

# Simultaneous embeddings with few bends and crossings

Fabrizio Frati   Michael Hoffmann   Vincent Kusters

Uni Roma Tre   ETH Zurich

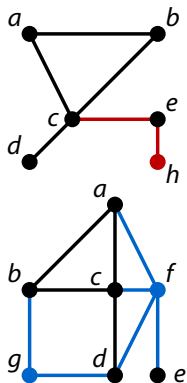
September 25, 2015

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A **simultaneous embedding with fixed edges** (SEFE) of  $G_1$  and  $G_2$  is a drawing of  $G_1 \cup G_2$  that is plane when restricted to  $G_1/G_2$ .

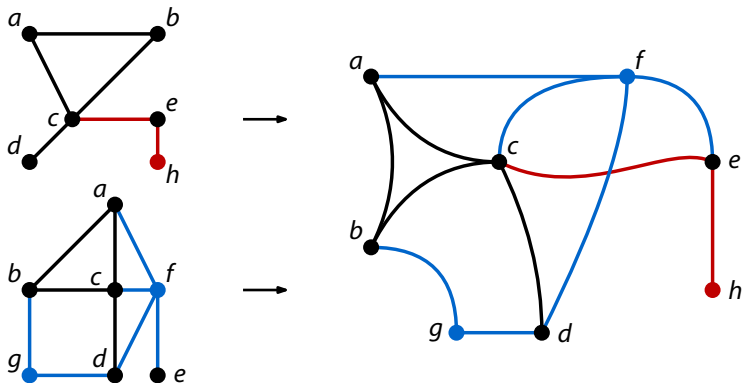
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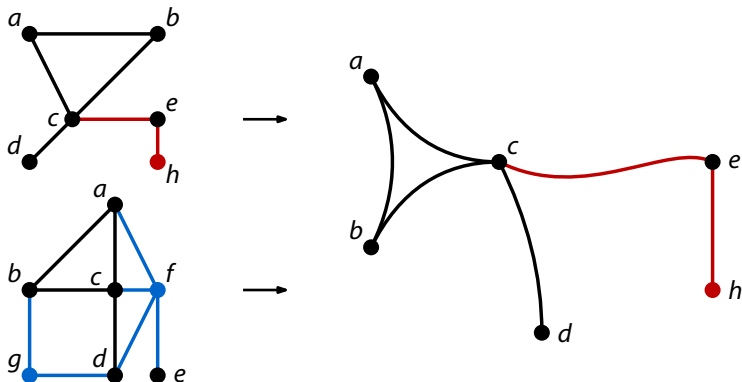
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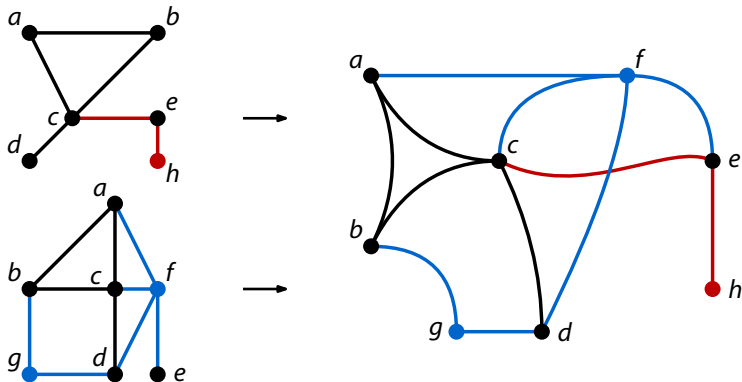
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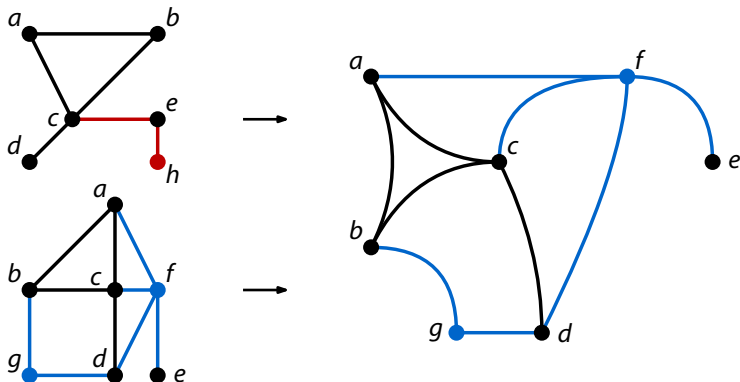
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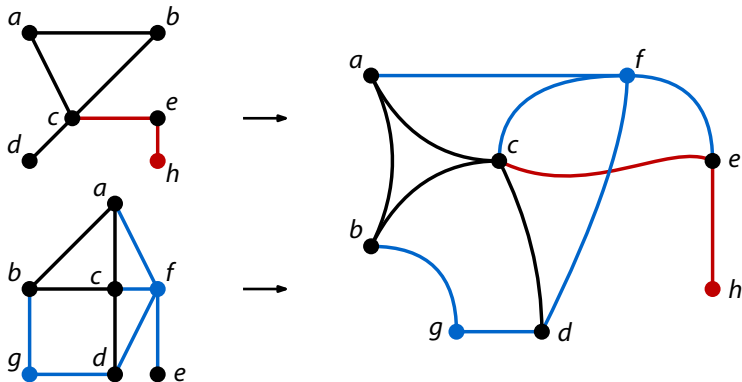
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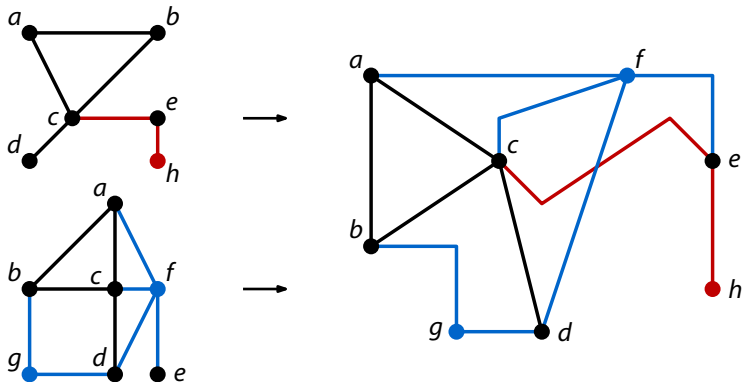
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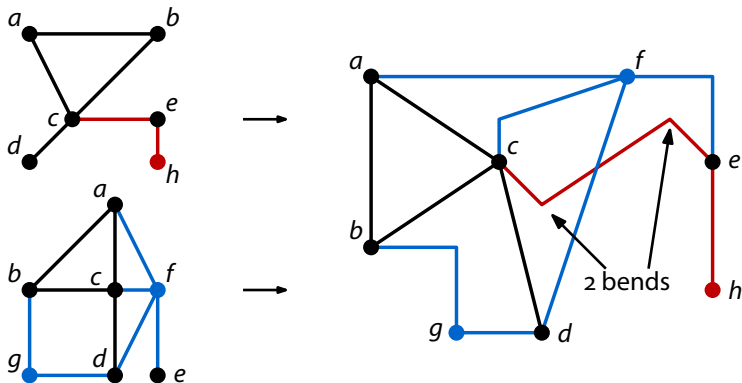
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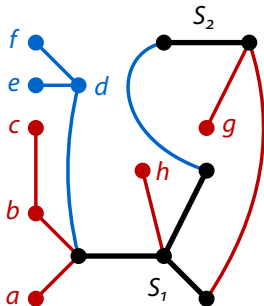
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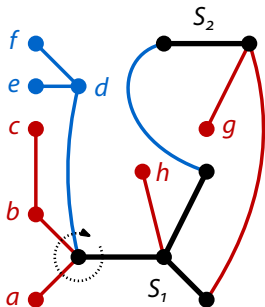
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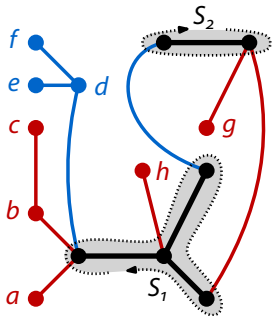
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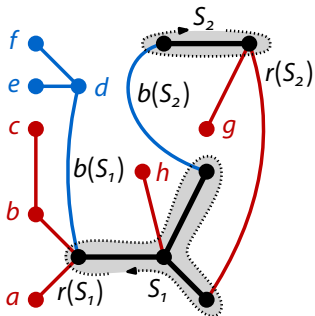
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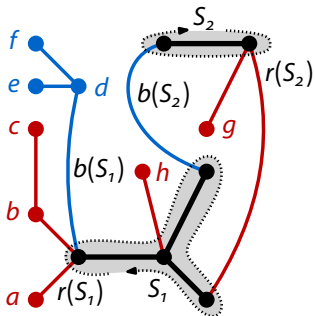
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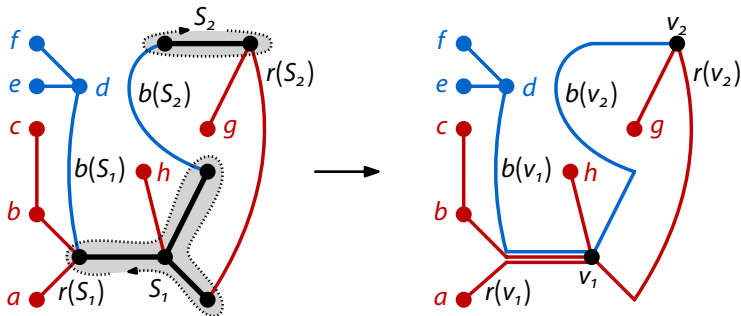


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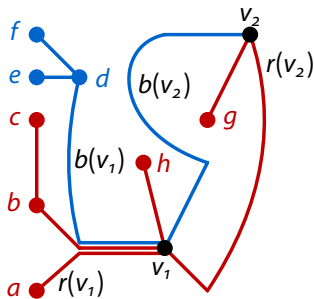




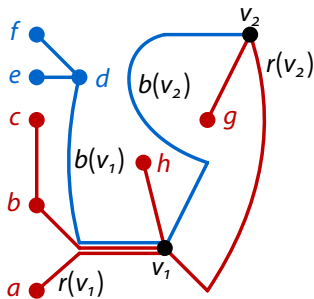
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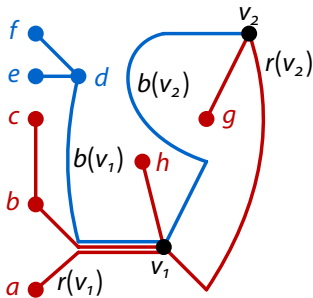
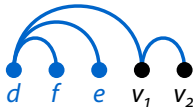
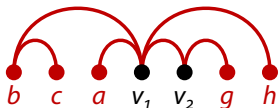
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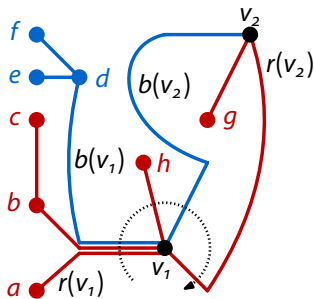
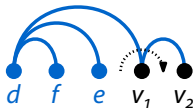
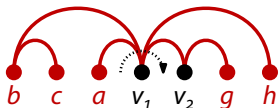
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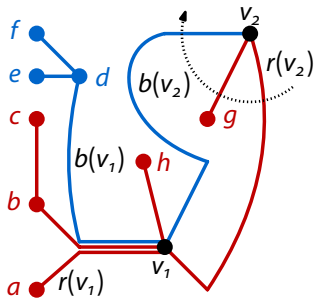
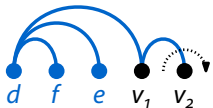
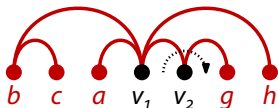
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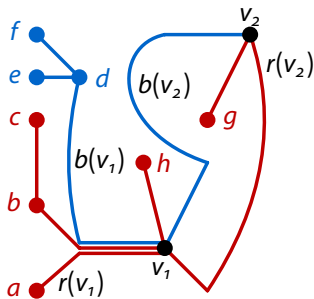
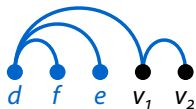
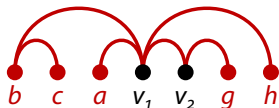
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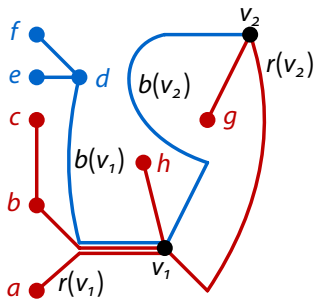
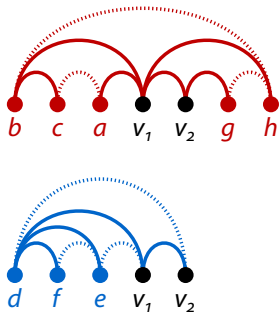
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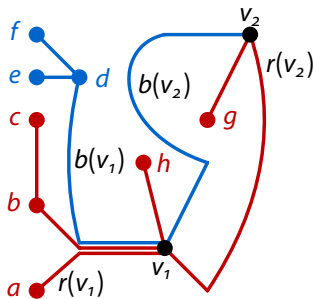
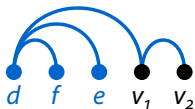
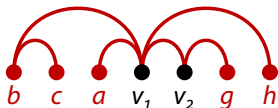


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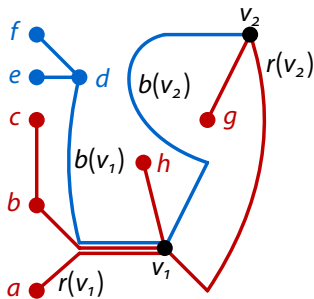
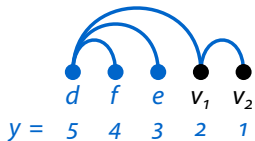
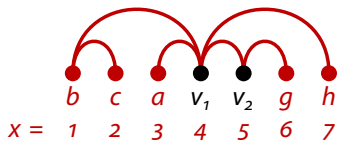




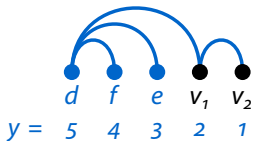
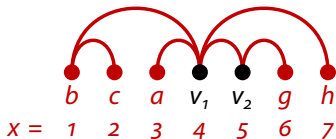
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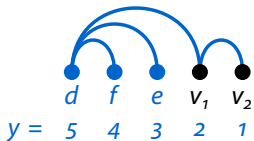
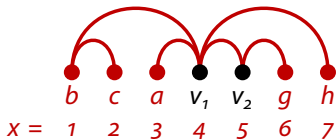
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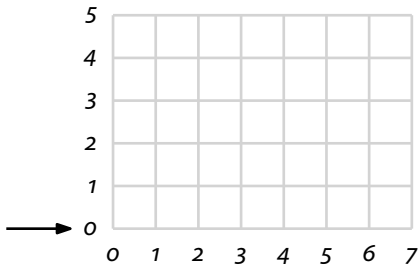
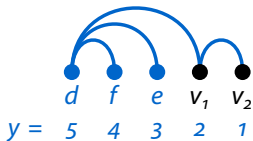
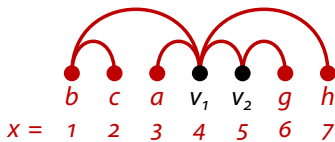
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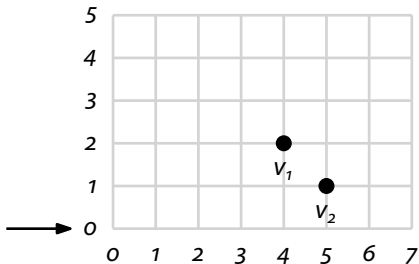
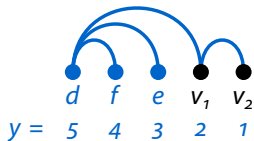
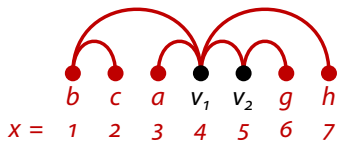
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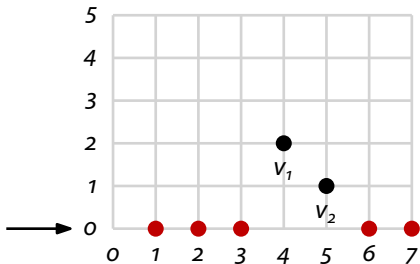
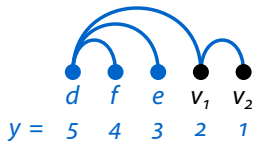
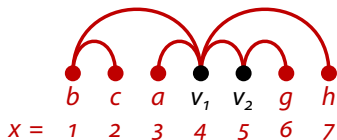
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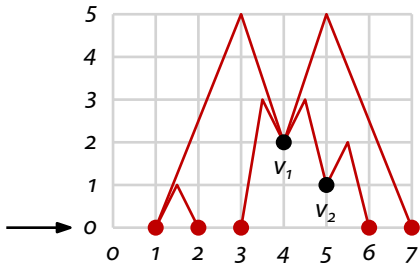
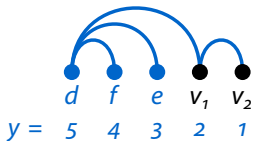
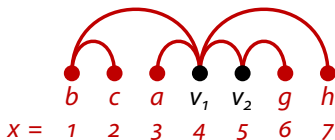
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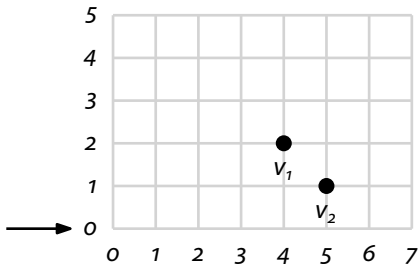
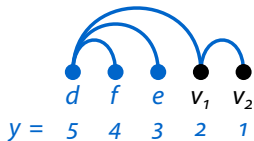
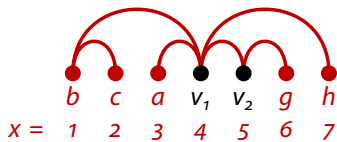


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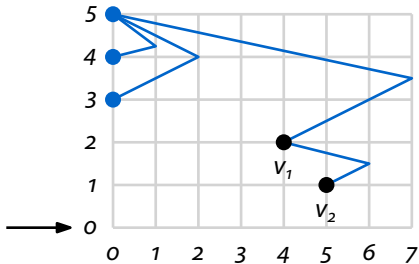
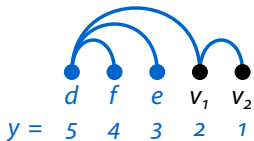
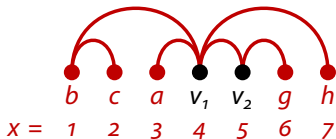




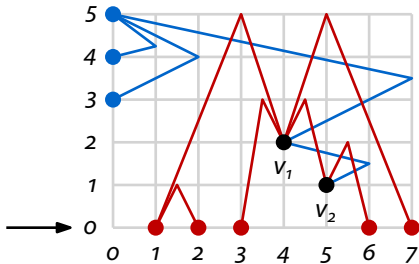
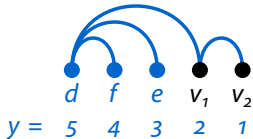
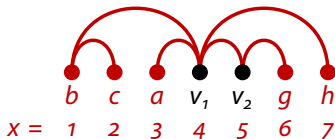
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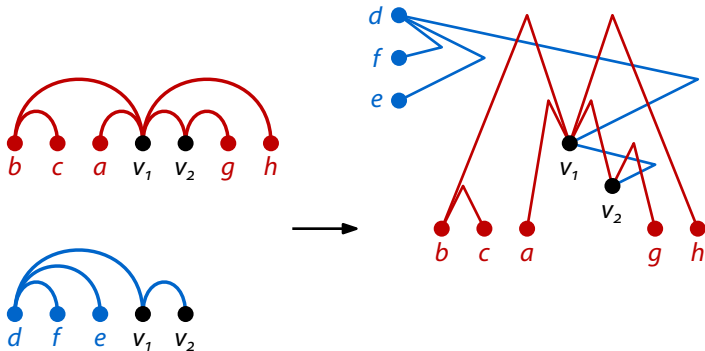
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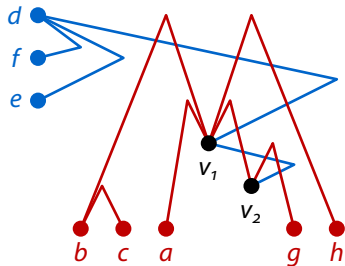
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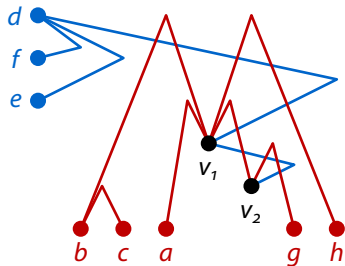
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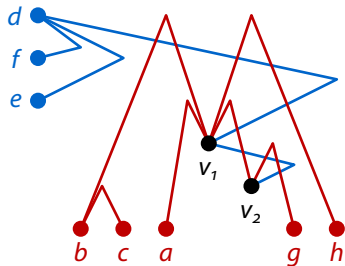
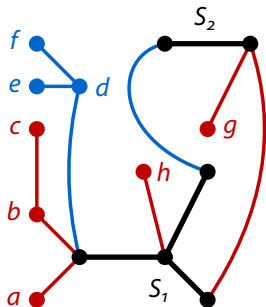
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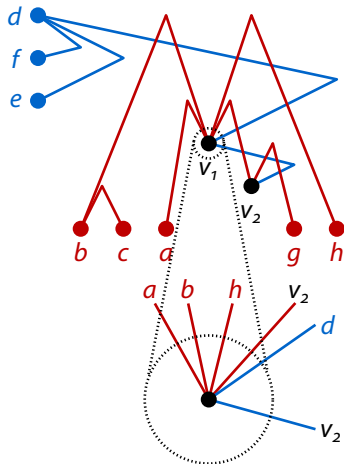
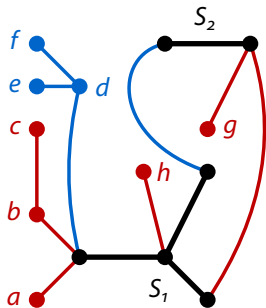
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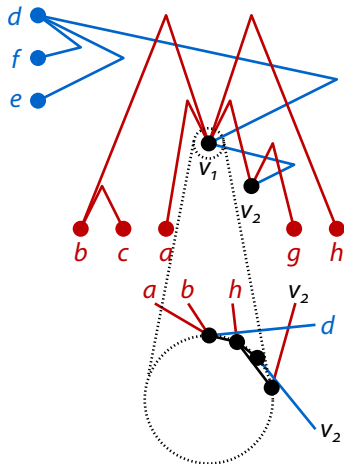
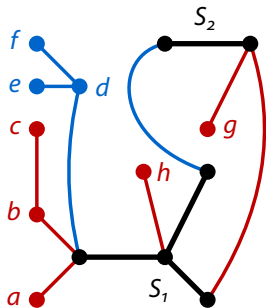


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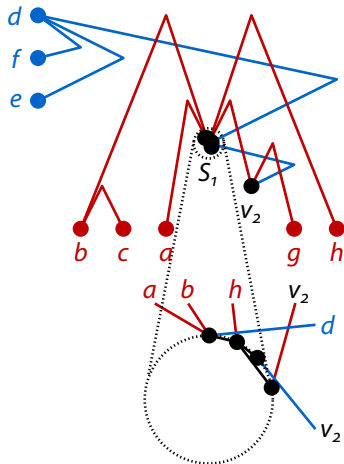
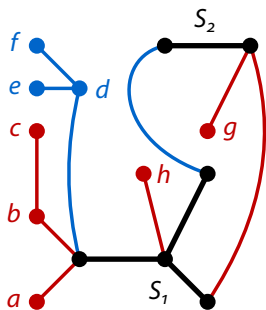




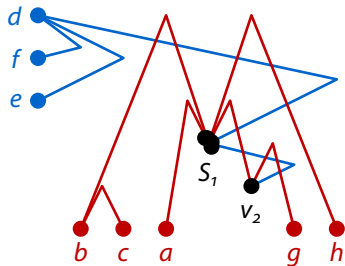
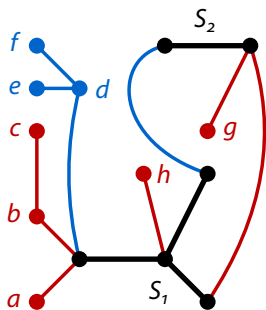
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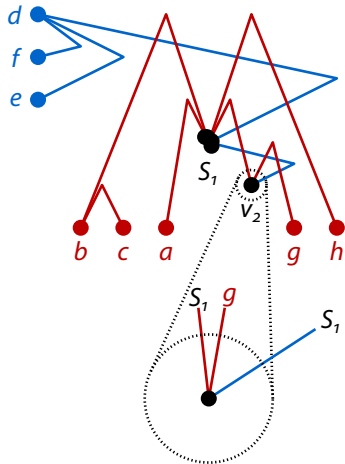
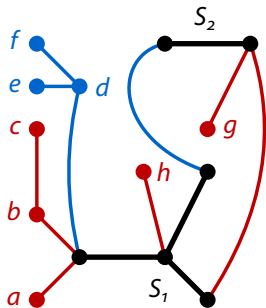
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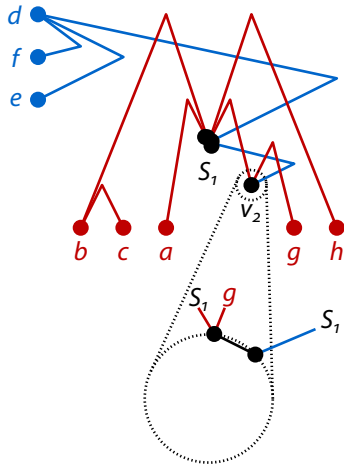
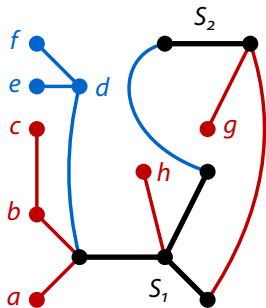
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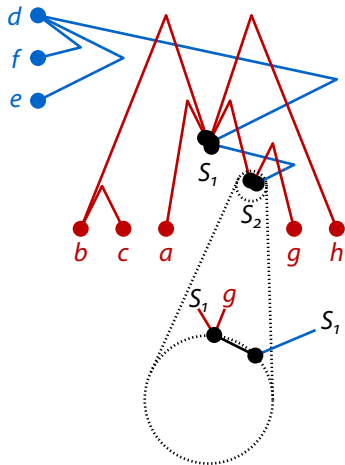
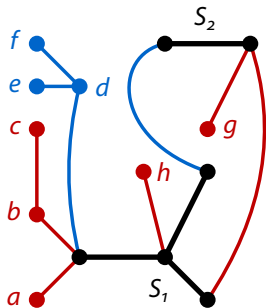
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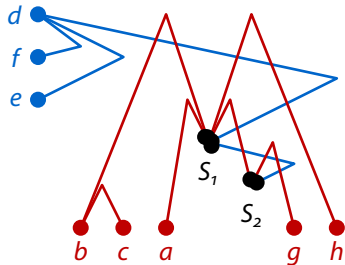
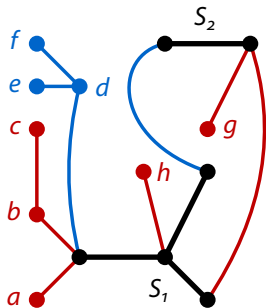
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