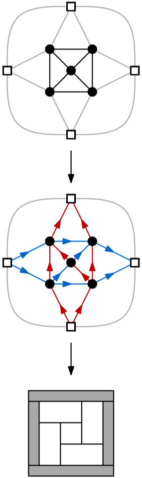


Characterizing sliceable graphs



An extended graph is sliceable if it has a sliceable rectangular dual.

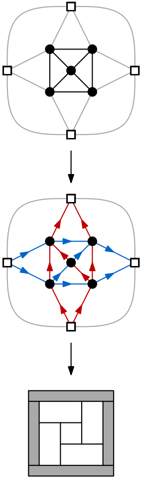
Characterizing sliceable graphs



An extended graph is sliceable if it has a sliceable rectangular dual.

- $E(G)$ has a separating 3-cycle

Characterizing sliceable graphs



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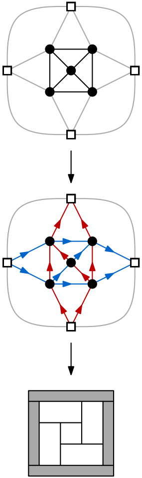
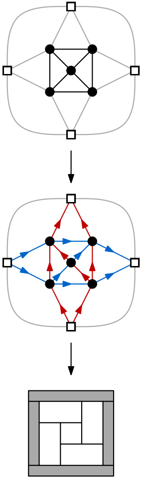


An extended graph is sliceable if it has a sliceable rectangular dual.

- $E(G)$ has a separating 3-cycle then not sliceable (Koźmiński and Kinnen 1985)

Characterizing sliceable graphs



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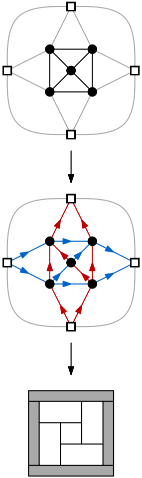
Characterizing sliceable graphs



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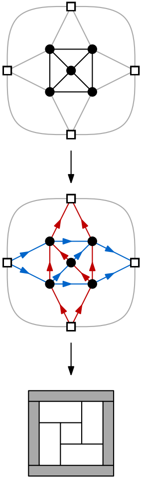

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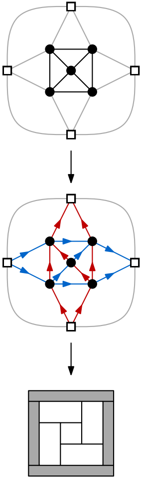
Characterizing sliceable graphs



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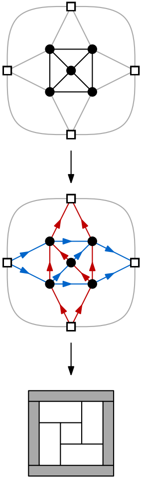
  then ???

Characterizing sliceable graphs



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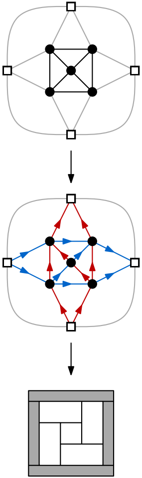
- G has exactly one separating 4-cycle

Characterizing sliceable graphs



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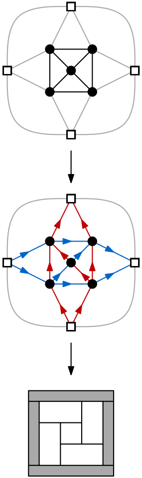
- G has a separating 4-cycle
  then ???
- G has exactly one separating 4-cycle
  then sliceable $\iff$ not rotating windmill

Characterizing sliceable graphs



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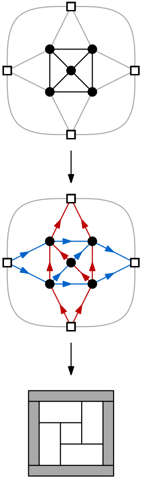
Characterizing sliceable graphs



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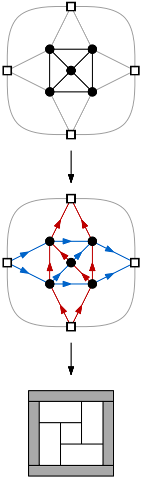
Proof.

$\Longrightarrow$: Show that rotating windmills are not sliceable.

□

Characterizing sliceable graphs



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- G has exactly one separating 4-cycle then sliceable $\Longleftrightarrow$ not rotating windmill

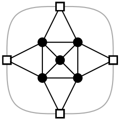
Proof.

$\Longrightarrow$: Show that rotating windmills are not sliceable.

$\Longleftarrow$: Given an extended graph $E(G)$ that is not a rotating windmill, show that we can find a slice that splits $E(G)$ into extended graphs that are not rotating windmills.

□

Rotating windmills



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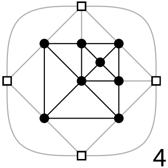

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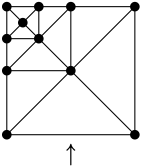
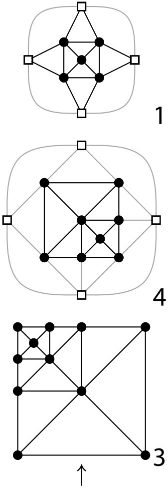


3

The following extended graphs are rotating windmills:

- the windmill,
- the four base rotating windmills,
- any extended graph obtained by applying one of three construction steps.

Rotating windmills



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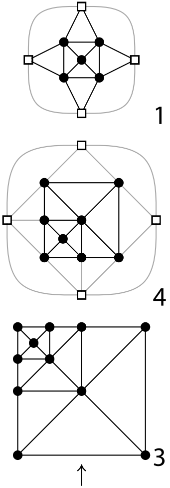

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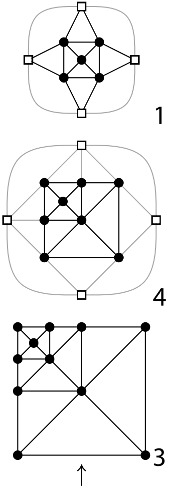

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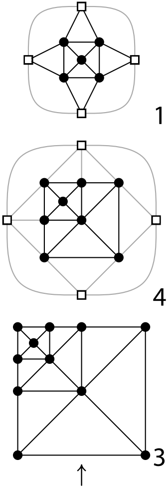

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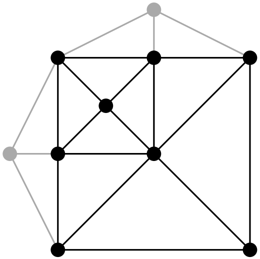
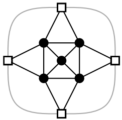


Rotating windmills



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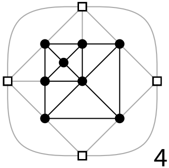

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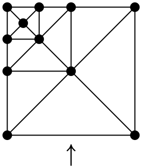
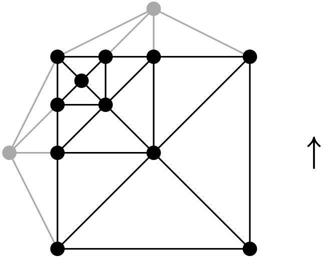


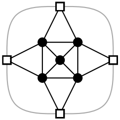
3

The following extended graphs are rotating windmills:

- the windmill,
- the four base rotating windmills,
- any extended graph obtained by applying one of three construction steps.



Rotating windmills



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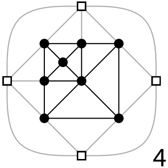

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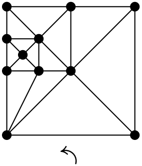

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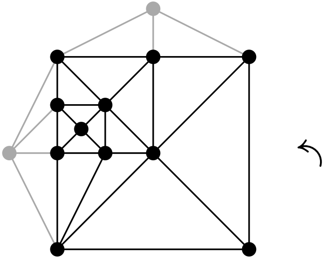
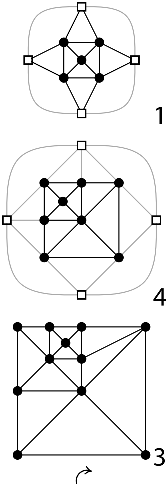


Rotating windmills



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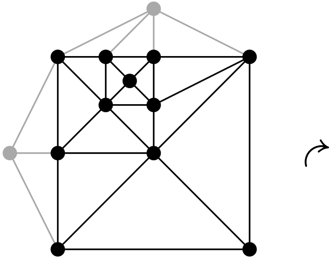
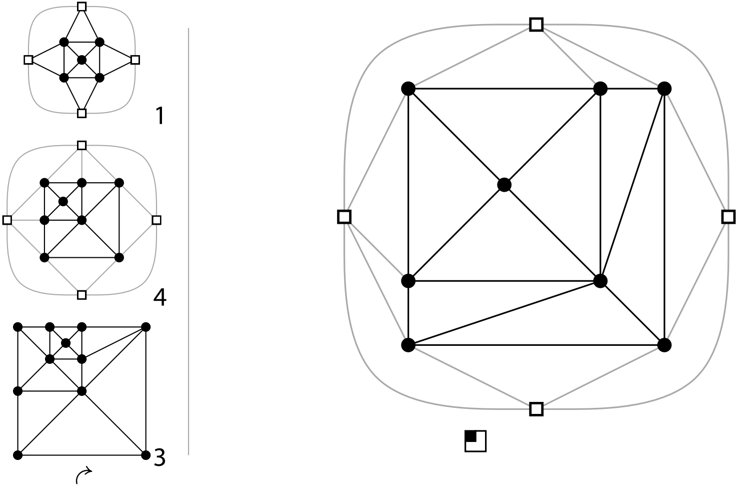


Rotating windmills



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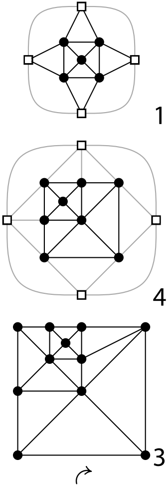
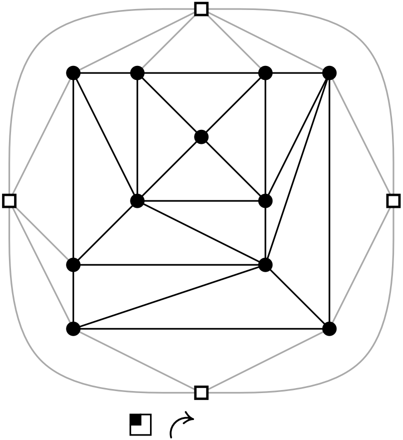
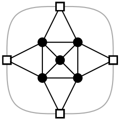




Rotating windmills



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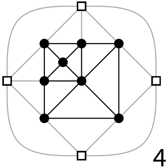

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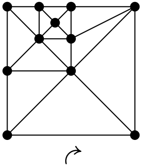

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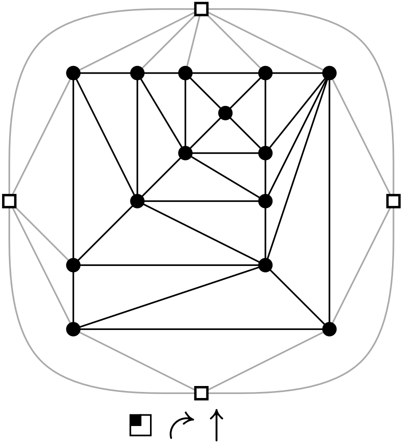

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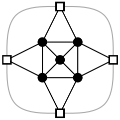

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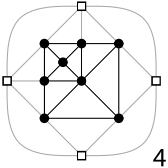

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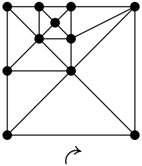

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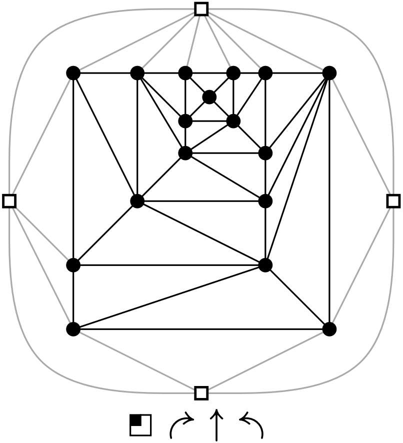

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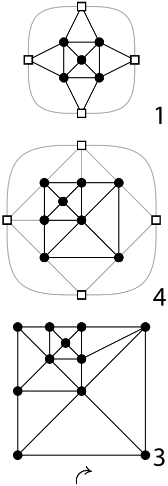
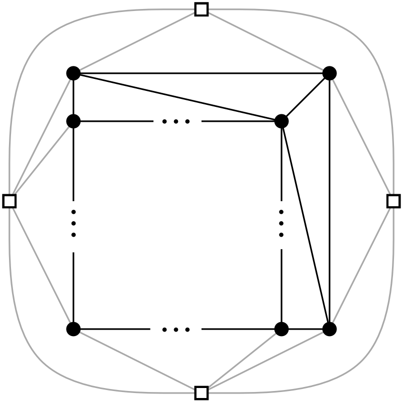


⟹: Rotating windmills are not sliceable.



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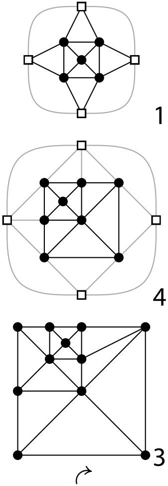

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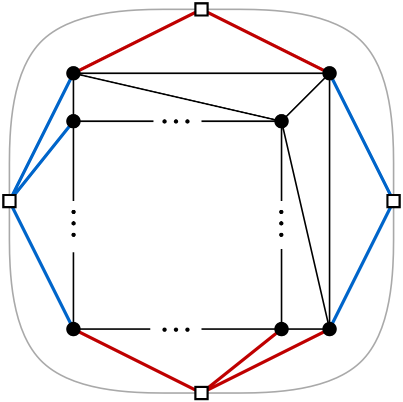

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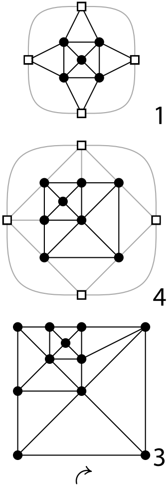

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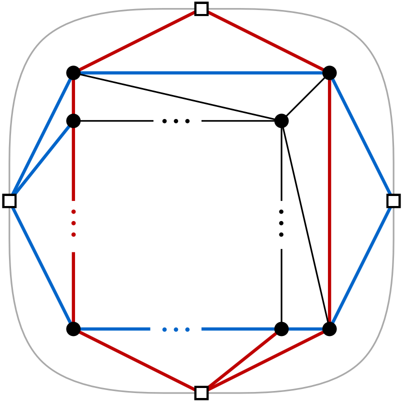

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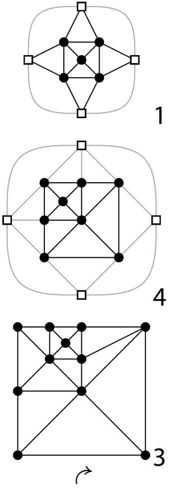

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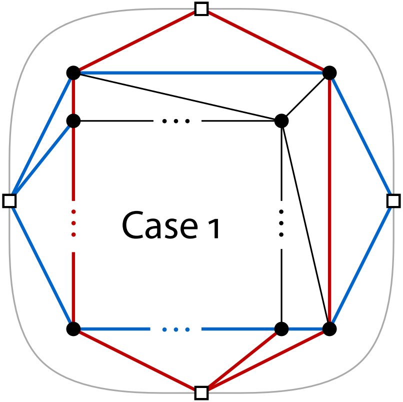

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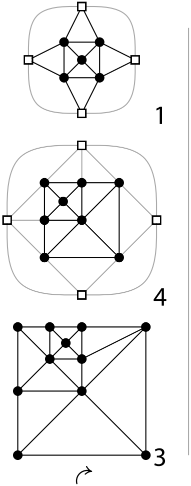

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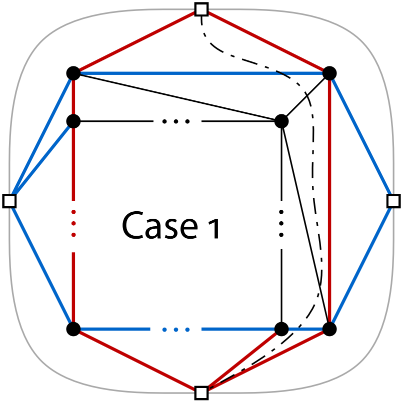

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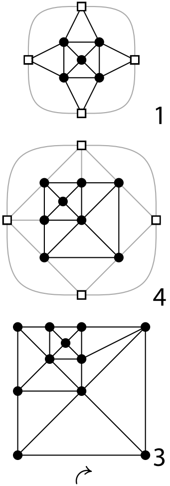

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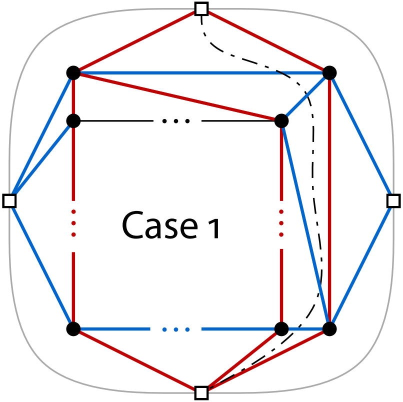

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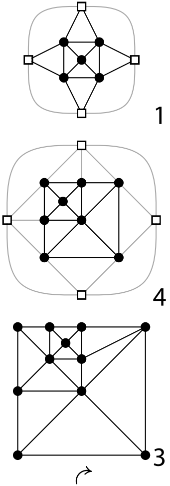

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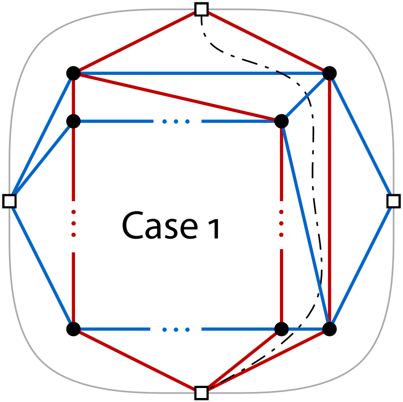

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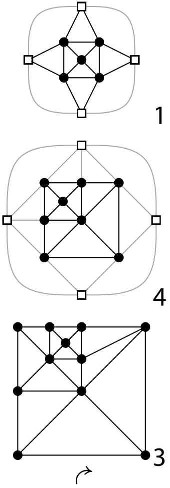

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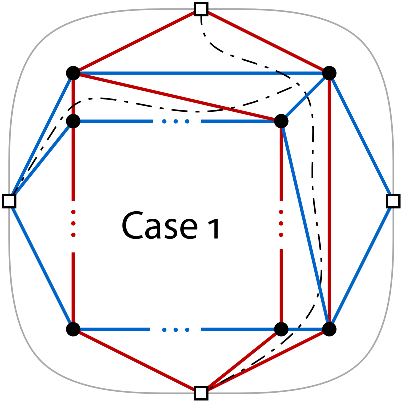

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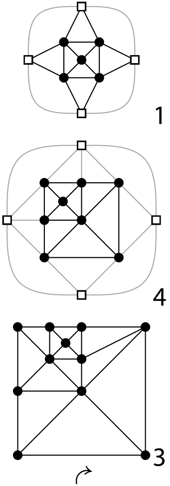

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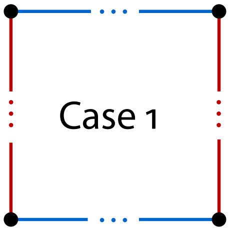

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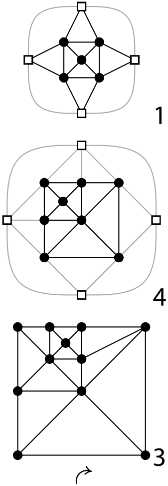

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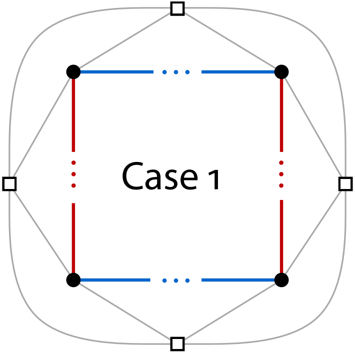

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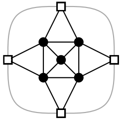

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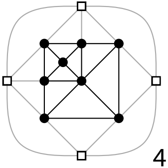

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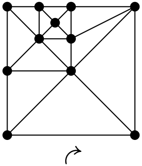

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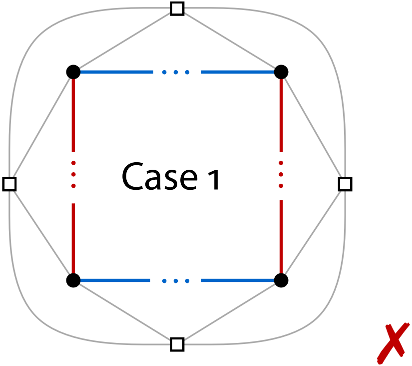

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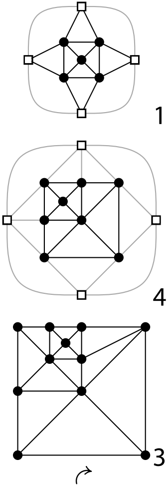

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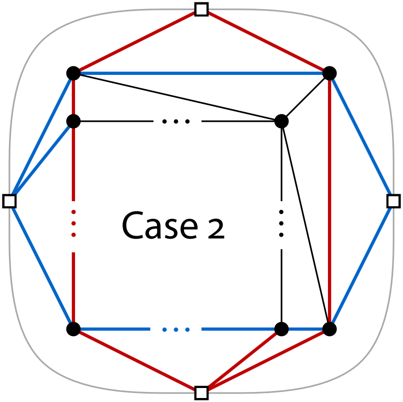

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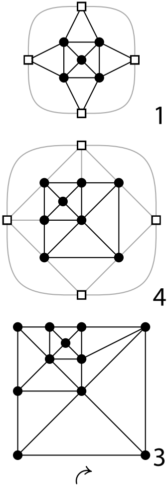

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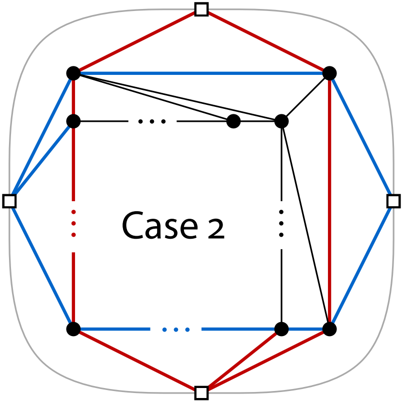

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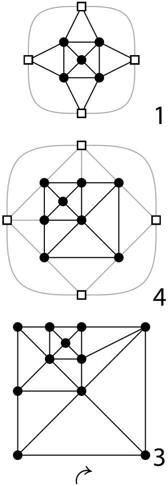

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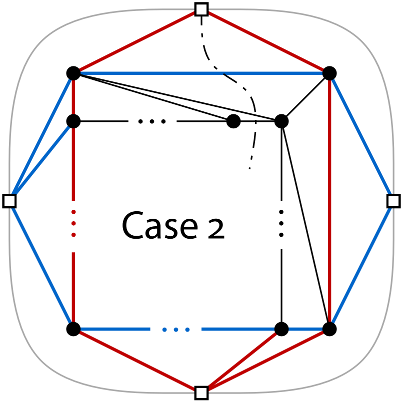

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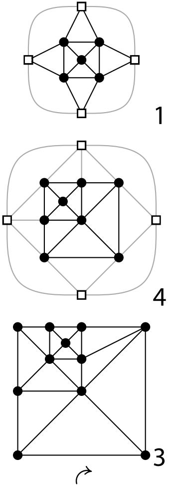

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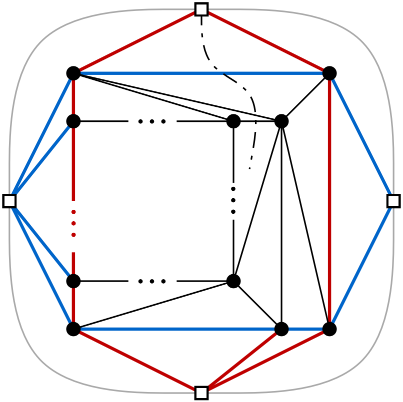

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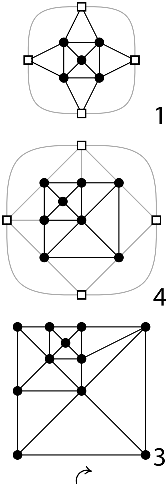

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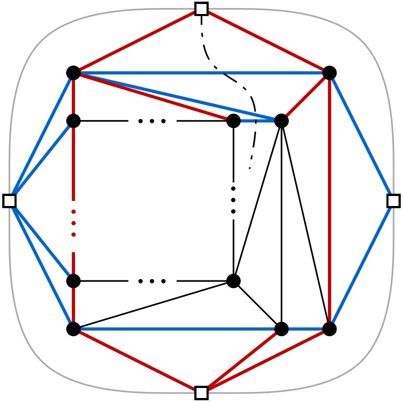

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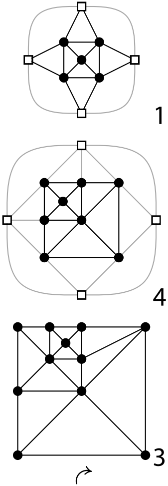

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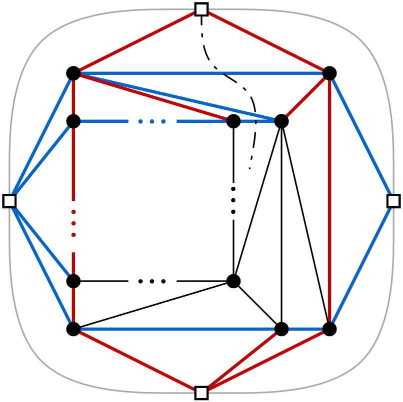

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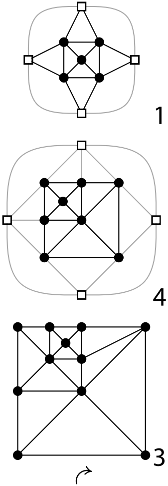

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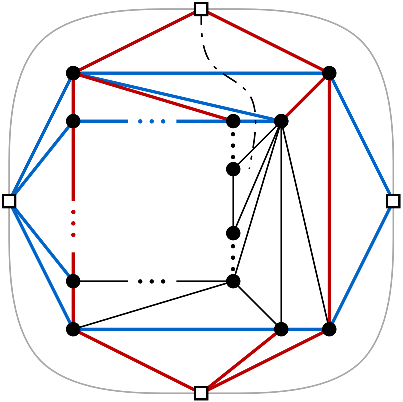

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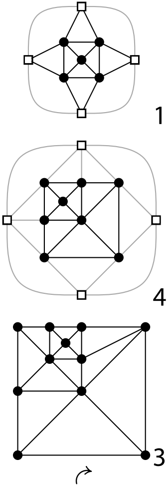

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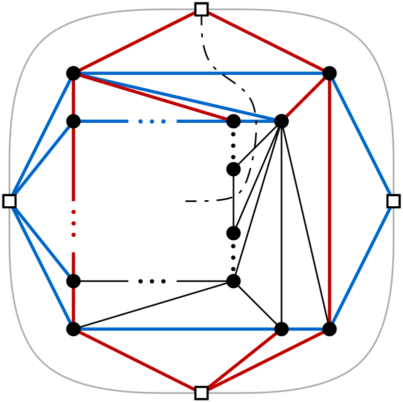

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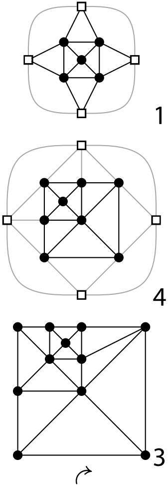

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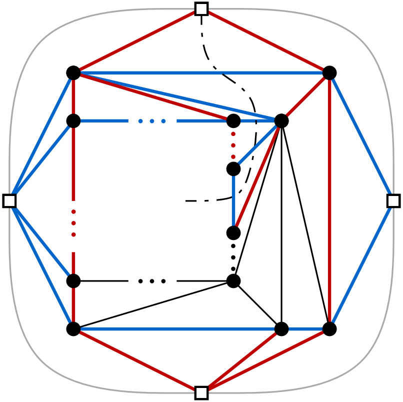

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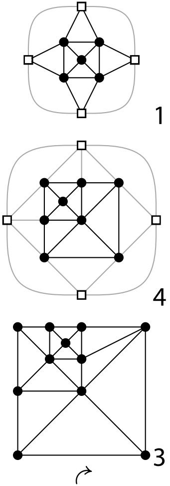

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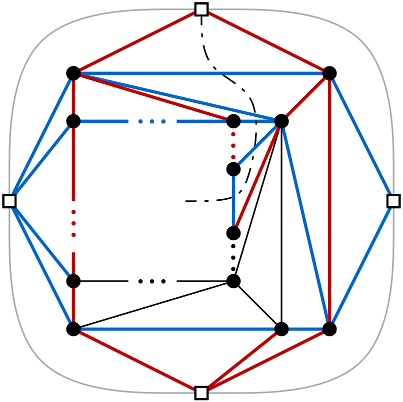

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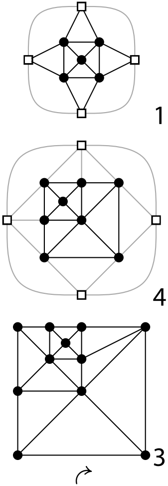

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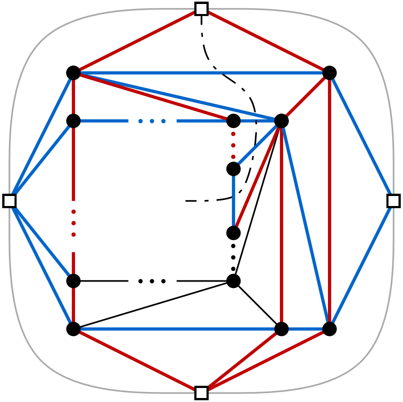

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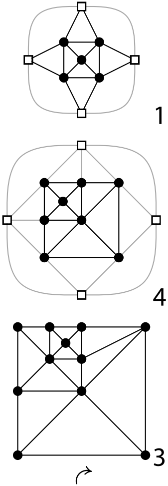

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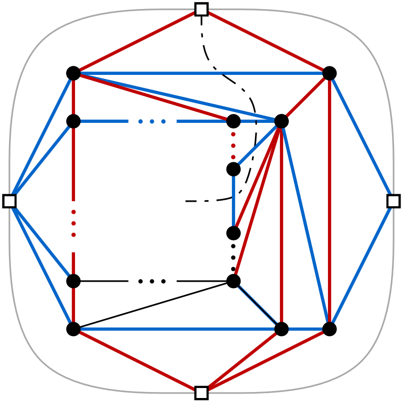

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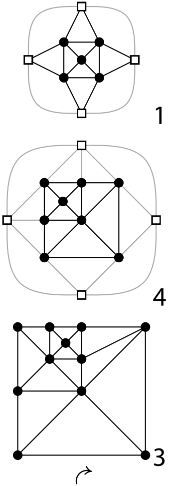

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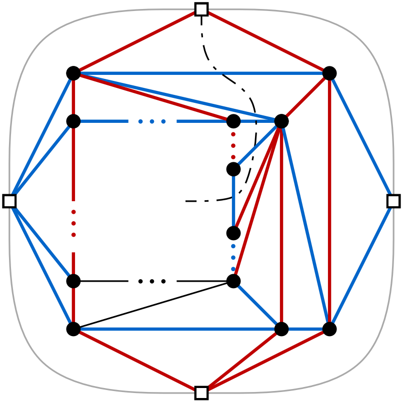

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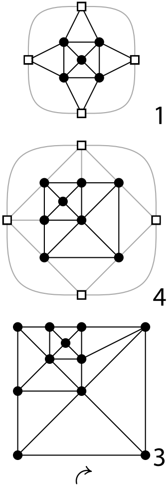

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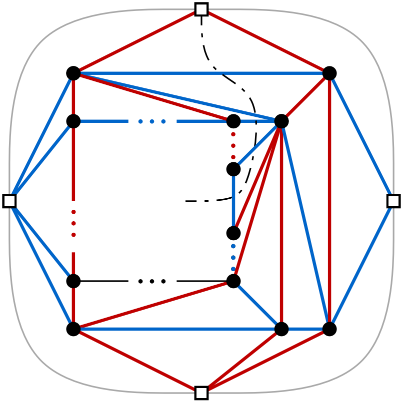

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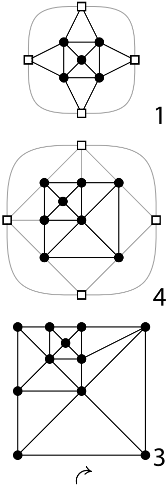

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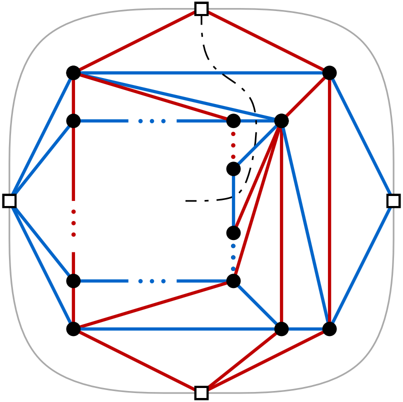

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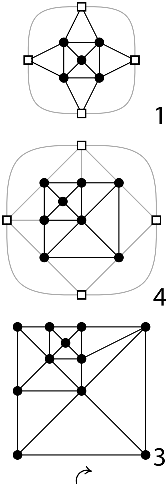

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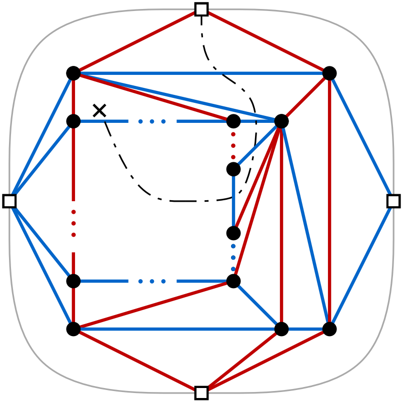

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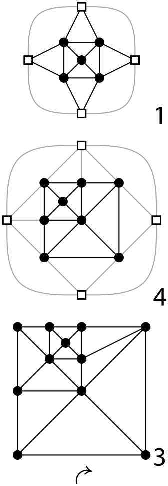

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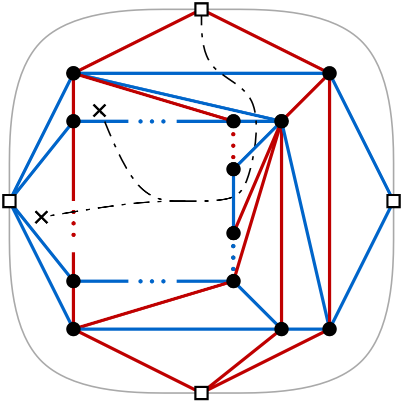

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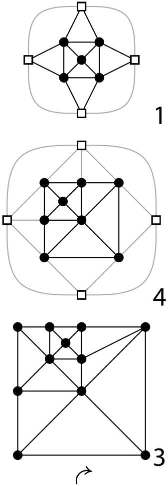

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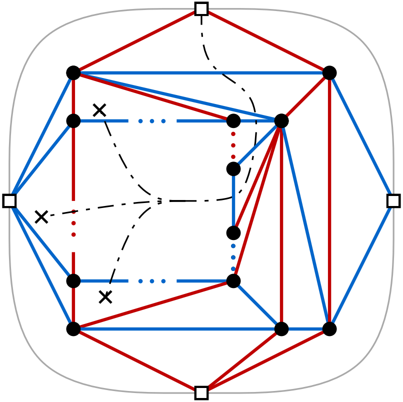

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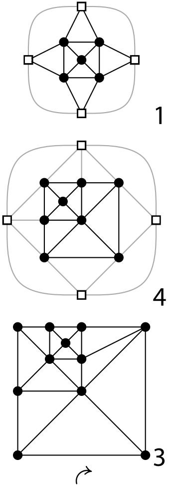

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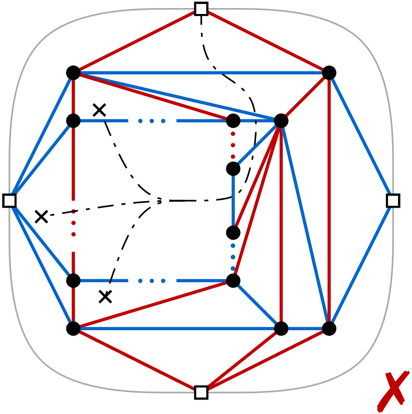

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Rotating windmills are exactly the nonsliceable graphs with exactly one separating 4-cycle.

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Thanks!

References I

- Koźmiński, K. and E. Kinnen (1985). "Rectangular duals of planar graphs". In: *Networks* 15.2, pp. 145–157. ISSN: 0028-3045.
- Mumford, E. (2008). "Drawing Graphs for Cartographic Applications". `http://repository.tue.nl/636963`. PhD thesis. TU Eindhoven.
- Yeap, G. and M. Sarrafzadeh (1995). "Sliceable floorplanning by graph dualization". In: *SIAM J. Disc. Math.* 8.2, pp. 258–280.