Drawing Planar Cubic 3-Connected Graphs with Few Segments: Algorithms & Experiments

Alex Igamberdiev
Wouter Meulemans
André Schulz
Graph complexity

Complexity of a graph $G = (V, E)$

Usually $|V|, |E|$, etc.
Graph complexity

Complexity of a graph $G = (V, E)$
Usually $|V|$, $|E|$, etc.

*Says nothing about how complex a drawing is*
Visual complexity

Planar graphs

Number of geometric objects for drawing
Visual complexity

Planar graphs

Number of geometric objects for drawing
Planar graphs

Number of geometric objects for drawing
Visual complexity

Planar graphs

Number of geometric objects for drawing

2
Visual complexity

Planar graphs

Number of geometric objects for drawing

3
Visual complexity

Planar graphs

Number of geometric objects for drawing

4
Visual complexity

Planar graphs

Number of geometric objects for drawing
Visual complexity

Planar graphs

Number of geometric objects for drawing
Visual complexity

Planar graphs

Number of geometric objects for drawing
Visual complexity

Planar graphs

Number of geometric objects for drawing
Visual complexity

Planar graphs

Number of geometric objects for drawing
Visual complexity

Planar graphs

Number of geometric objects for drawing

9 line segments for 18 edges
## Known results

<table>
<thead>
<tr>
<th>Segments</th>
<th>Class</th>
<th>Lower</th>
<th>Upper</th>
<th>References</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Tree</td>
<td>$K/2$</td>
<td>$K/2$</td>
<td>[Durocher et al, 2013]</td>
</tr>
<tr>
<td>2- and 3-trees</td>
<td>2$V$</td>
<td>2$V$</td>
<td>2$V$</td>
<td>[Dujmović et al, 2007]</td>
</tr>
<tr>
<td>3-connected</td>
<td>2$V$</td>
<td>5$V/2$</td>
<td>2$V$</td>
<td>[Dujmović et al, 2007]</td>
</tr>
<tr>
<td>Triangulation</td>
<td>2$V$</td>
<td>7$V/3$</td>
<td>2$V$</td>
<td>[Durocher, Mondal, 2014]</td>
</tr>
<tr>
<td>Planar</td>
<td>2$V$</td>
<td>16$V/3 - E$</td>
<td></td>
<td>[Durocher, Mondal, 2014]</td>
</tr>
</tbody>
</table>
## Known results

<table>
<thead>
<tr>
<th>Segments</th>
<th>Class</th>
<th>Lower</th>
<th>Upper</th>
<th>References</th>
</tr>
</thead>
<tbody>
<tr>
<td>Tree</td>
<td>$K/2$</td>
<td>$K/2$</td>
<td></td>
<td>[Durocher et al, 2013]</td>
</tr>
<tr>
<td>2- and 3-trees</td>
<td>$2V$</td>
<td>$2V$</td>
<td></td>
<td>[Dujmović et al, 2007]</td>
</tr>
<tr>
<td>3-connected</td>
<td>$2V$</td>
<td>$5V/2$</td>
<td></td>
<td>[Dujmović et al, 2007]</td>
</tr>
<tr>
<td>Triangulation</td>
<td>$2V$</td>
<td>$7V/3$</td>
<td></td>
<td>[Durocher, Mondal, 2014]</td>
</tr>
<tr>
<td>Planar</td>
<td>$2V$</td>
<td>$16V/3 - E$</td>
<td></td>
<td>[Durocher, Mondal, 2014]</td>
</tr>
<tr>
<td>Circ. arcs</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>3-trees</td>
<td>$E/6$</td>
<td>$11E/18$</td>
<td></td>
<td>[Schulz, 2013]</td>
</tr>
<tr>
<td>3-connected</td>
<td>$E/6$</td>
<td>$2E/3$</td>
<td></td>
<td>[Schulz, 2013]</td>
</tr>
</tbody>
</table>
Our results

Line-segment drawings
Planar cubic 3-connected graphs
Our results

Line-segment drawings

Planar cubic 3-connected graphs

Two new algorithms

\[ n/2 + 3 \] segments

[Mondal et al, 2013]

Resolve flaw & improved
Our results

Line-segment drawings

Planar cubic 3-connected graphs

Two new algorithms

\[ n/2 + 3 \] segments

[Mondal et al, 2013]

Resolve flaw & improved

Experimental comparison
Deconstruction algorithm
Theorem.

Every graph can be constructed from the triangular prism with insertions maintaining a given outer face.
Deconstruction algorithm

Algorithm

1. Draw triangular prism
Deconstruction algorithm

Algorithm

1. Draw triangular prism
2. Construct graph, maintaining drawing

Inner faces are convex
No insertions on outer face
Deconstruction algorithm

Algorithm

1. Draw triangular prism
2. Construct graph, maintaining drawing

![Diagram showing the process of insertion]

**Insertion**
Deconstruction algorithm

Algorithm

1. Draw triangular prism
2. Construct graph, maintaining drawing
Algorithm

1. Draw triangular prism
2. Construct graph, maintaining drawing
Deconstruction algorithm

Algorithm

1. Draw triangular prism
2. Construct graph, maintaining drawing
Algorithm

1. Draw triangular prism
2. Construct graph, maintaining drawing
Windmill algorithm
Windmill algorithm

Algorithm

**Pre:** cycle $C$ drawn convex

**Post:** inside of $C$ drawn
Windmill algorithm

Algorithm

Pre: cycle $C$ drawn convex

Post: inside of $C$ drawn
Algorithm

*Pre:* cycle $C$ drawn convex

*Post:* inside of $C$ drawn
Windmill algorithm

Algorithm

*Pre:* cycle $C$ drawn convex

*Post:* inside of $C$ drawn
Windmill algorithm

Algorithm

*Pre:* cycle $C$ drawn convex

*Post:* inside of $C$ drawn
Windmill algorithm

Algorithm

\textit{Pre:} cycle \(C\) drawn convex

\textit{Post:} inside of \(C\) drawn
Windmill algorithm

Algorithm

Pre: cycle $C$ drawn convex

Post: inside of $C$ drawn
Windmill algorithm

Algorithm

$Pre$: cycle $C$ drawn convex

$Post$: inside of $C$ drawn
Set of harmonic equations

\[ u = \lambda v + (1 - \lambda)w, \text{ for } \lambda \in (0, 1) \]
Postprocessing

Set of harmonic equations

\[ u = \lambda v + (1 - \lambda)w, \text{ for } \lambda \in (0, 1) \]

Solve for uniform edge length, i.e. \( \lambda = 1/2 \)

[Aerts & Felsner, 2013]
"Grid"

\( n/2 + 4 \) segments

6 slopes

\( (n/2 + 1)^2 \) grid
“Grid”

\( n/2 + 4 \) segments

6 slopes

\( (n/2 + 1)^2 \) grid

Resolved flaw in algorithm
<table>
<thead>
<tr>
<th>“Grid”</th>
<th>“Min”</th>
</tr>
</thead>
<tbody>
<tr>
<td>( \frac{n}{2} + 4 ) segments</td>
<td>( \frac{n}{2} + 3 ) segments</td>
</tr>
<tr>
<td>6 slopes</td>
<td>7 slopes</td>
</tr>
<tr>
<td>((n/2 + 1)^2) grid</td>
<td>Not on a grid</td>
</tr>
<tr>
<td>Resolved flaw in algorithm</td>
<td></td>
</tr>
<tr>
<td><strong>“Grid”</strong></td>
<td><strong>“Min”</strong></td>
</tr>
<tr>
<td>------------</td>
<td>-----------</td>
</tr>
<tr>
<td>( \frac{n}{2} + 4 ) segments</td>
<td>( \frac{n}{2} + 3 ) segments</td>
</tr>
<tr>
<td>6 slopes</td>
<td>7 slopes</td>
</tr>
<tr>
<td>((\frac{n}{2} + 1)^2) grid</td>
<td>Not on a grid</td>
</tr>
<tr>
<td>Resolved flaw in algorithm</td>
<td>Reduced to 6 slopes</td>
</tr>
<tr>
<td></td>
<td>On a grid</td>
</tr>
</tbody>
</table>
Three algorithms

- Deconstruction
- Windmill
- [Mondal et al, 2013]
Measuring layout quality

2000 graphs with $24 \ldots 30$ vertices using plantri

Six measures for each graph-algorithm pair
2000 graphs with 24...30 vertices using plantri

Six measures for each graph-algorithm pair

Angular resolution
Measuring layout quality

2000 graphs with 24...30 vertices using plantri

Six measures for each graph-algorithm pair

Angular resolution

Edge length
Measuring layout quality

2000 graphs with 24...30 vertices using plantri

Six measures for each graph-algorithm pair

Angular resolution

Edge length

Face aspect ratio
Measuring layout quality

2000 graphs with 24...30 vertices using plantri

Six measures for each graph-algorithm pair

Angular resolution

Edge length

Face aspect ratio

Average and worst-case
Angular resolution

Average

Minimum

WIN
DEC
DEC-ALT
MON-GRID
MON-MIN

0
\(\pi/2\)

0
\(\pi/2\)
Face aspect ratio

**Average**

- WIN
- DEC
- DEC-ALT
- MON-GRID
- MON-MIN

**Minimum**

- WIN
- DEC
- DEC-ALT
- MON-GRID
- MON-MIN
Experiment summary

“Wins”

“Wins” minus “Losses”

-6 -5 -4 -3 -2 -1 0 1 2 3 4 5 6
Conclusion

Minimal visual complexity

Two new algorithms

Fixed and improved [Mondal et al, 2013]

Experiments

Best depends on measure
Conclusion

Minimal visual complexity

Two new algorithms

Fixed and improved [Mondal et al, 2013]

Experiments

Best depends on measure

Future work

Closing gap for other classes

Circular arcs

Visual complexity $\sim$ observer’s assessment?

Visual complexity $\sim$ cognitive load?