The Utility of Untangling

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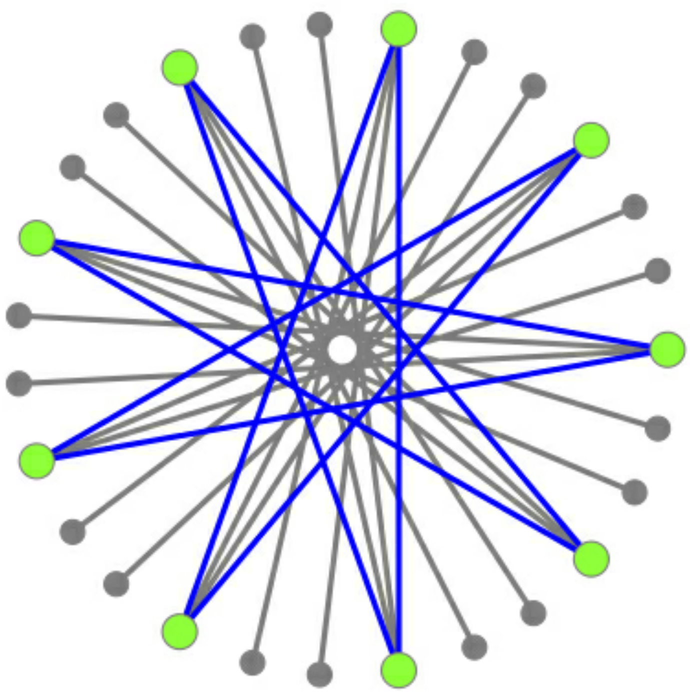
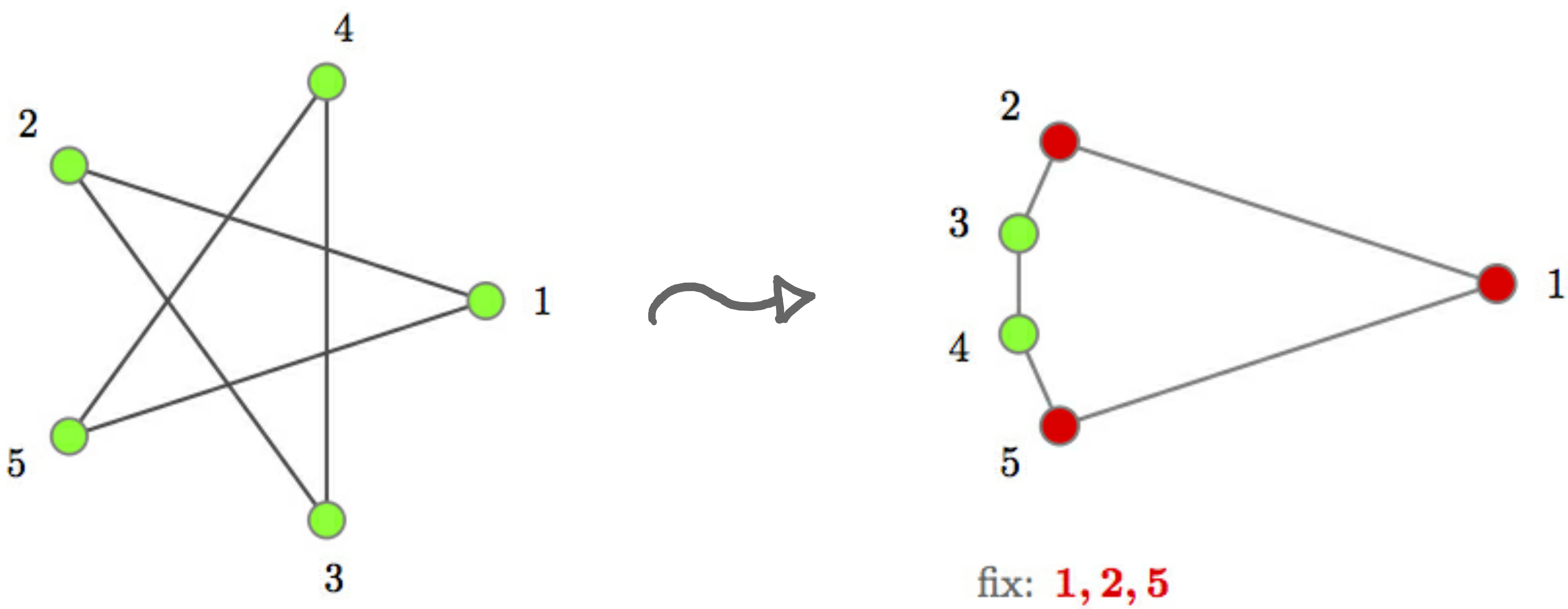


Dedicated to dearly missed
Ferran Hurtado

what this talk is not about → UNTANGLING

GIVEN : a geometric planar graph G

TASK : move as few vertices as possible s.t the resulting geom. planar graph is crossing-free i.e untangle G



fix: **1, 2, 5**

AN OLD RESULT

th: Every n-vertex geom planar graph can be untangled while keeping $\Omega(n^{1/4})$

[Bose, Dujmović, Hurtado, Langerman, Morin, Wood] 2007

Q: Why do we care?

AN OLD RESULT

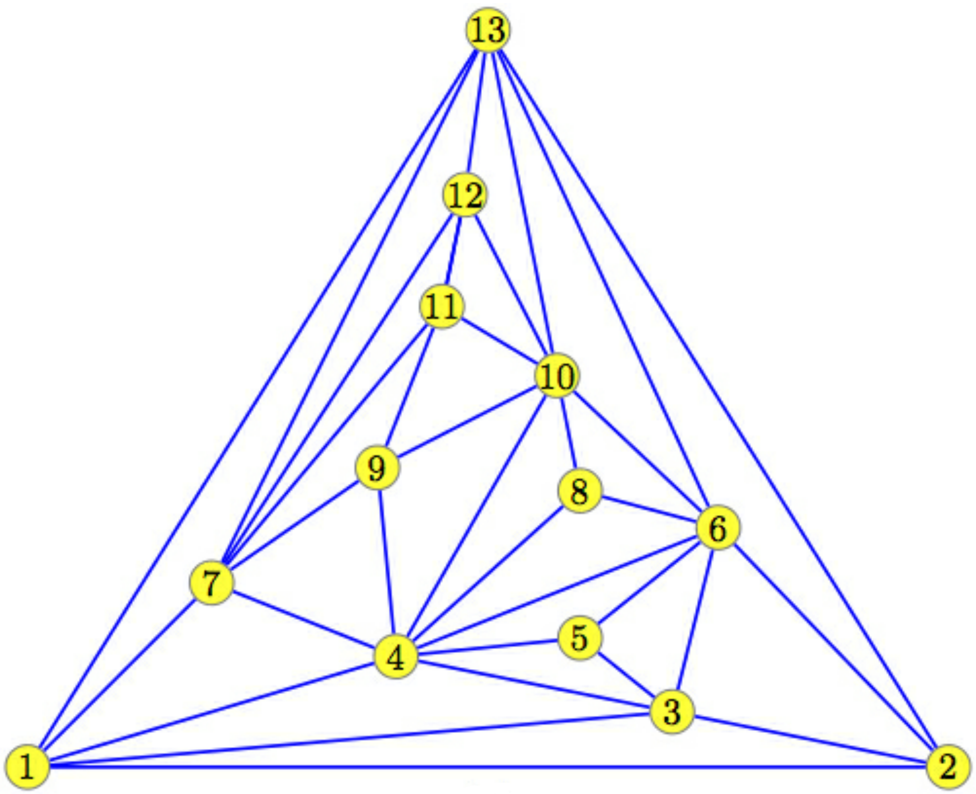
Q: Why do we care?

Tools developed by to attack that problem implicitly give 2 lemmas that can be used to answer open problems in several new GD models.

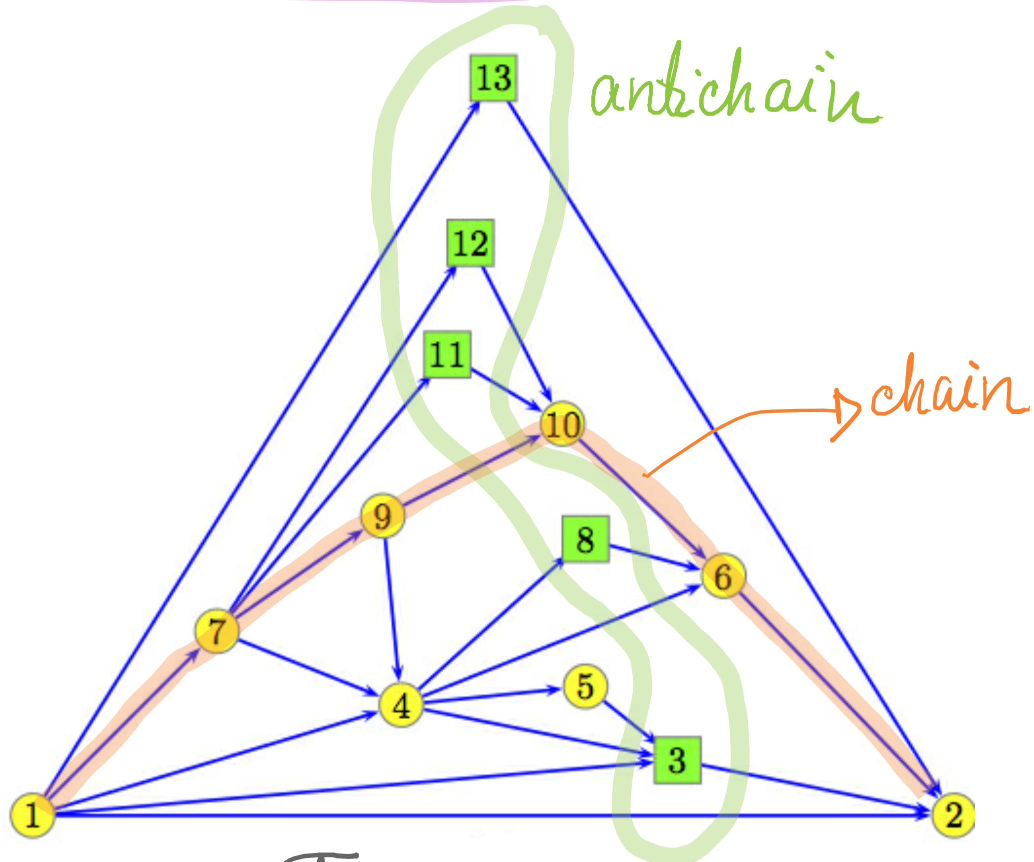
[Bose, Dujmović, Hurtado, Langerman, Morin, Wood 2007]

[Goaoc, Kratochvil, Okamoto, Shin, Spillner, Wolff 2007]

Tools

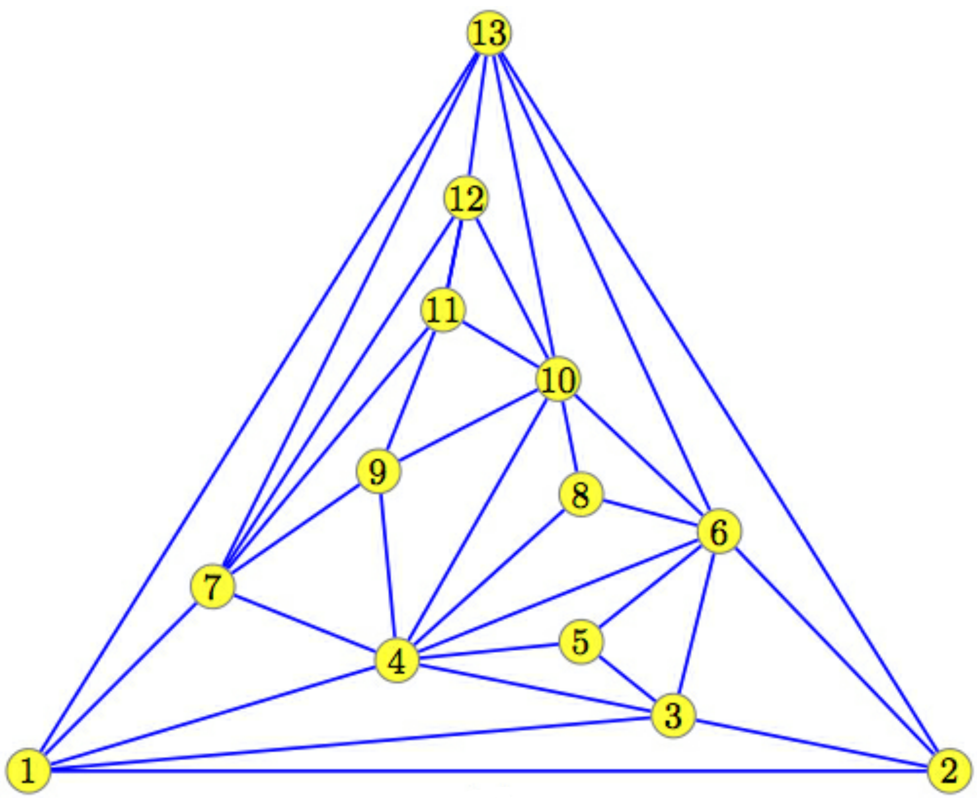


Canonical Ordering

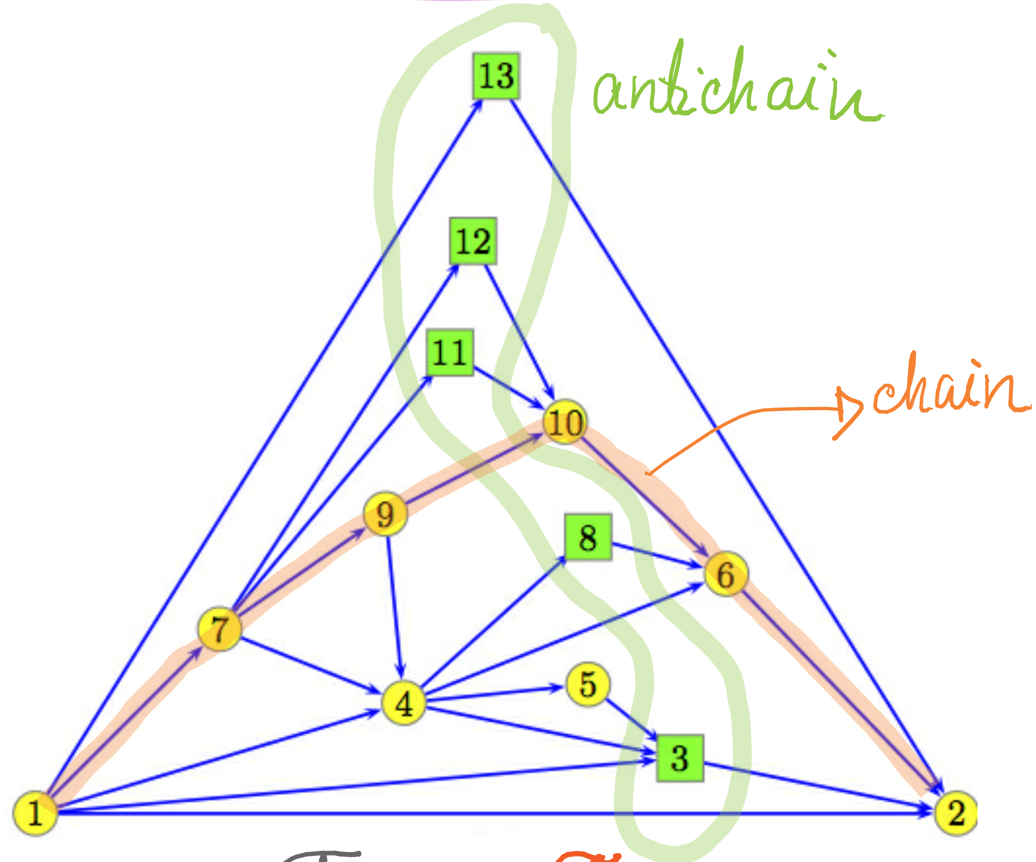


Frame

Tools



Canonical Ordering



Frame $\mathcal{F}$

is also a union of 2 trees in Schnyder 3-tree decomposition with edge directions in one of the trees reversed as observed by [Giacomo, Liotta, Mchedlidze 2012]

Tools : lemma 1

G plane triangulation
$<_{\mathcal{F}}$ a partial order defined by $\mathcal{F}$
H a graph induced in G by a maximal chain in $<_{\mathcal{F}}$

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Lemma 1

[Gaoc, Kratochvil, Okamoto, Shin, Spillner, Wolff] 2007

$\exists$ an independent set $I \subseteq V(H)$ where $|I| \in \Omega(|V(H)|)$
s.t. given any set of points P, where $|P| = |I|$
every mapping from I to P can be extended
to a straight-line drawing of G, as long as
$\forall v, w \in I$, $x(v) < x(w)$ iff $v <_{\mathcal{F}} w$

x-coord

crossing-free

Tools : Lemma 2

G plane triangulation
$<_\sigma$ canonical ordering used to derive $\mathcal{F}$

Lemma 2

[Bose, Dujmović, Hurtado, Langerman, Morin, Wood]
2007

Let $A \subseteq V(G)$ induce an antichain in $<_\mathcal{F}$
given any set of points P, where $|P| = |A|$,
every mapping from A to P can be extended
to a straight-line drawing of G as long as
$\forall v, w \in A$, $x(v) < x(w)$ iff $v <_\sigma w$

ONE APPLICATION

Def:

bigger one

G_1, G_2 s.t $|V(G_1)| \geq |V(G_2)|$ have geom. simultaneous embedding with no mapping (GSEnoM) if $\exists$ a pointset P of size $|V(G_1)|$ such that both G_1 & G_2 have a straight-line cross-face drawing on P.

Q: [Angelini, Evans, Frati, Gudmundsson,] 2013

What is the largest k s.t every n-vertex planar graph and every k-vertex planar graph have GSEnoM?

ONE APPLICATION

Th $\forall n$ and every $k \le \sqrt{n/2}$, every n-vertex planar graph G_1 and every k-vertex planar graph G_2 admit GSEnoM

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Th $\forall n$ and every $k \leq \sqrt{n/2}$, every n-vertex planar graph G_1 and every k-vertex planar graph G_2 admit GSEnoM

Proof sketch

Fáry Th

straight-line crossing-free

1. G_2 has SLCF drawing on some pointset P where $|P| = k \leq \sqrt{n/2}$

2. A frame of G_1 has either a chain or an antichain of size $\sqrt{n}$

Lm 1 or Lm 2 $\Longrightarrow$ G_1 has a SLCF drawing where each point of P is taken by some vertex of G_1. QED

OTHER APPLICATIONS

- Column planar sets
- Universal point subsets
- Other variant of Simultaneous Geometric Embeddings with or without mappings

- anything else ?

Thank you Patrizio!

not a hat but
a chocolat cake

All the mistakes
are made by Hida

.... Enjoy the dinner