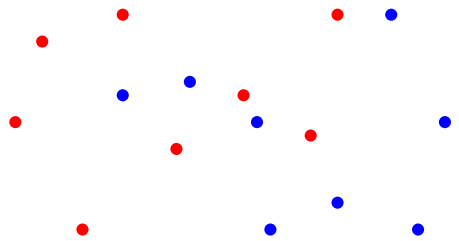
On Embeddability of Buses in Point Sets

Till Bruckdorfer, Michael Kaufmann, Stephen Kobourov, Sergey Pupyrev

26-09-2015

Motivation: Set Membership Visualization



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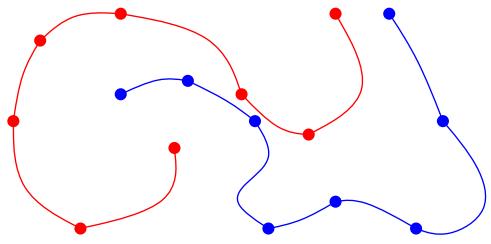

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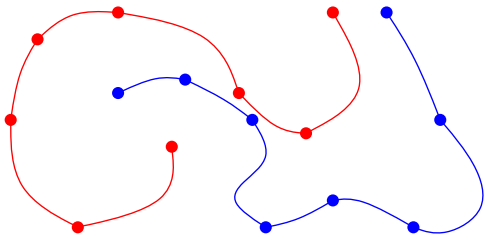

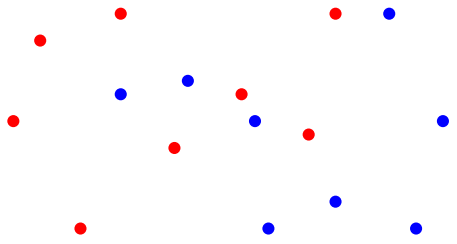

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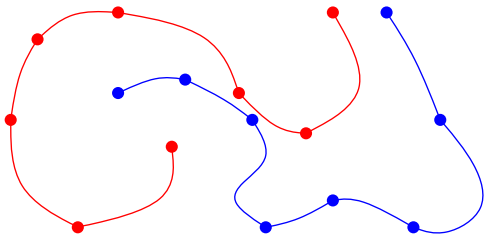

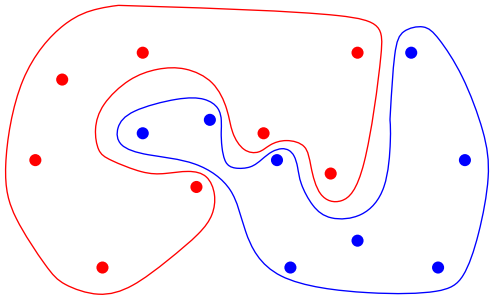

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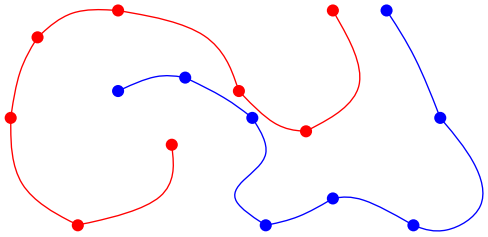

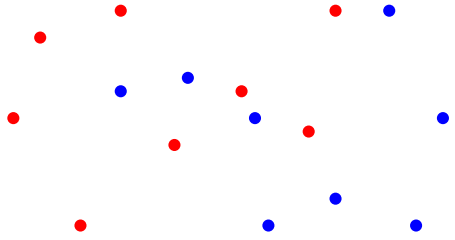

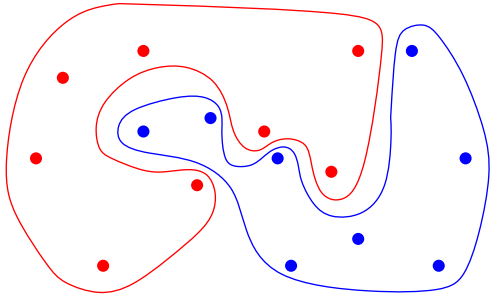

Eulerdiagrams

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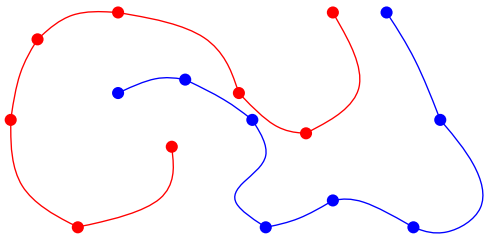

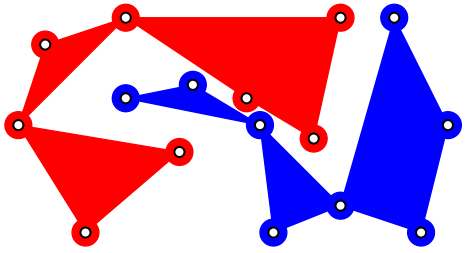

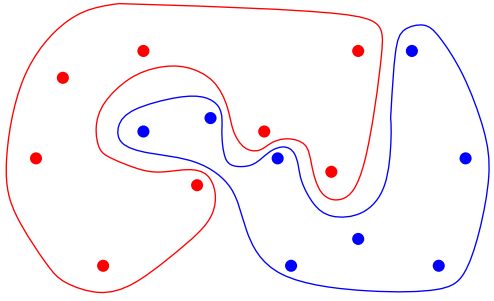

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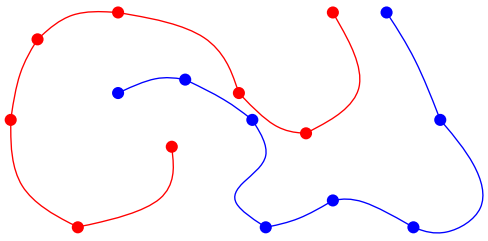
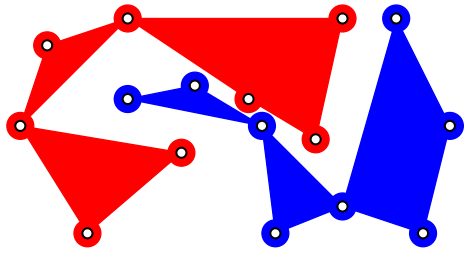
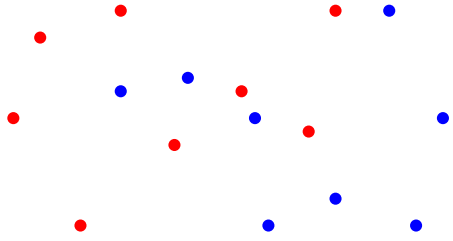
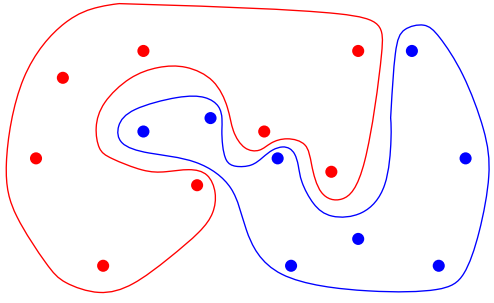


KelpFusion [Meulemans et al., 2013]







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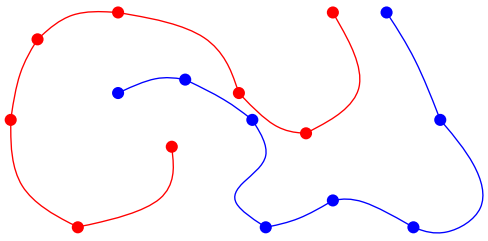

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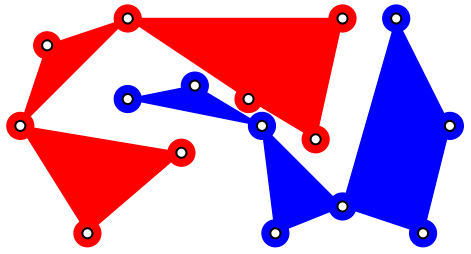

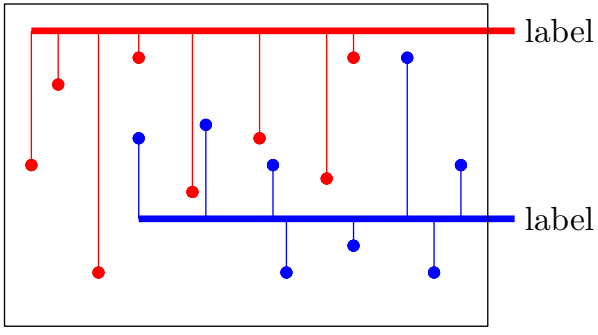

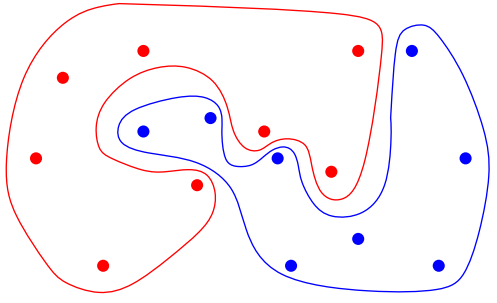

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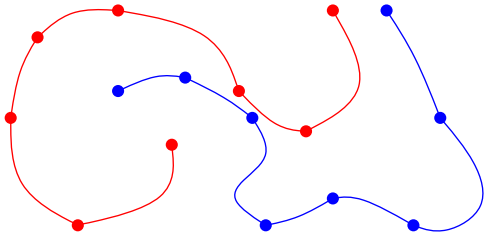

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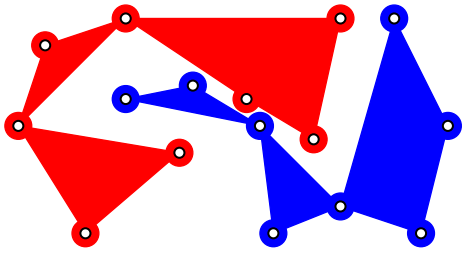
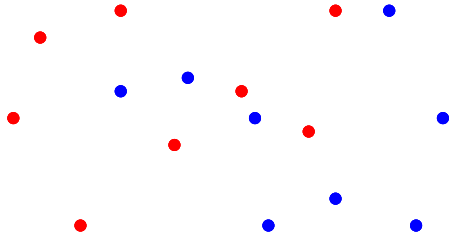
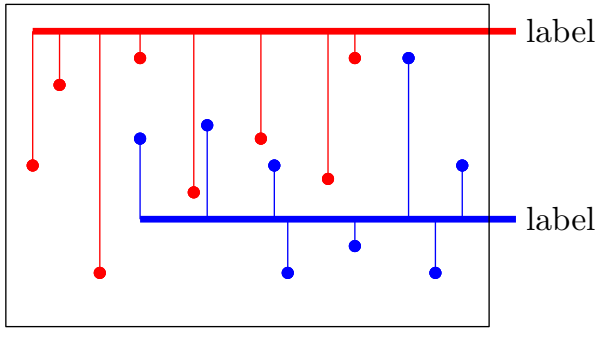
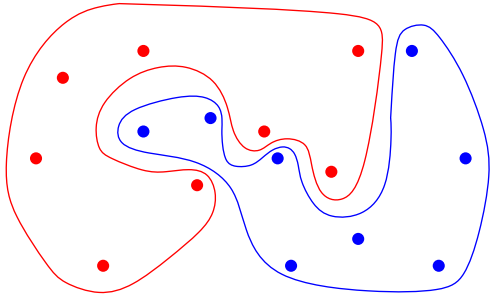






Backbone Boundary Labeling [Bekos et al., 2013]



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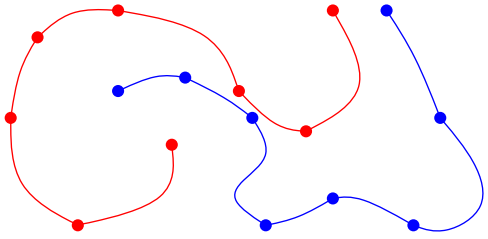

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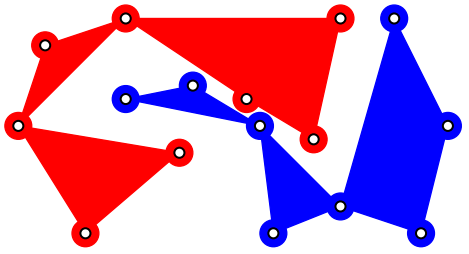
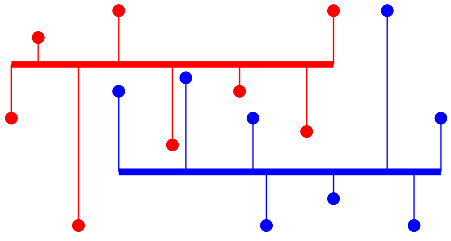
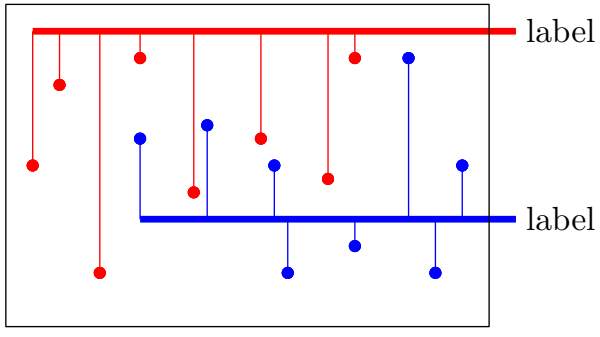


Bus [Ada et al., 2007]





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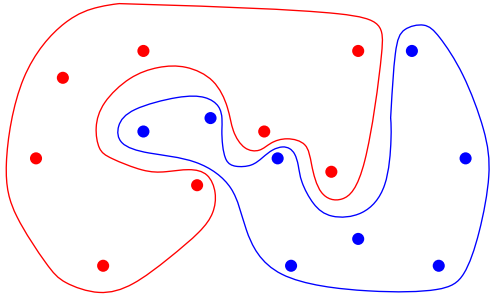

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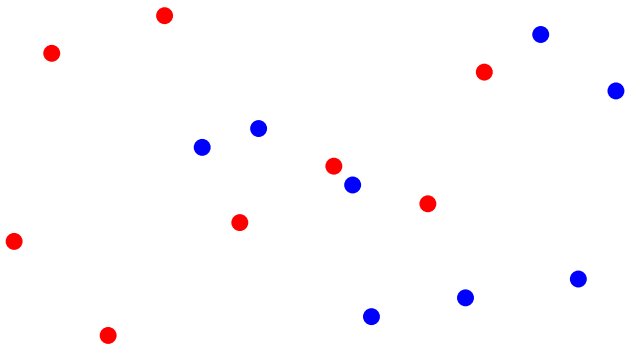
Bus Embeddability Problem (BEP)



Input: Set of colored points

Output: Bus = colored horiz. segment

All points of the same color are orthogonally connected with their bus without crossings.

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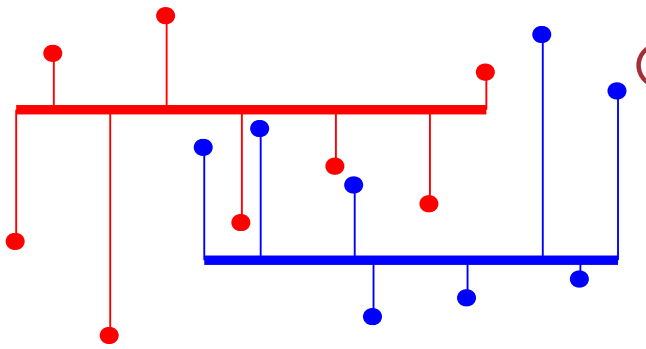

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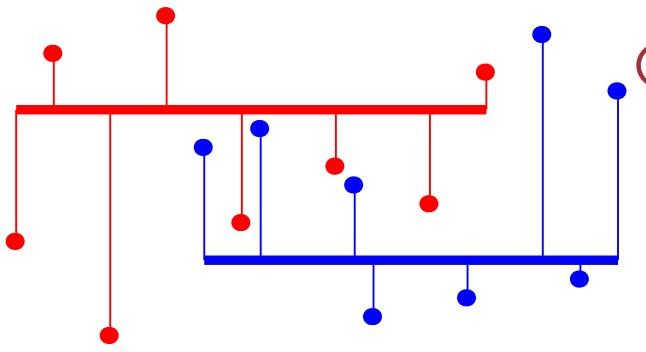

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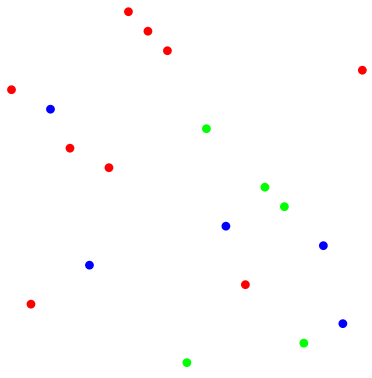

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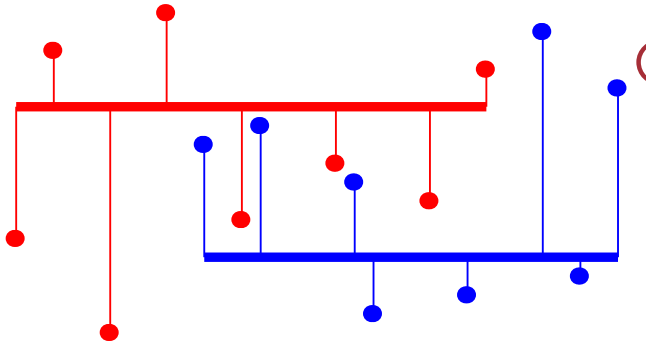

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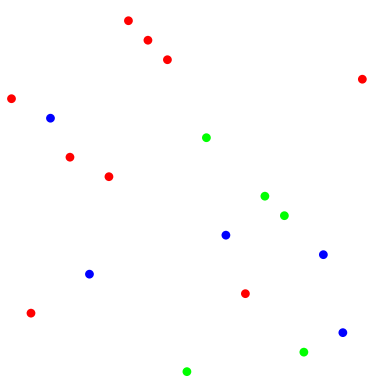

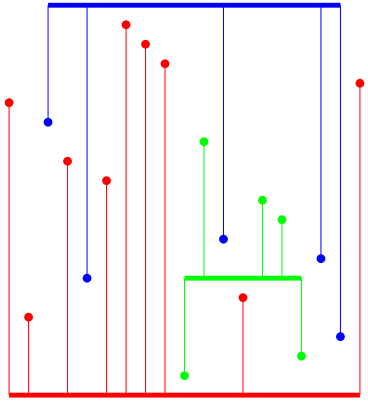

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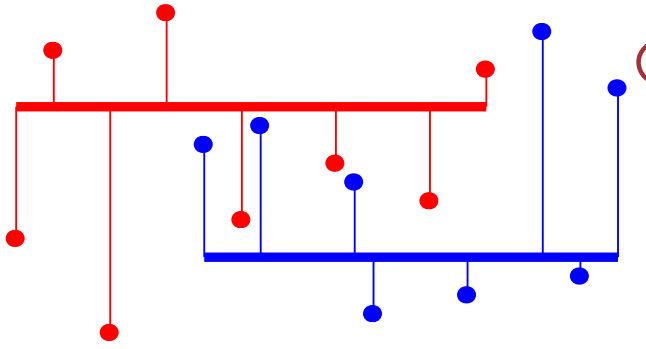

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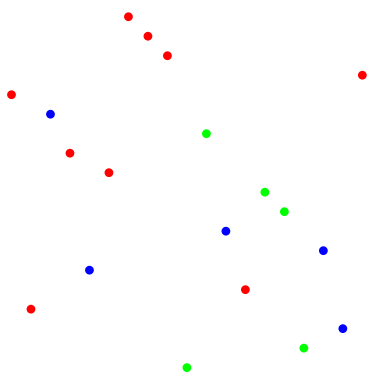

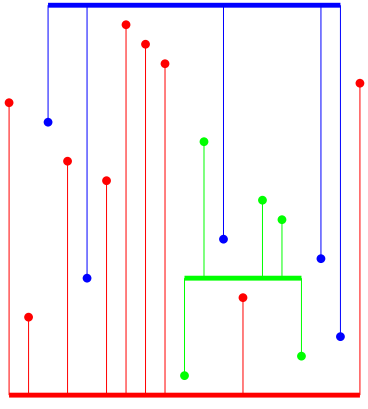

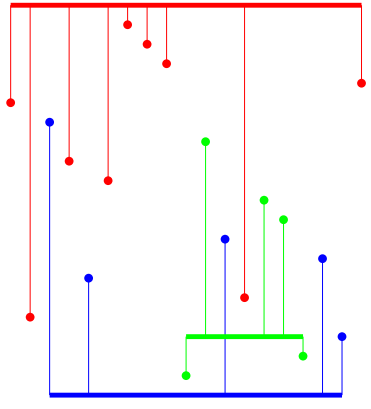

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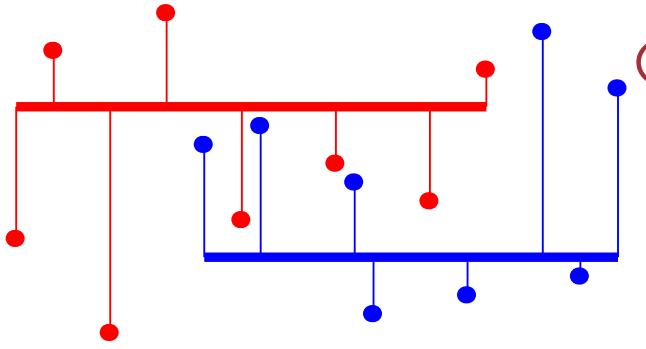

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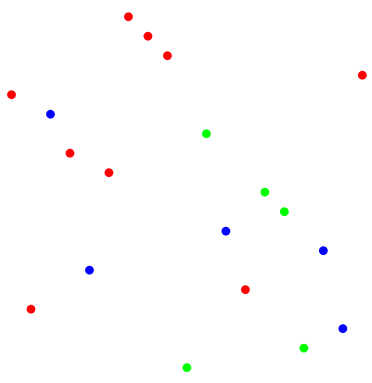

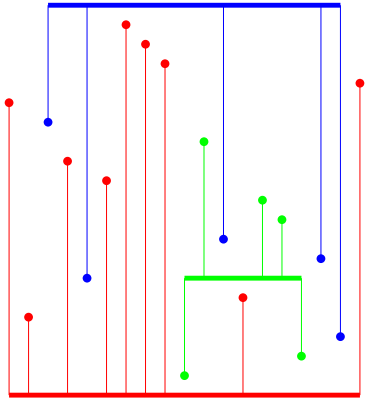

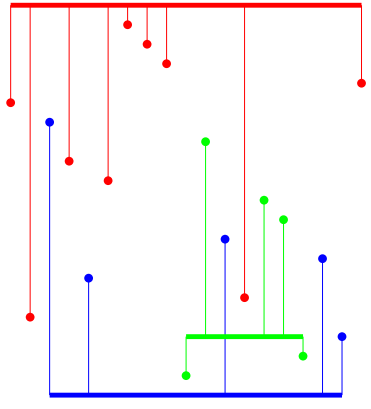

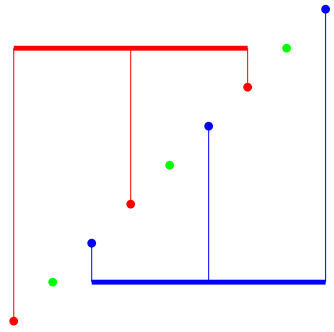

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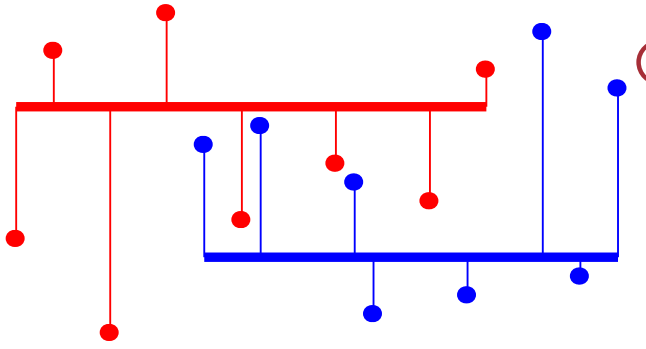

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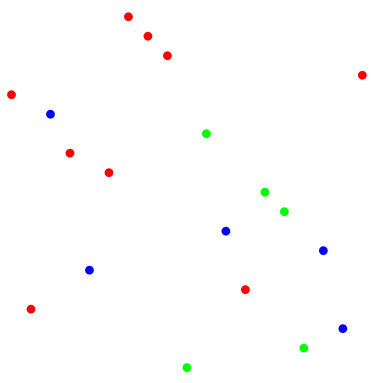
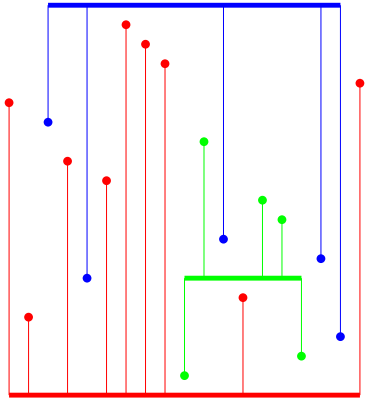
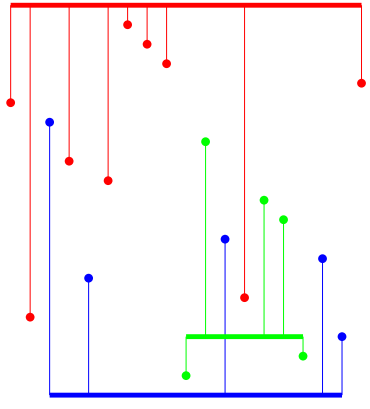
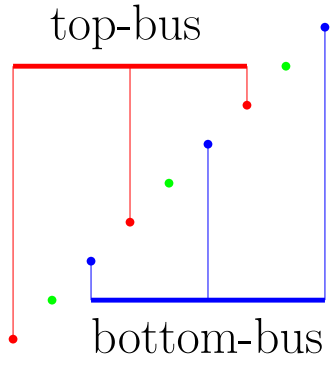








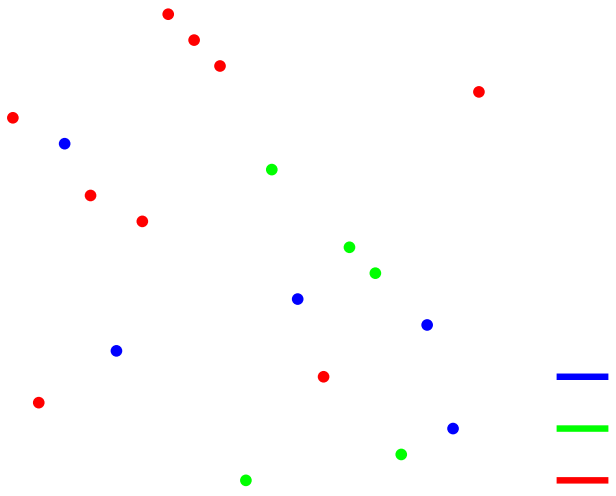
1. General Type of Buses: Given Relative Order

Input: n colored points

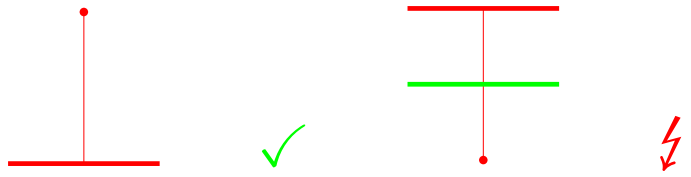
Output: y-coord. of buses

Constr.: given relative order

$\Rightarrow$ Test $O(n \log n)$ time.



- scan from bottom to top
- draw bus as early as possible



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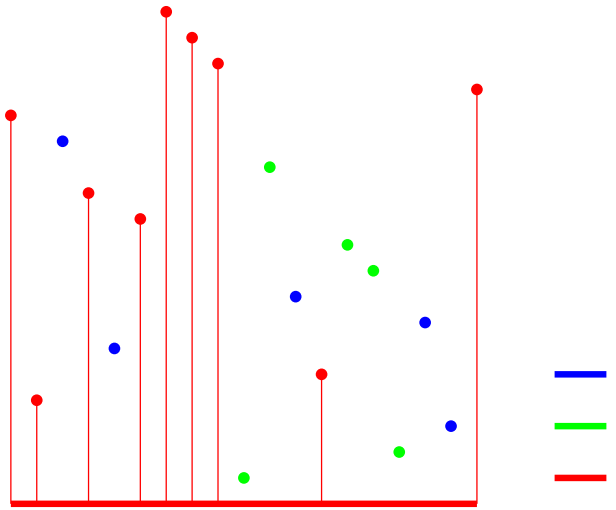

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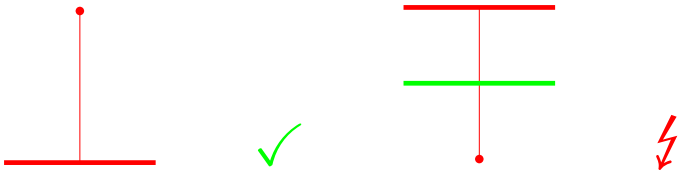

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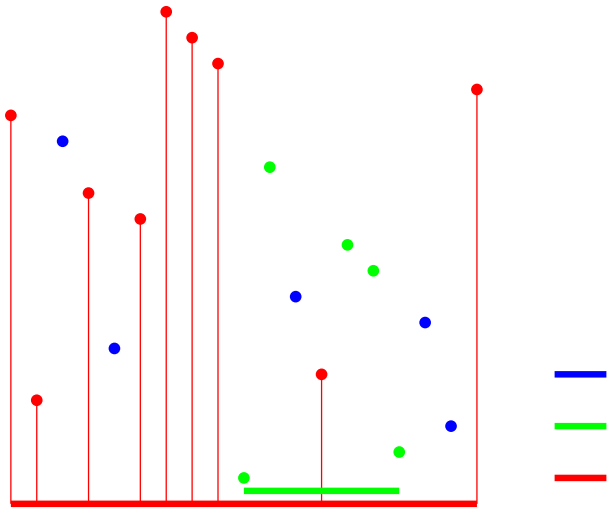

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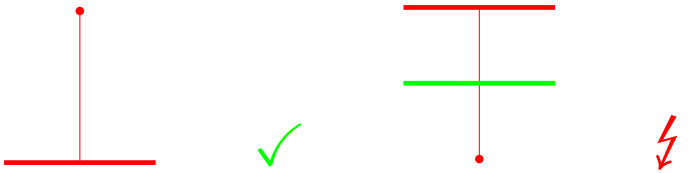
- draw bus as early as possible



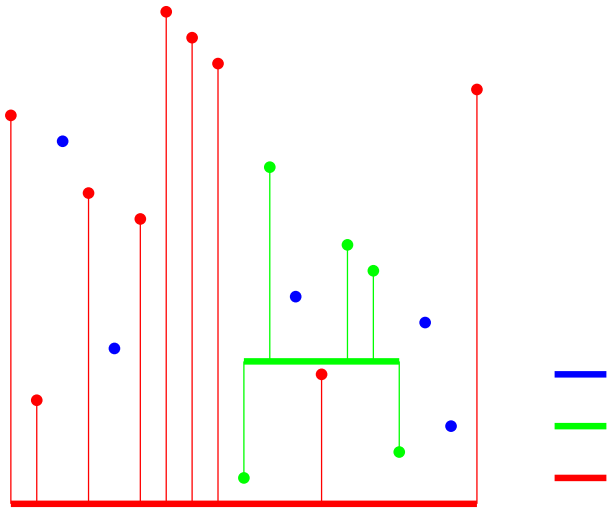
1. General Type of Buses: Given Relative Order

Input: n colored points

Output: y-coord. of buses

Constr.: given relative order

$\Rightarrow$ Test $O(n \log n)$ time.



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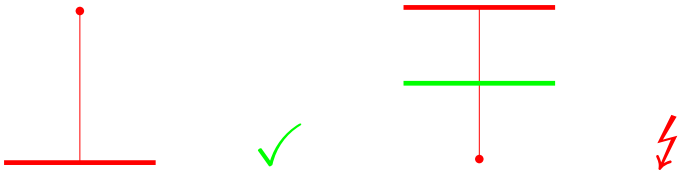

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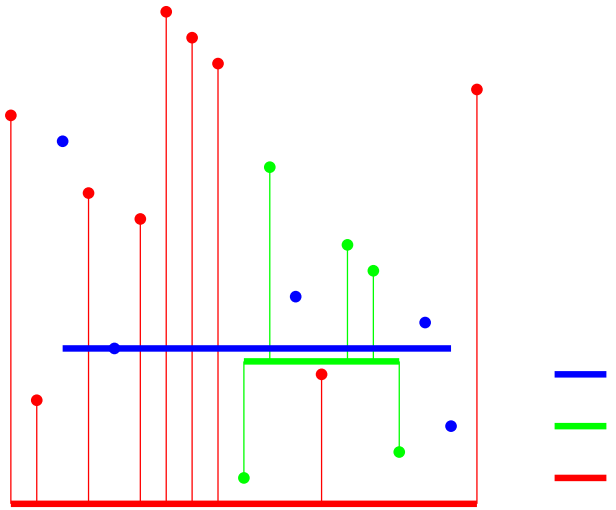

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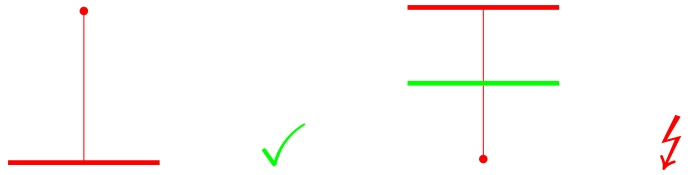

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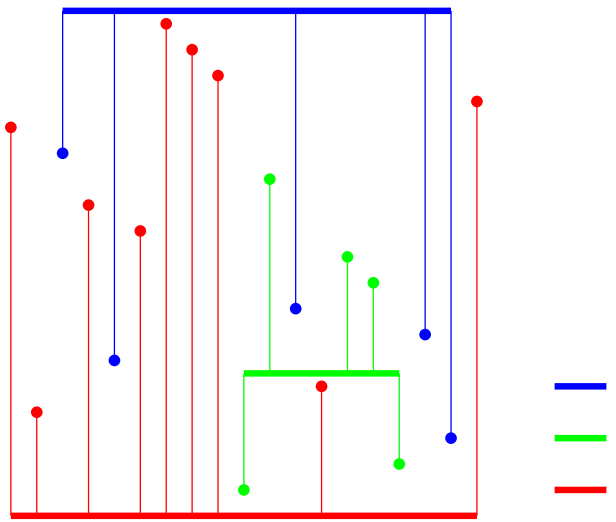

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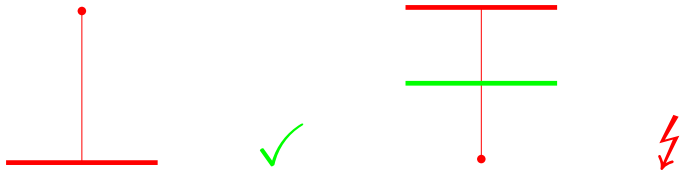

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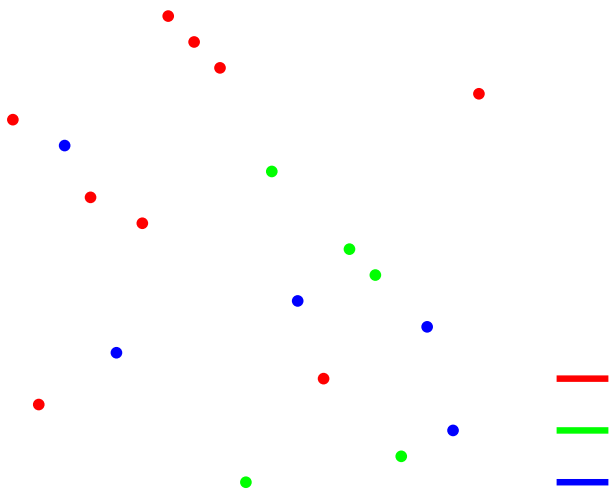

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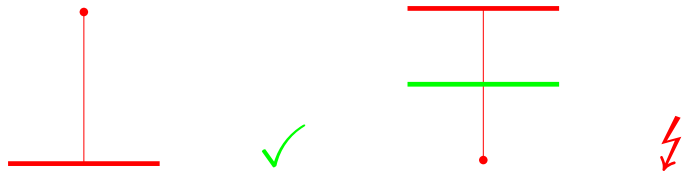

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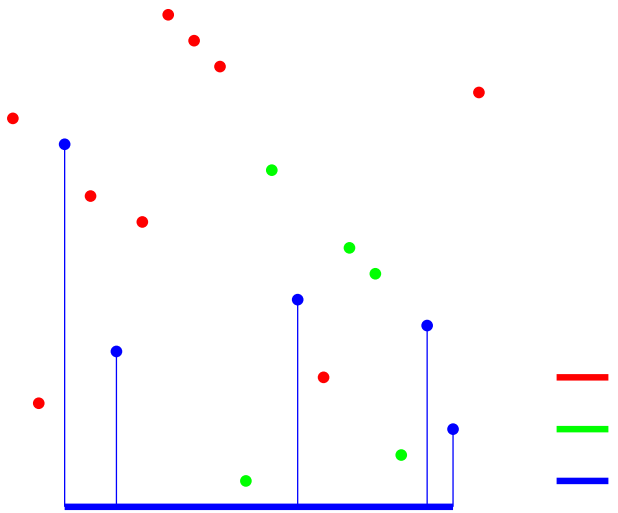

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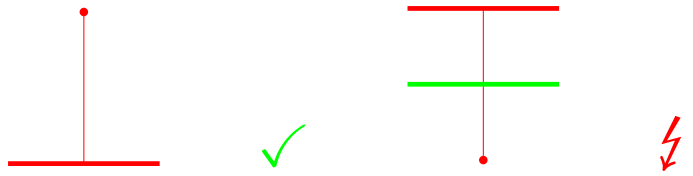

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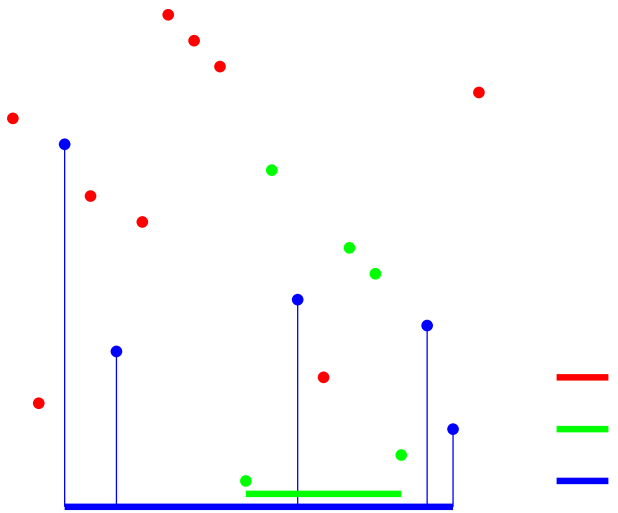

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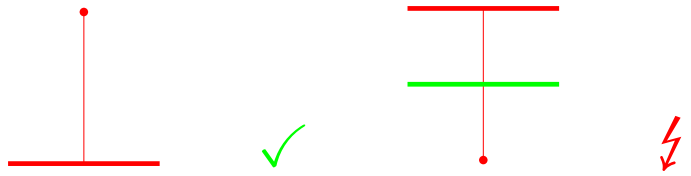

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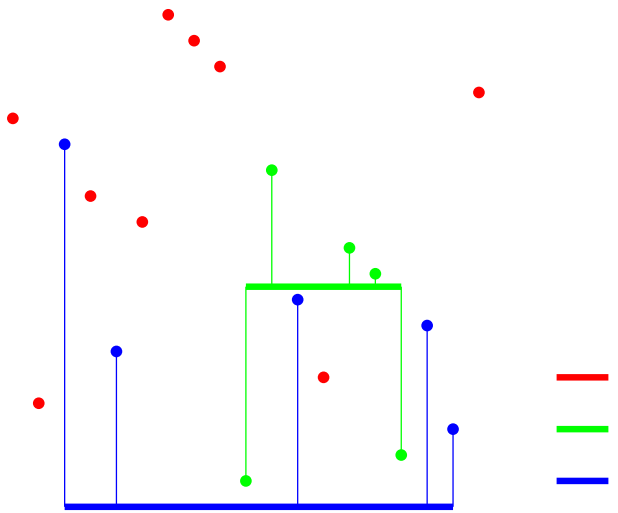

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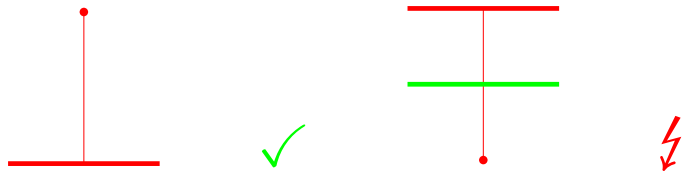

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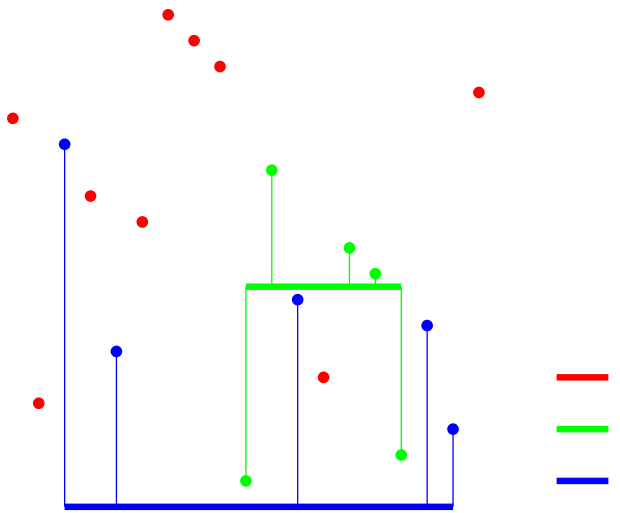

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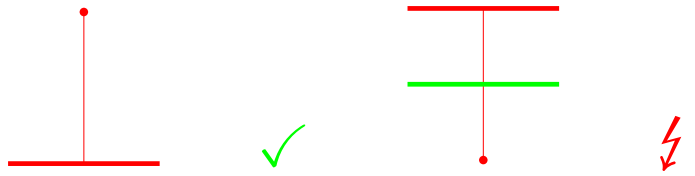


- use segment tree: support extract max. y-coordinate of processed points on range

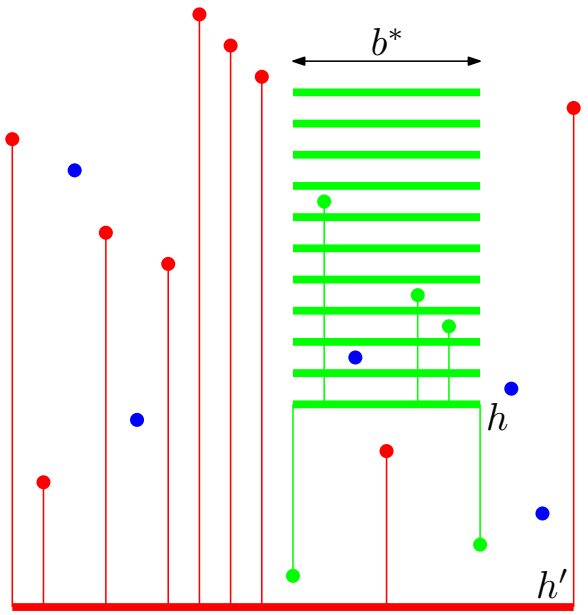
2. General Type of Buses: no Restriction

Input: n colored points

Output: y-coord. of buses

Constr.: —

$\Rightarrow$ Test $O(p(n)2^k)$ time.



- Dynamic Programming
- state pair (h,B), $B \subseteq \{b_1,\ldots,b_k\}$, uses all buses of B + topmost bus at y-coord. $0 \le h \le n+1$
- $F(h,B)$ =true $\Leftrightarrow$ $\exists$ solution using only buses of B
- $F(h,B)$:=true for all $|B| = 1$
- $F(h,B)$:=true, if $\exists\ b^* \in B$ with $F(h',B \setminus \{b^*\})$ =true for $h' < h$ and $h \ge$ topmost point "$\in b^*$"
- there are $\approx n2^k$ state pairs
- compute state pair in $O(p(n))$

2. General Type of Busese: ILP with Experiments

Input: n colored points

Output: y-coord. of buses

Constr.: —

$\Rightarrow$ ILP

$$\begin{aligned} \min\ & 1 \\ s.t.\ & \forall\ (b,p), c(p) \neq b, \text{“}p \in b\text{”}: \\ & y(p) < y(b) \wedge y(c(p)) < y(b) \\ \vee\ & y(p) > y(b) \wedge y(c(p)) > y(b) \end{aligned}$$

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Input: n colored points

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$$\min \ 1$$

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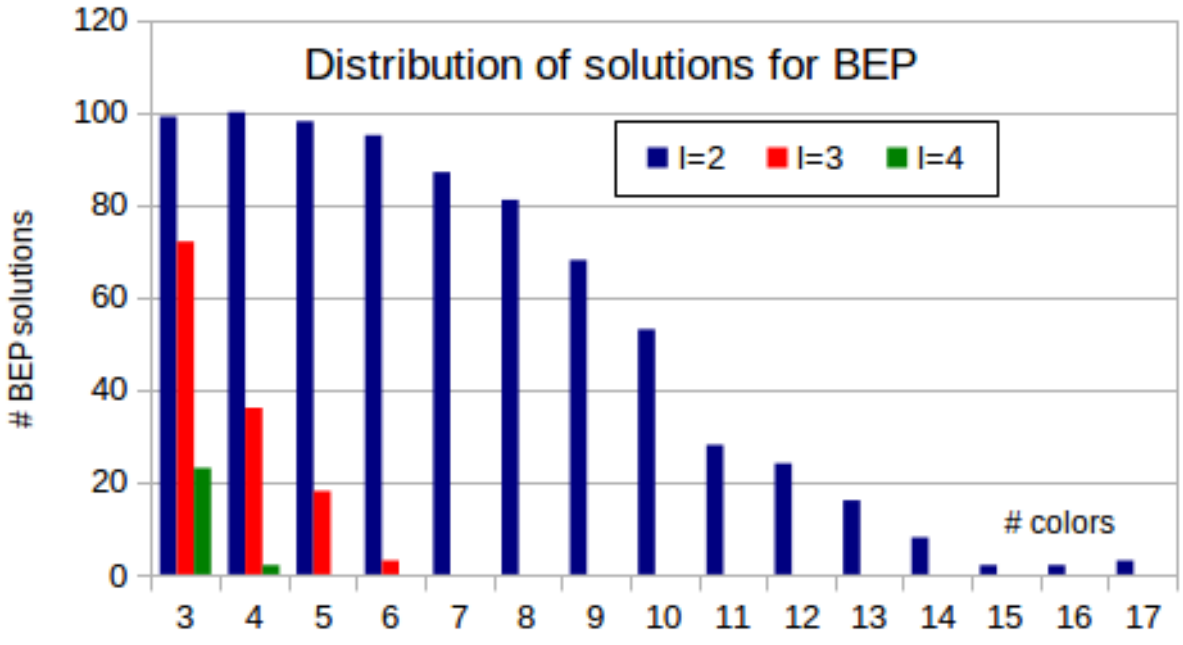

3. Restricted Type of Buses: ⊓-BEP

Input: n colored points

Output: y-coord. of buses

Constr.: using only ⊓-buses

$\Rightarrow$ Test $O(n \log n)$ time

- scan from bottom to top
- draw bus as early as possible
- store scanned points in array A
- $\exists$ pattern yxxy $\in A$

$\Rightarrow$ draw bus x & remove xx from A

- in the end $A = \varnothing \Rightarrow \exists$ solution
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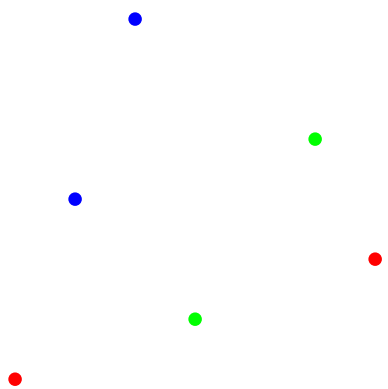

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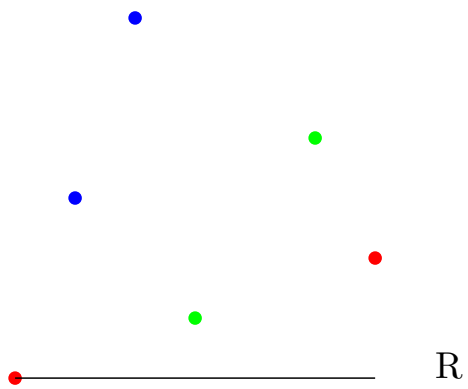

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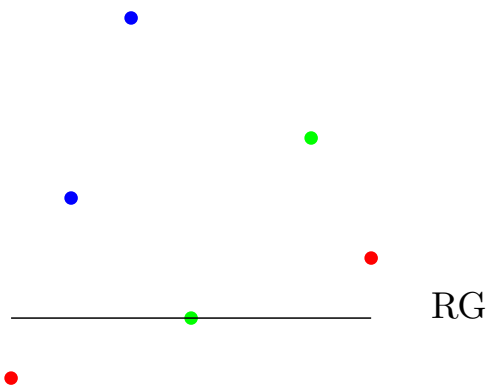

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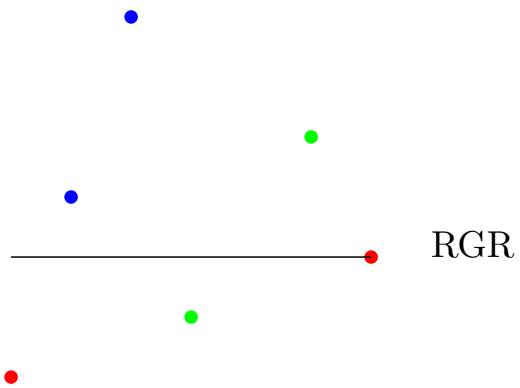

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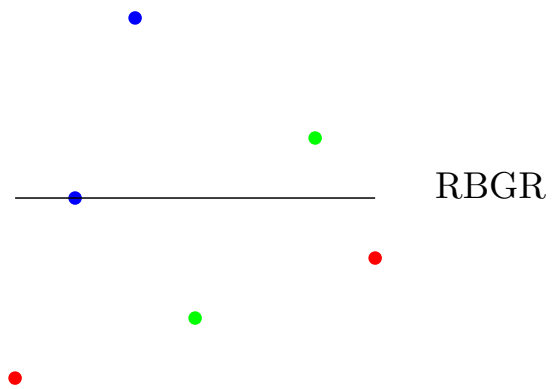

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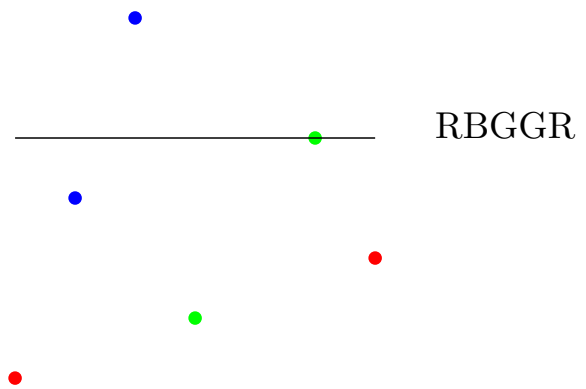

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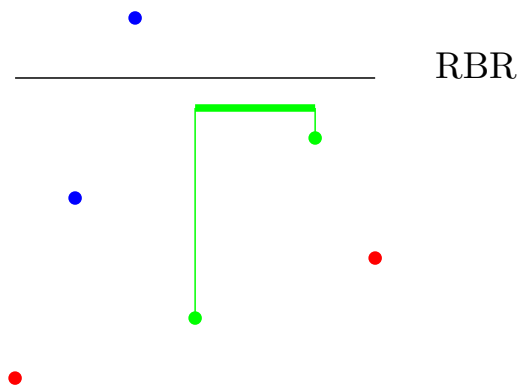

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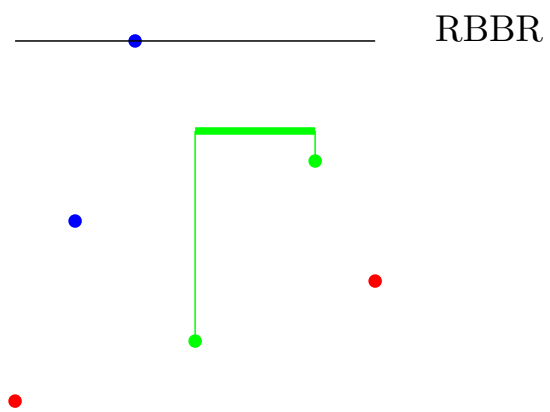

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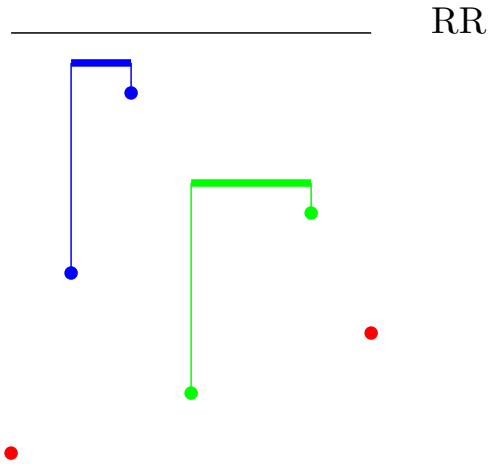

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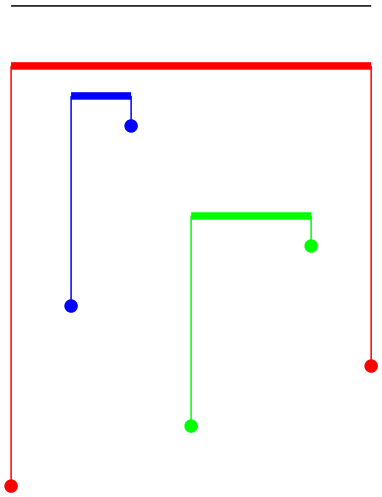

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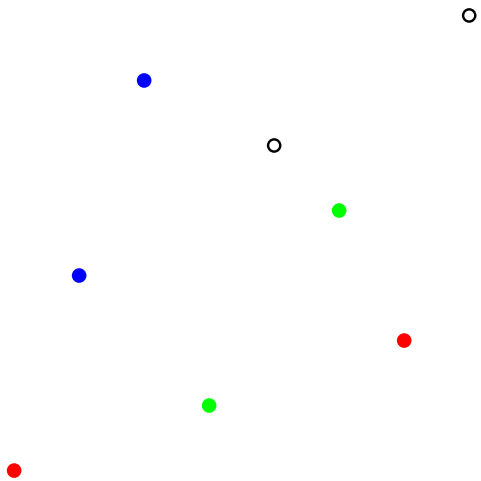

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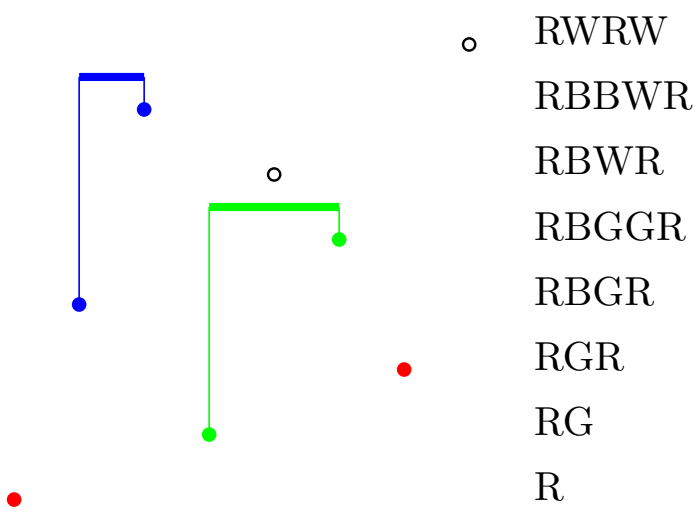

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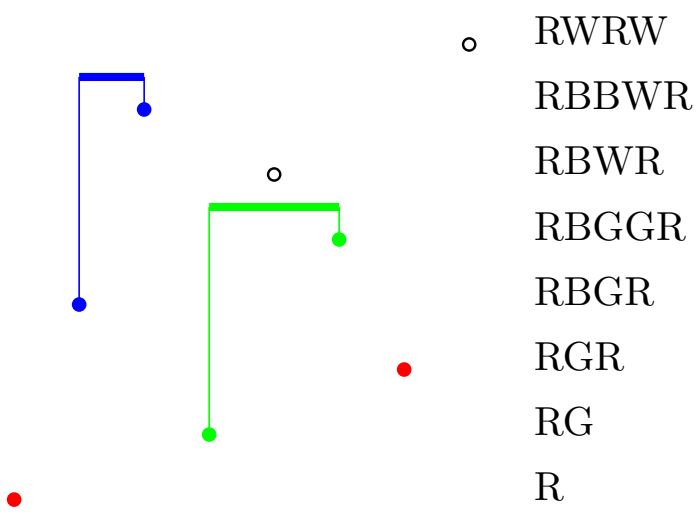

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- use balanced binary tree: support insert/remove in $O(\log n)$ time

4. Restricted Type of Buses: (⌜,⌞)-BEP

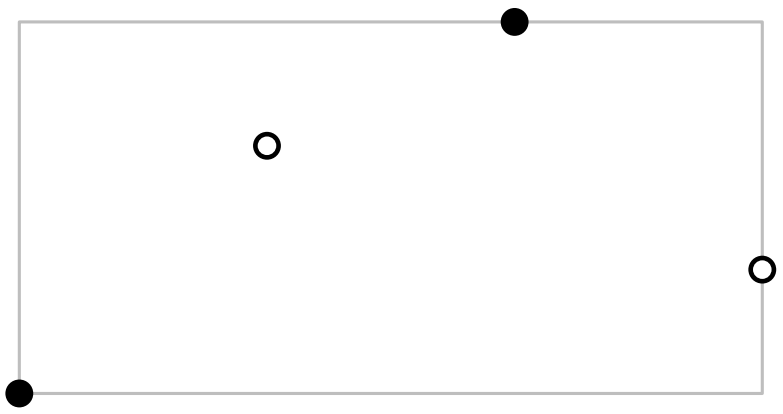
Input: n colored points

Output: y-coord. of buses

Constr.: using only ⌜,⌞-buses

$\Rightarrow$ Test $O(n^2)$ time

- via 2-SAT: variable $x_b \forall$ bus b
- $b = ⌜ \Leftrightarrow x_b =$ true
- $b = ⌞ \Leftrightarrow x_b =$ false
- build a clause for every pair of buses b, b':



$x_{\bullet} = true$

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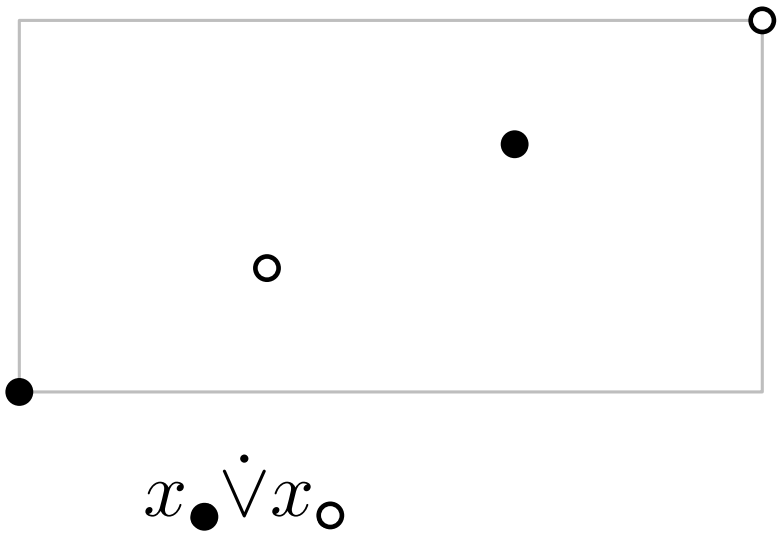

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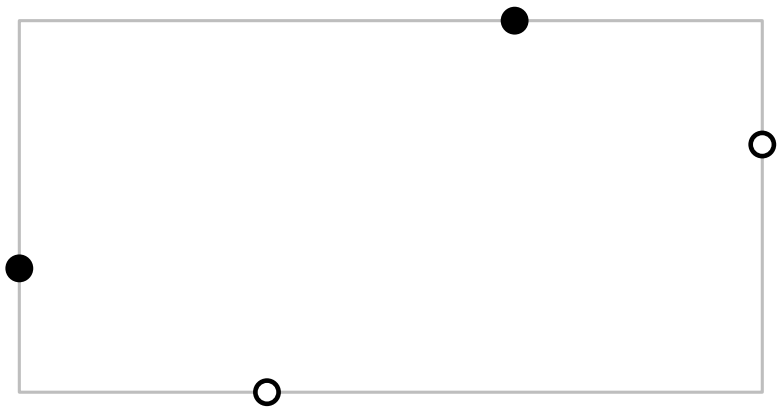
- build a clause for every pair of buses b, b':



$x_{\circ} \Rightarrow x_{\bullet}$

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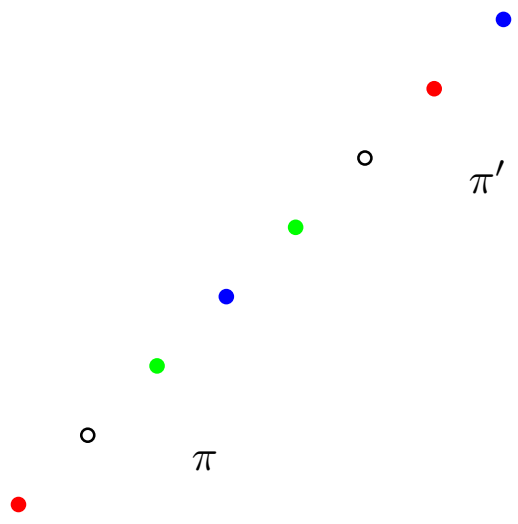
Input: 2 points per color

Output: y-coord. of buses

Constr.: on diagonal (π, π')

$\Rightarrow$ Test $O(n^2)$ time



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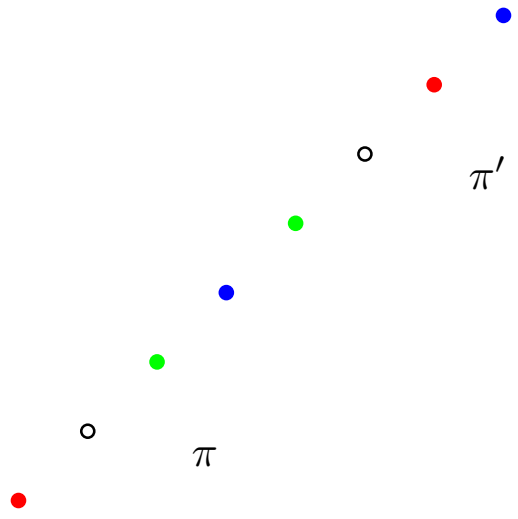

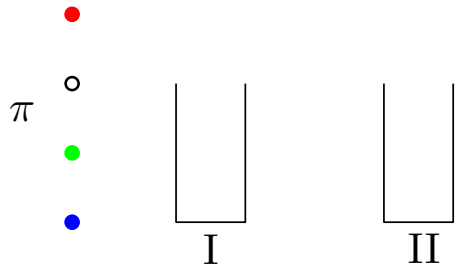

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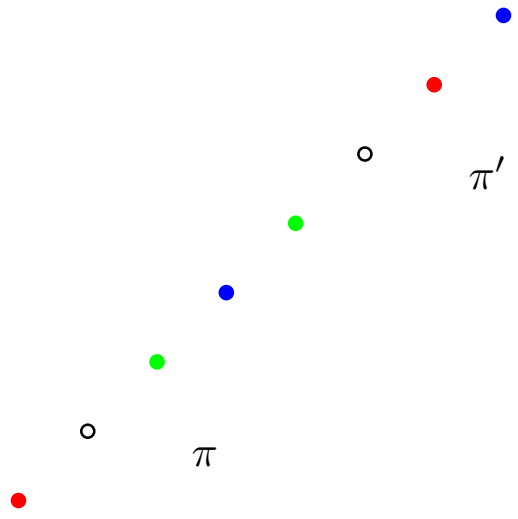

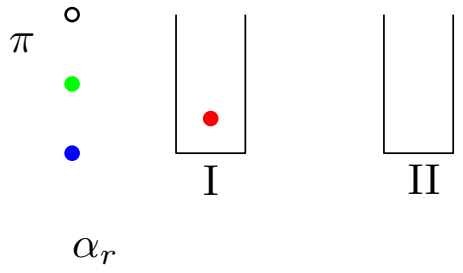

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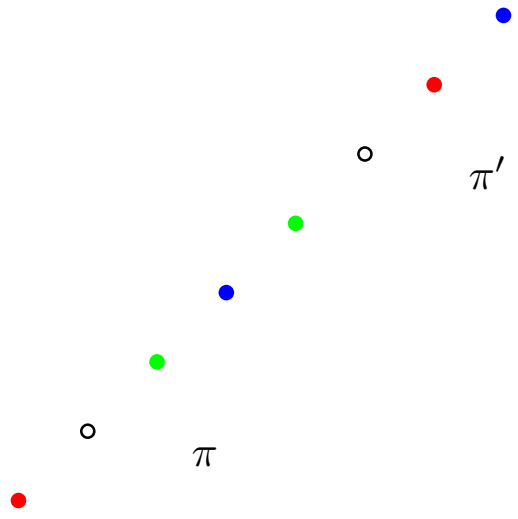

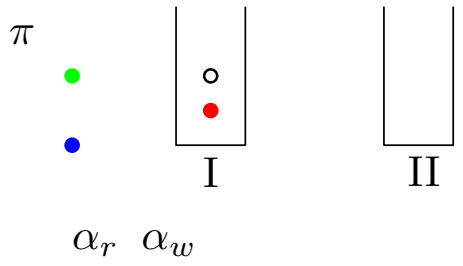

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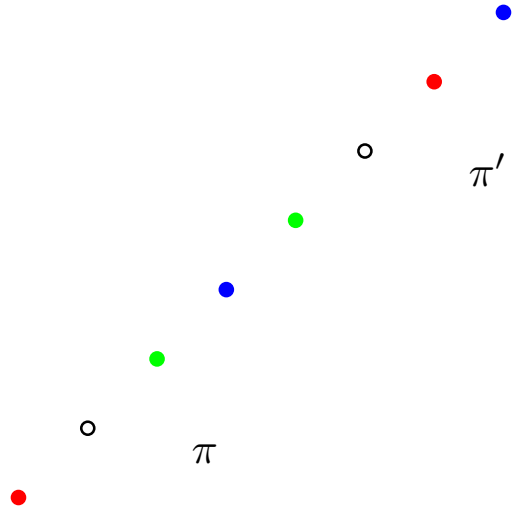

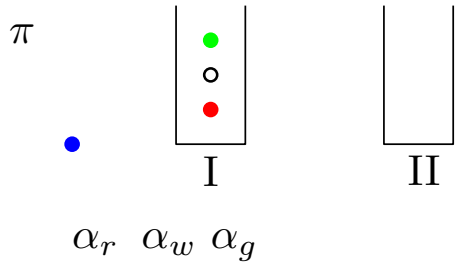

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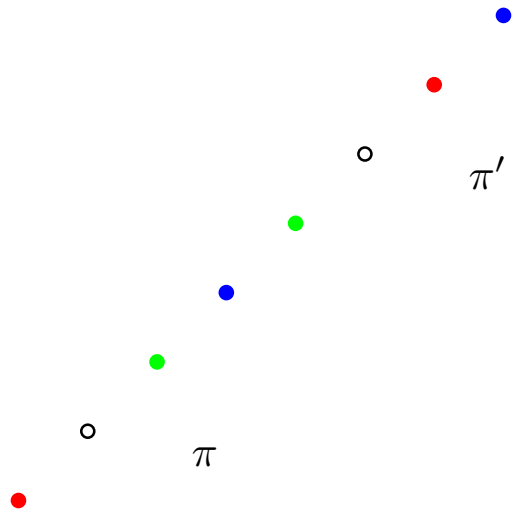

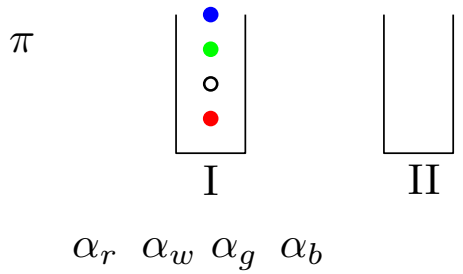

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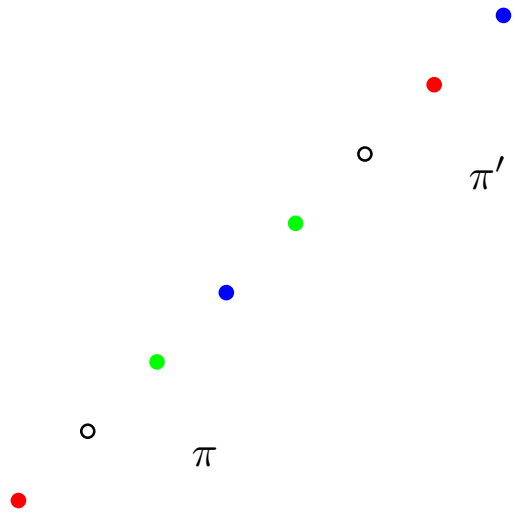

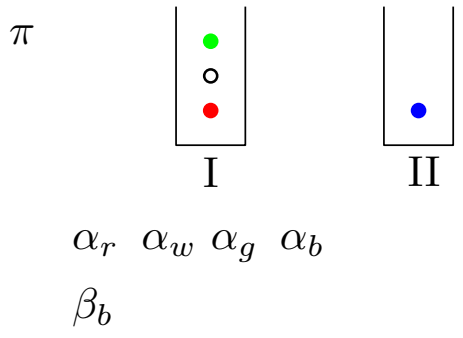

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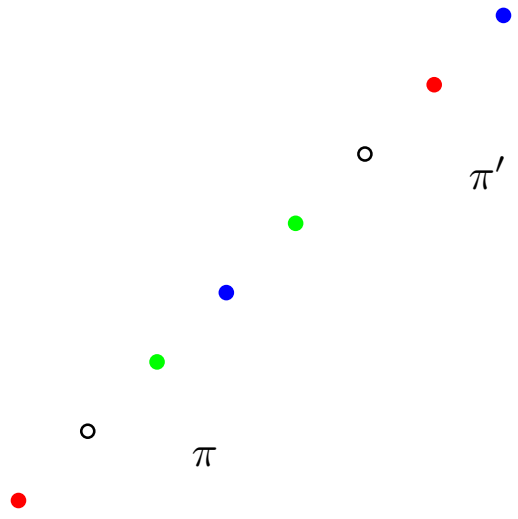

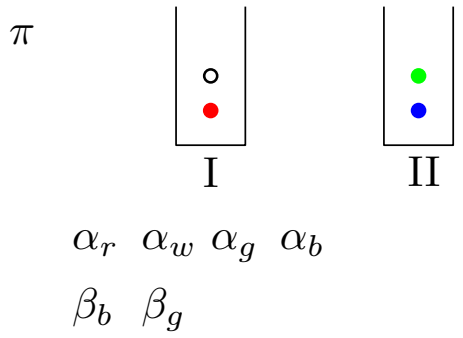

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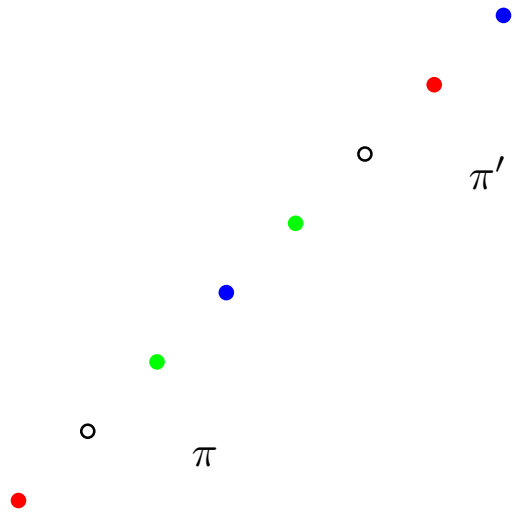

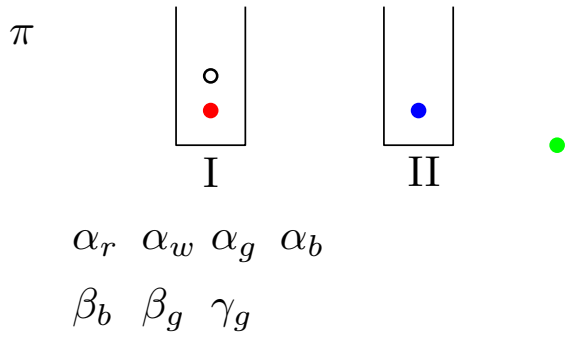

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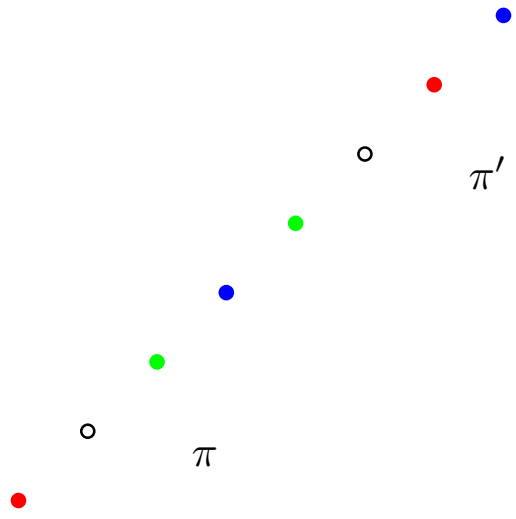

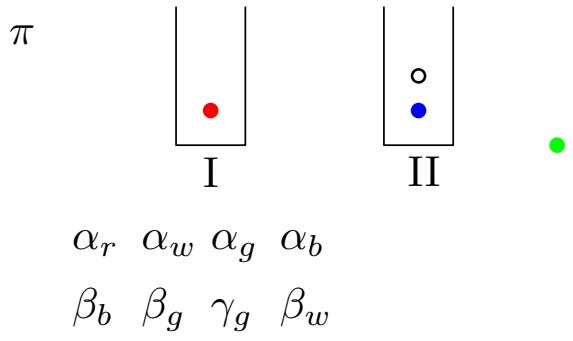

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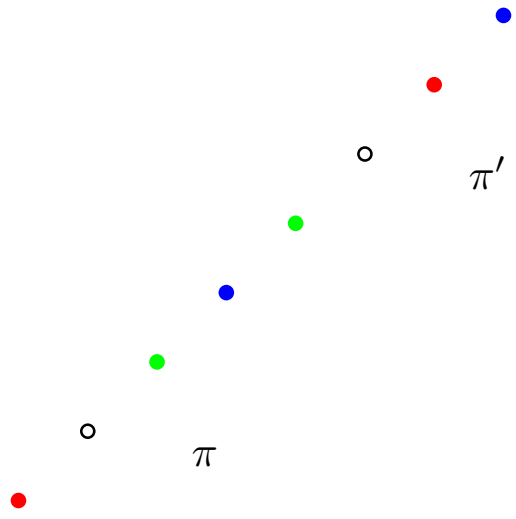

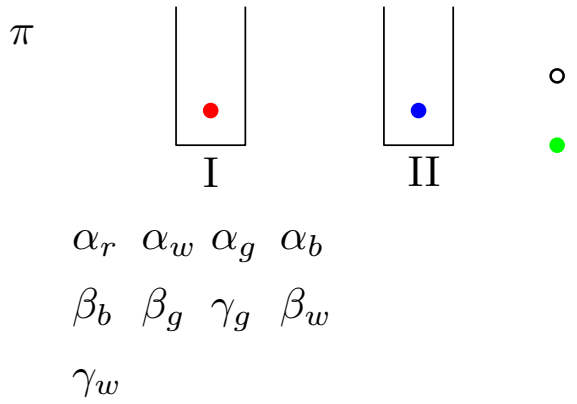

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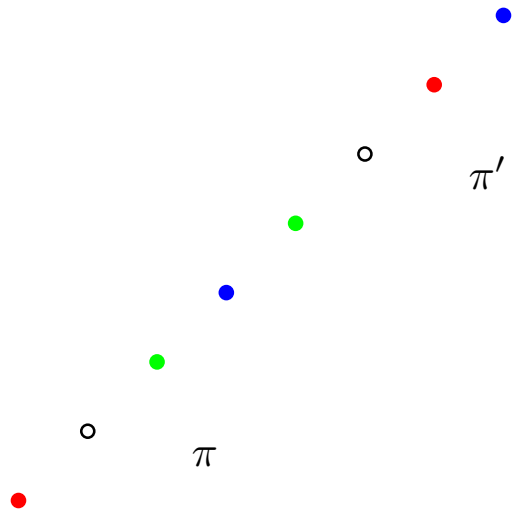

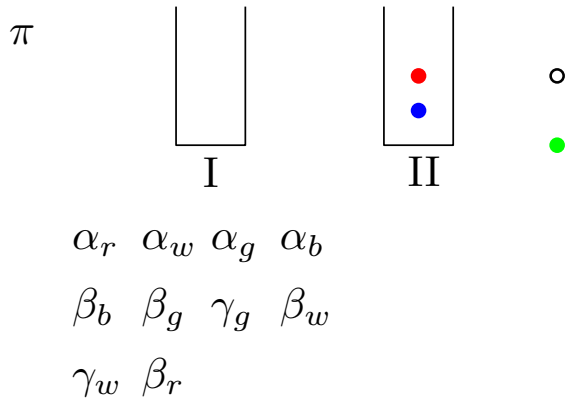

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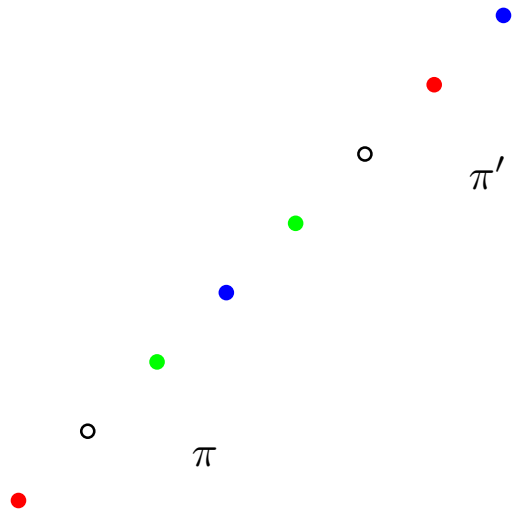

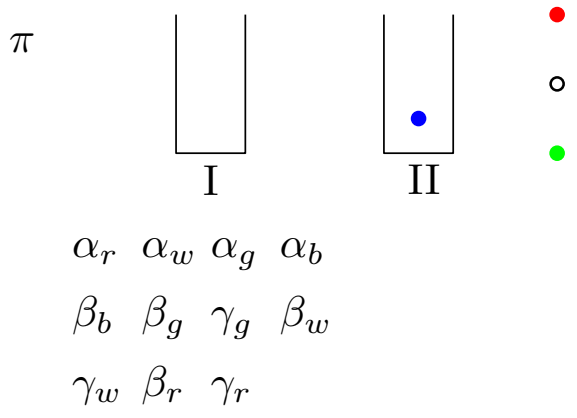

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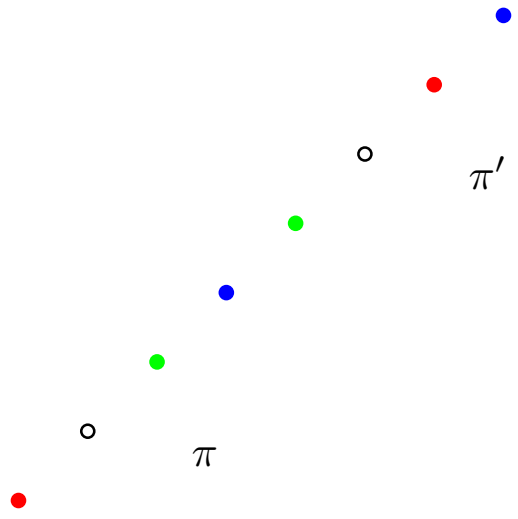

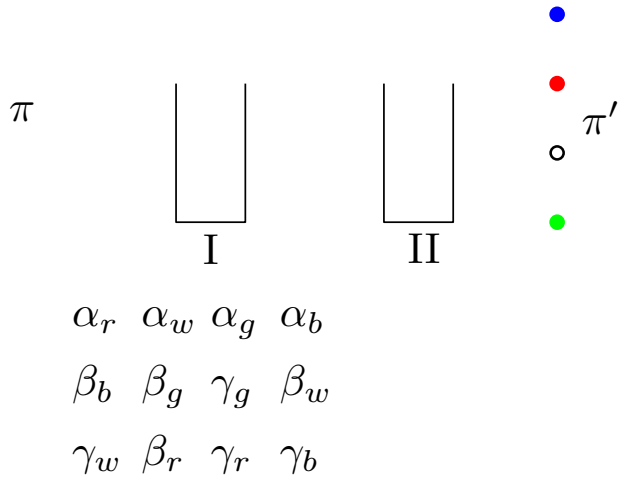

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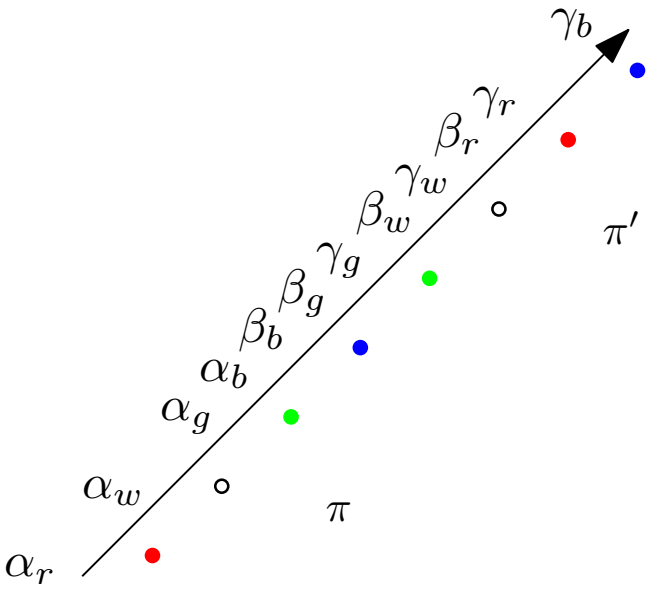

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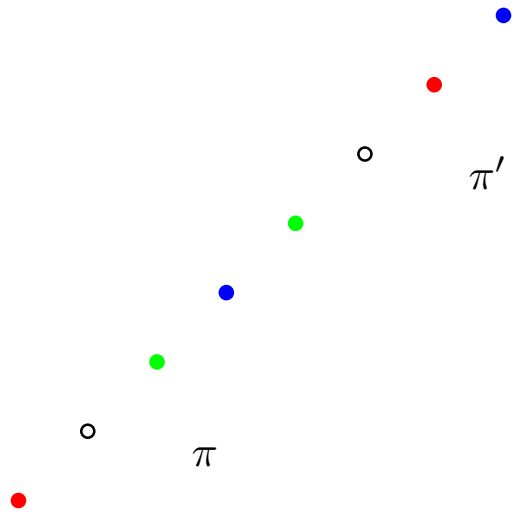

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$\alpha_r \ \alpha_w \ \alpha_g \ \alpha_b \ \beta_b \ \beta_g \ \gamma_g \ \beta_w \ \gamma_w \ \beta_r \ \gamma_r \ \gamma_b$

α = pop from input and push on stack I
$\hat{=}$ connection up

β = pop from stack I and push on stack II
$\hat{=}$ cross diagonal from left to right

γ = pop from stack II and push on putput
$\hat{=}$ connection up

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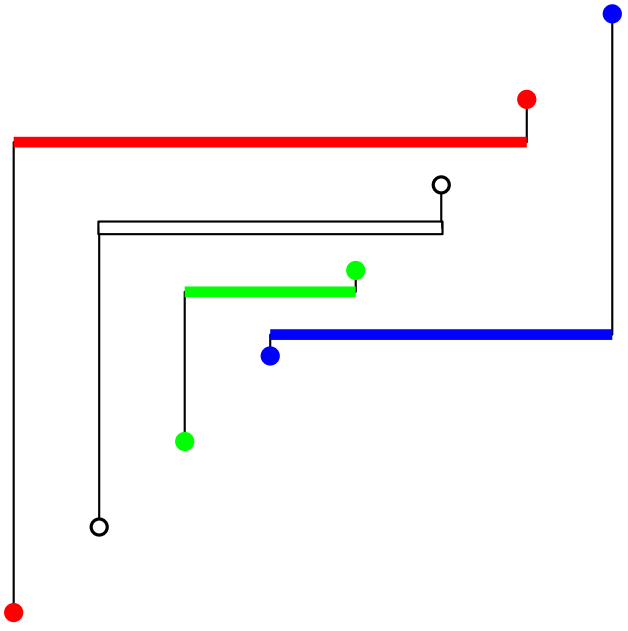

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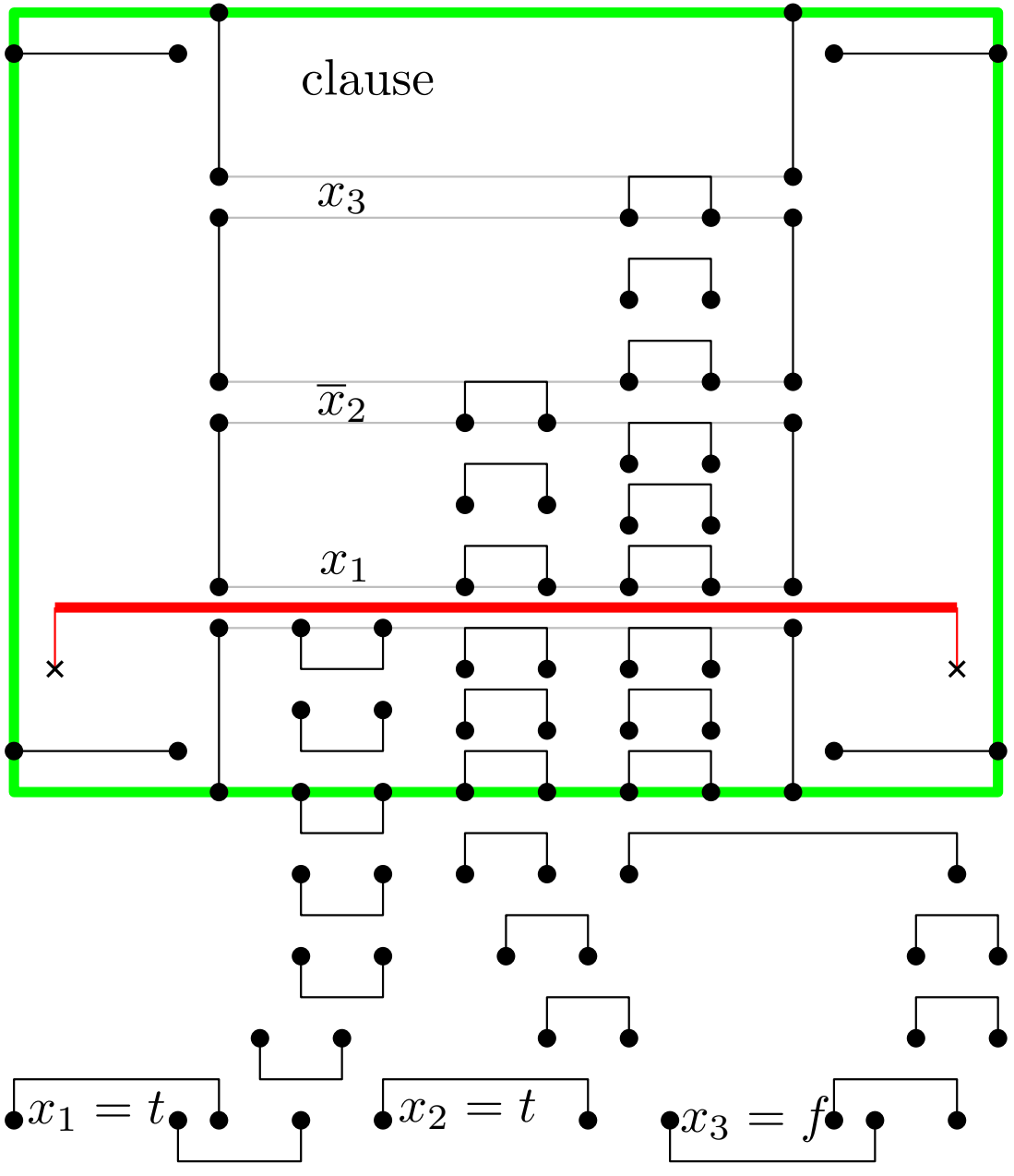
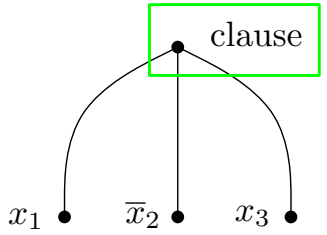
6. Hardness of BEP$^{\varepsilon}$ ($\varepsilon \hat{=} \exists$ min. dist. buses $\leftrightarrow$ points)



- reduction from planar 3-SAT



→ ($\sqcap$,$\sqcup$)-BEP$^{\varepsilon}$

→ points may share a coordinate

→ 2 points per color

Conclusions and Future Work

- we used buses for visualization of set membership
- some variants of BEP are efficiently solvable
  (e.g. with 2-SAT or 2-stack pushall sorting)
- BEP seems to be hard in general

- we want to study non-planar settings:
  - minimize number of crossings
    minimize number of buses (stay planar)