# Drawing Planar Cubic 3-Connected Graphs with Few Segments: Algorithms & Experiments

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# Graph complexity

Complexity of a graph $G = (V, E)$

Usually $|V|$, $|E|$, etc.

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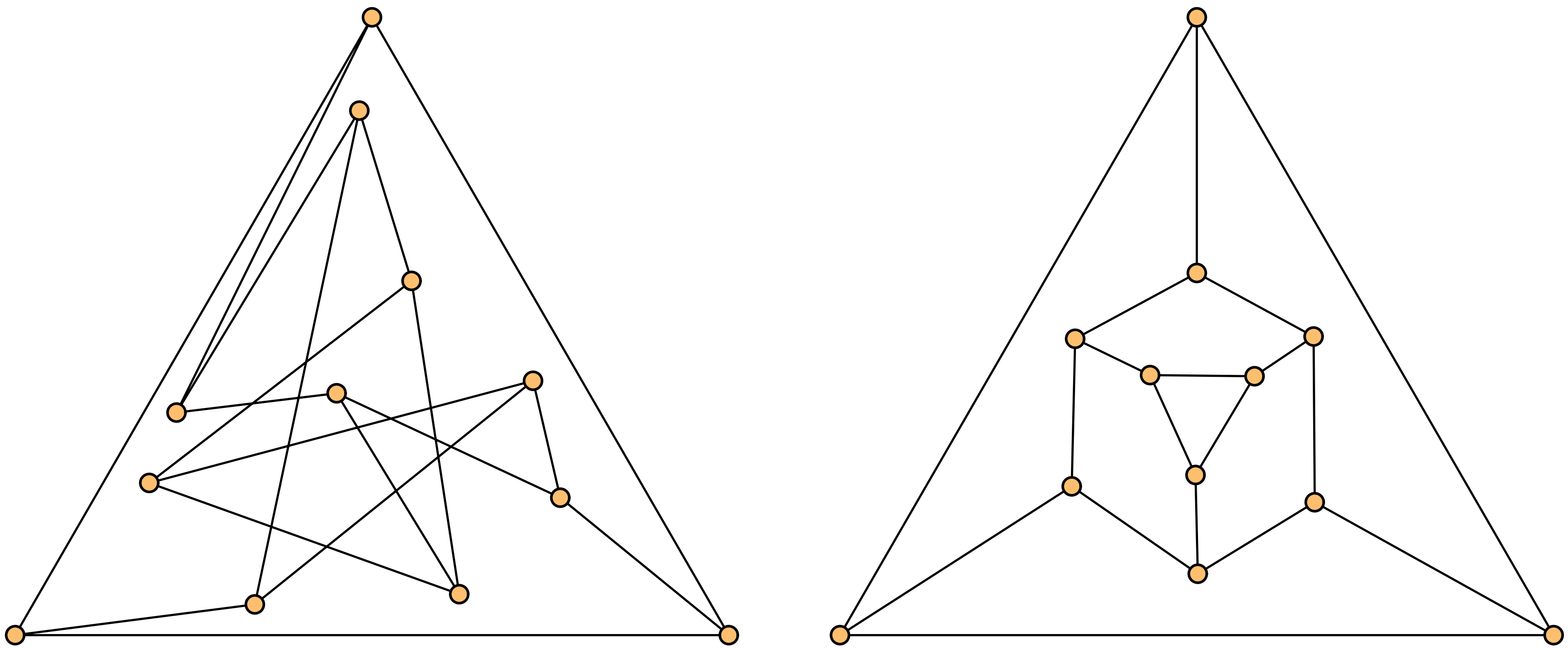
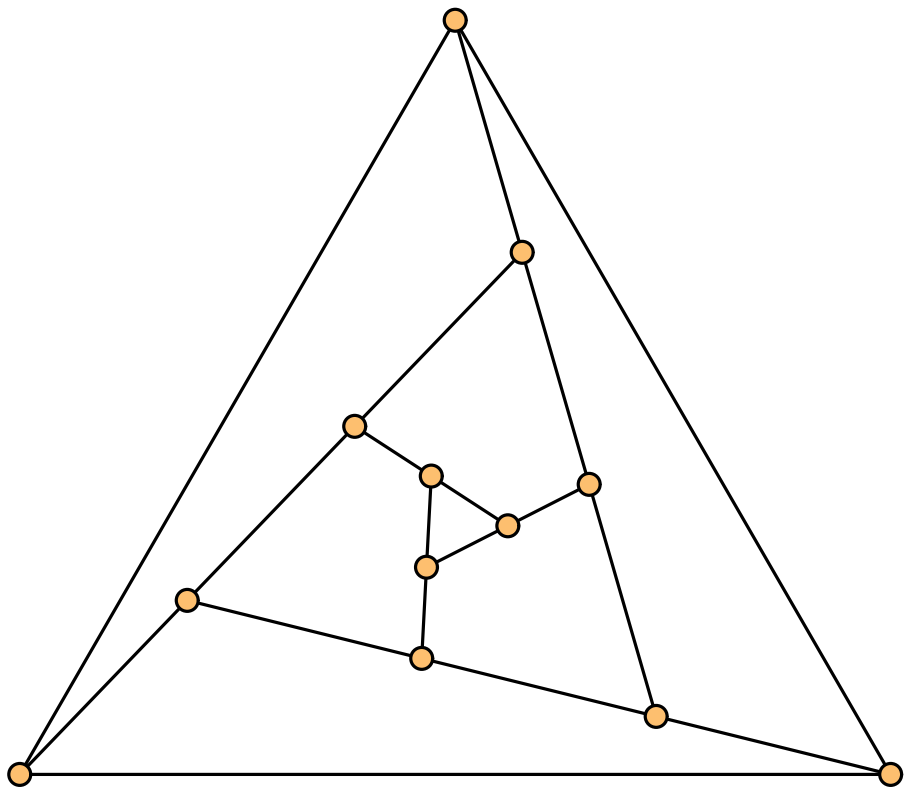
*Says nothing about how complex a drawing is*



# Visual complexity

Planar graphs

Number of geometric objects for drawing



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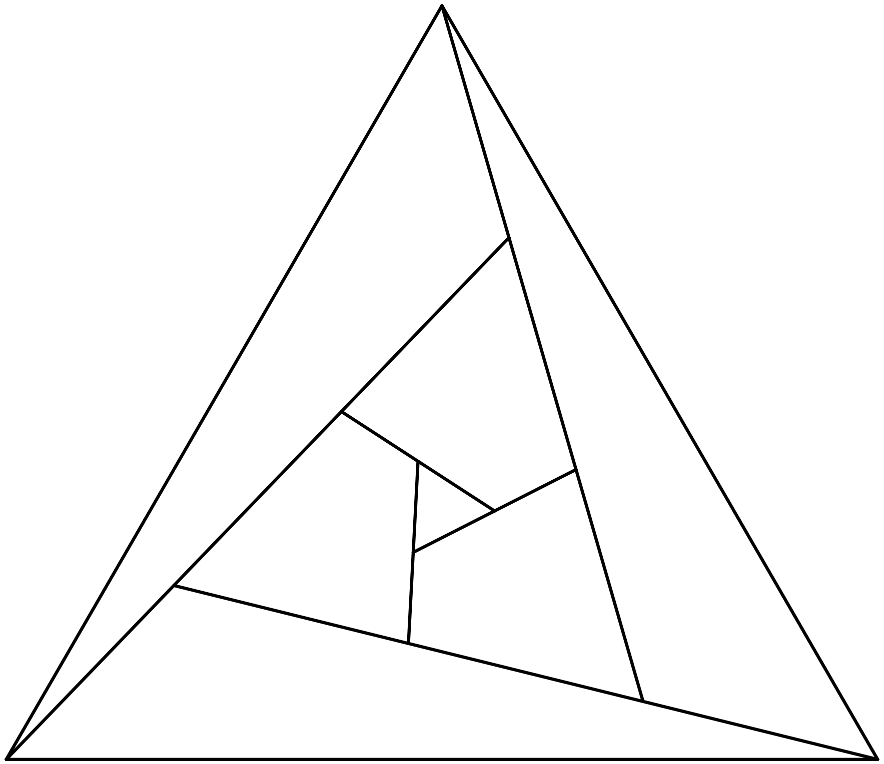

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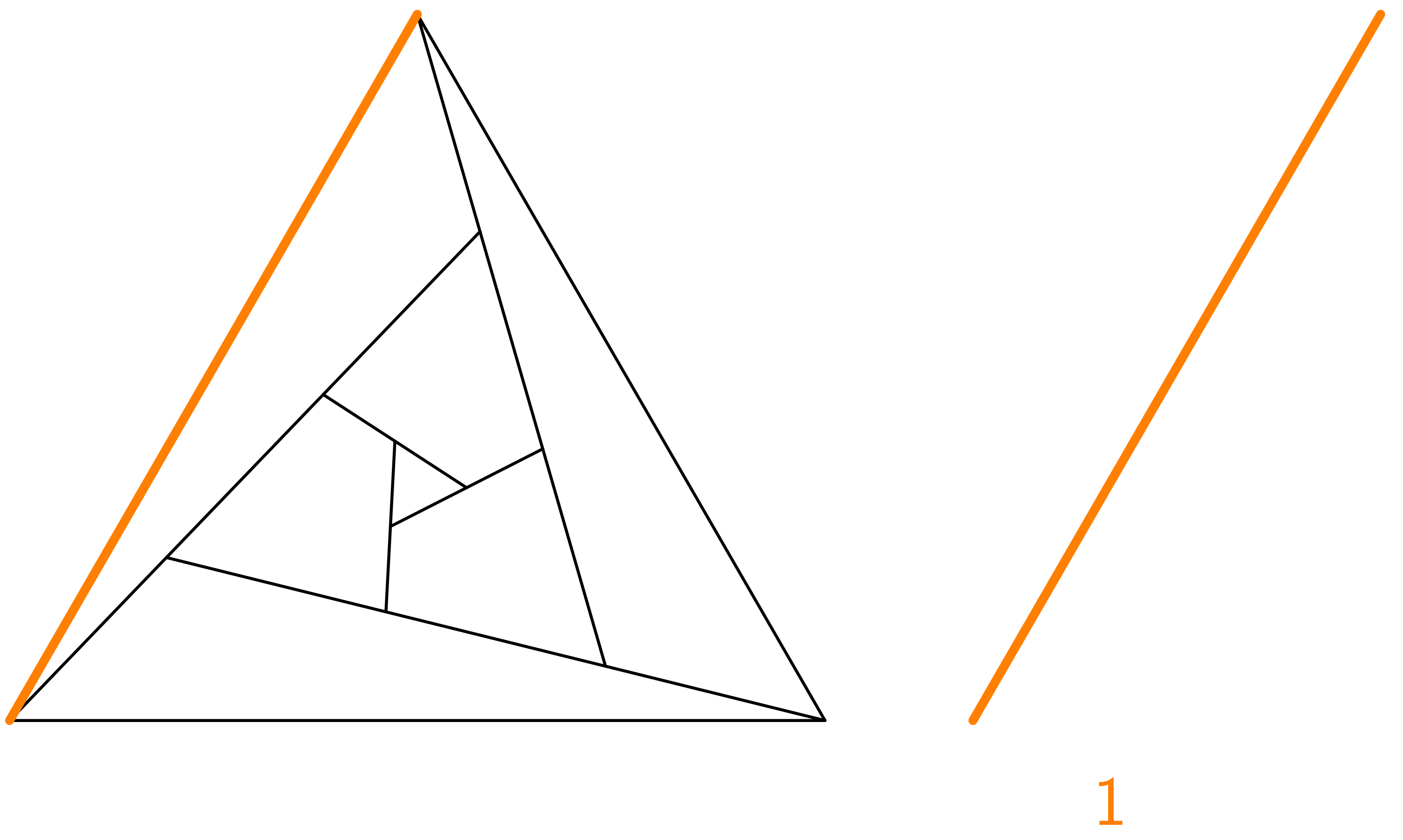

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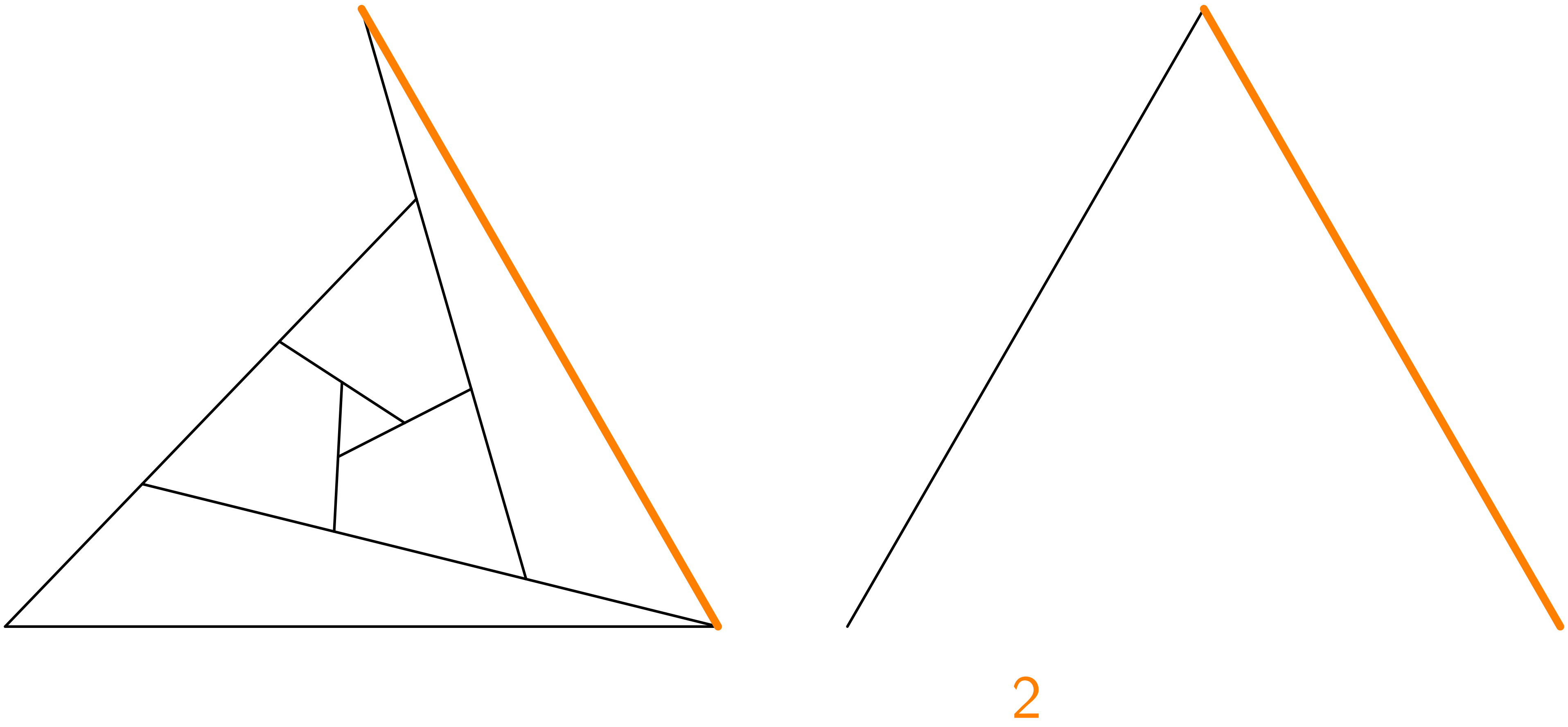

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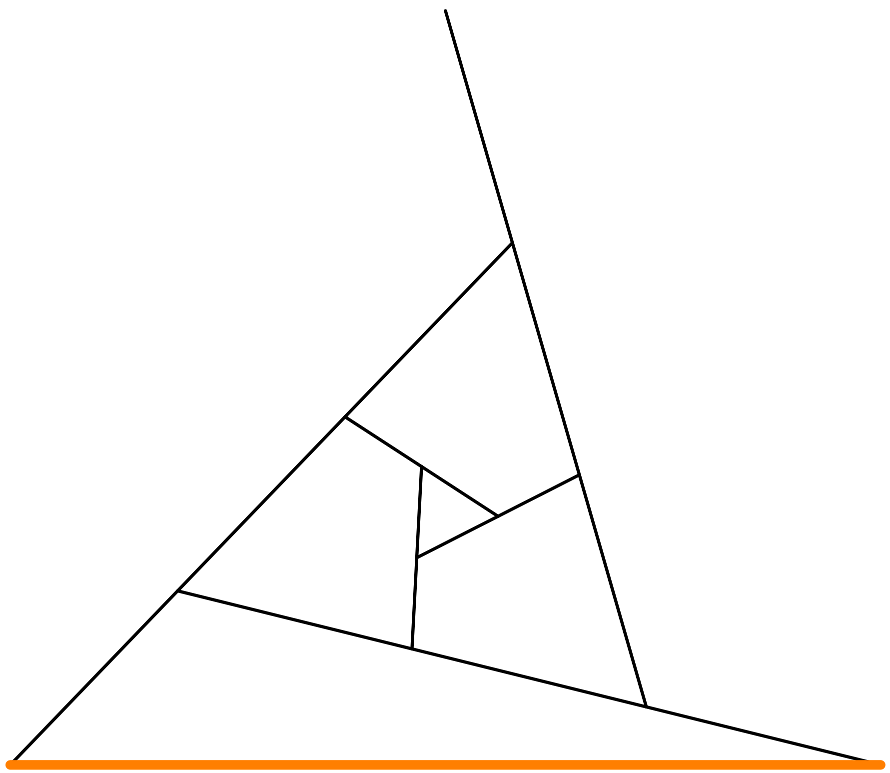

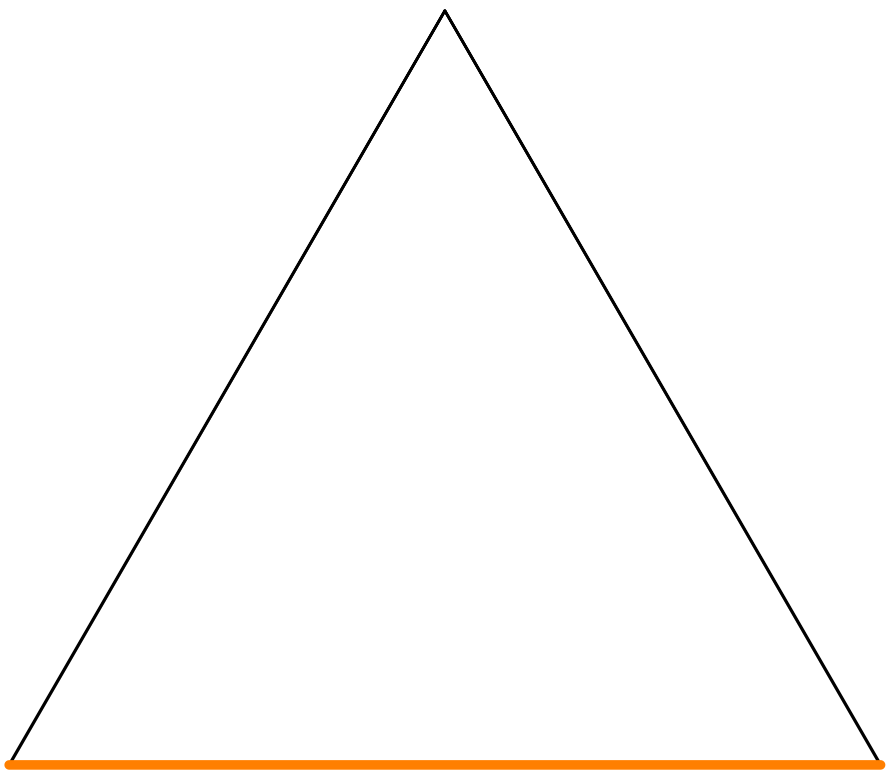

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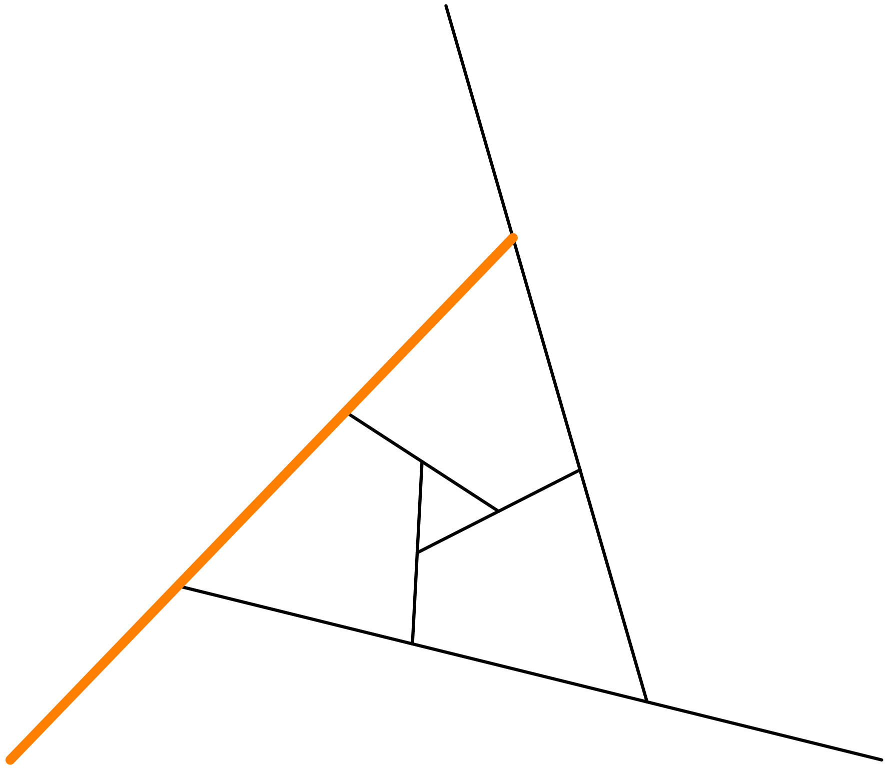

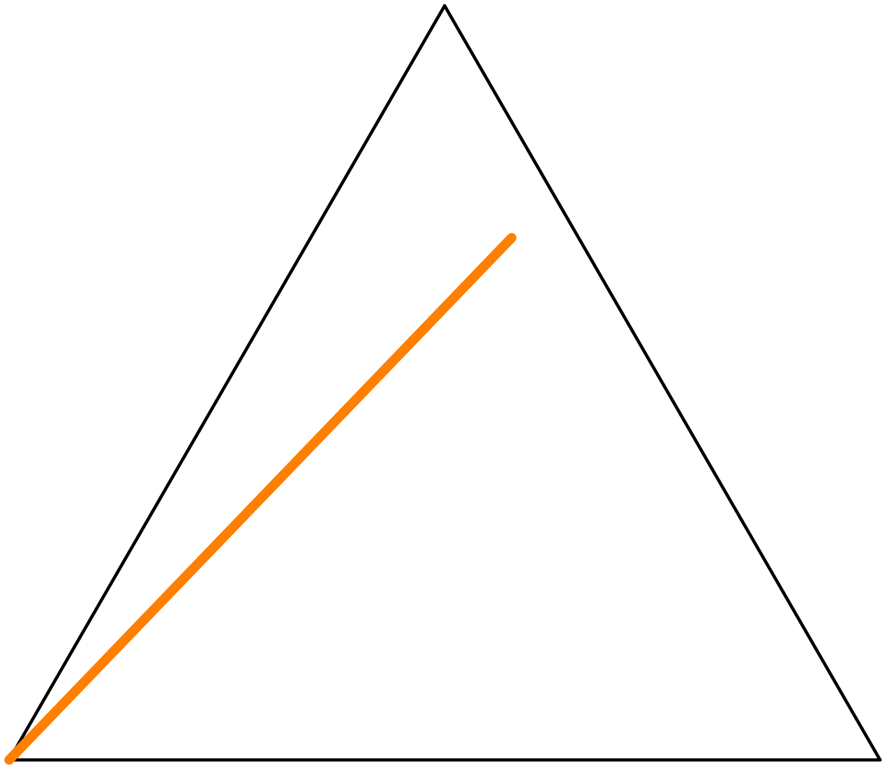

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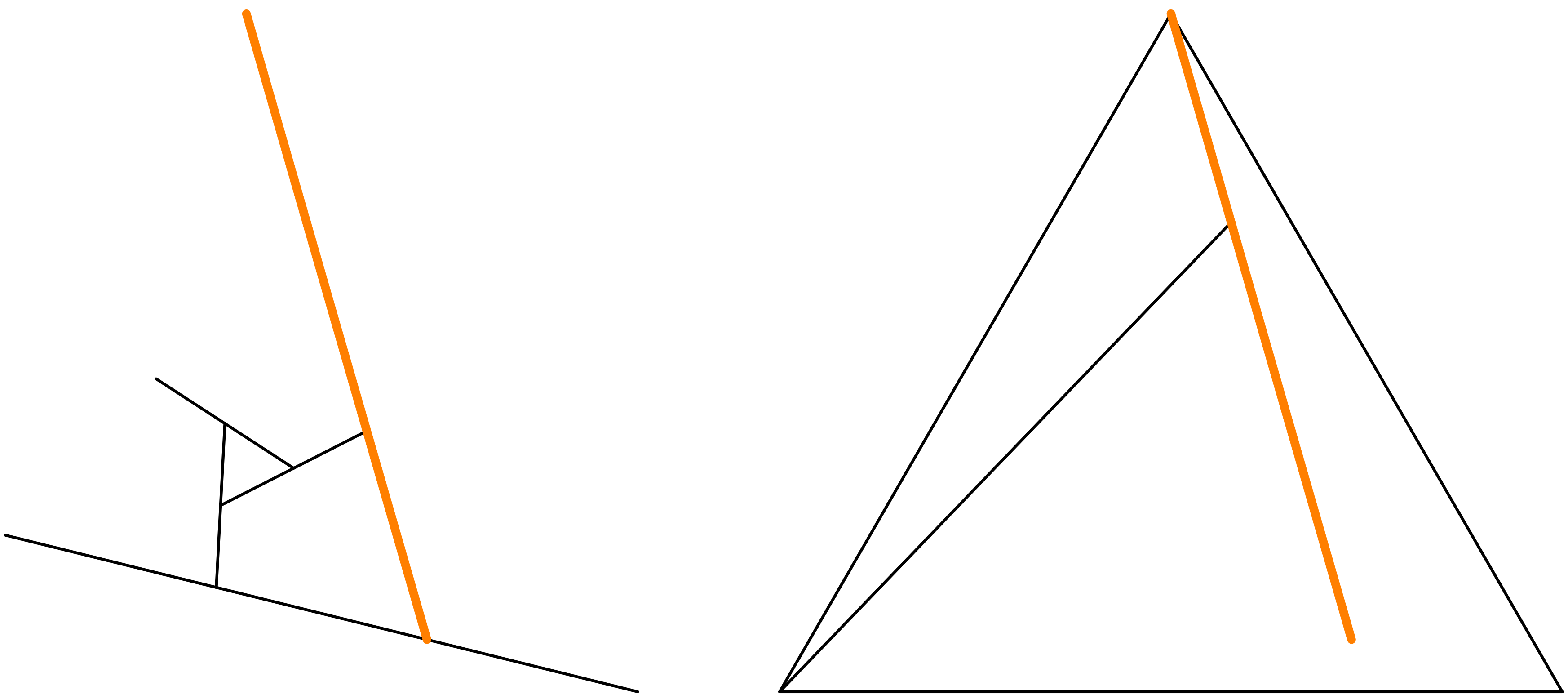

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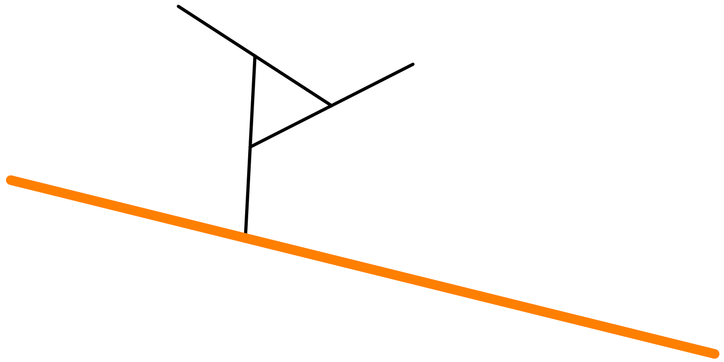

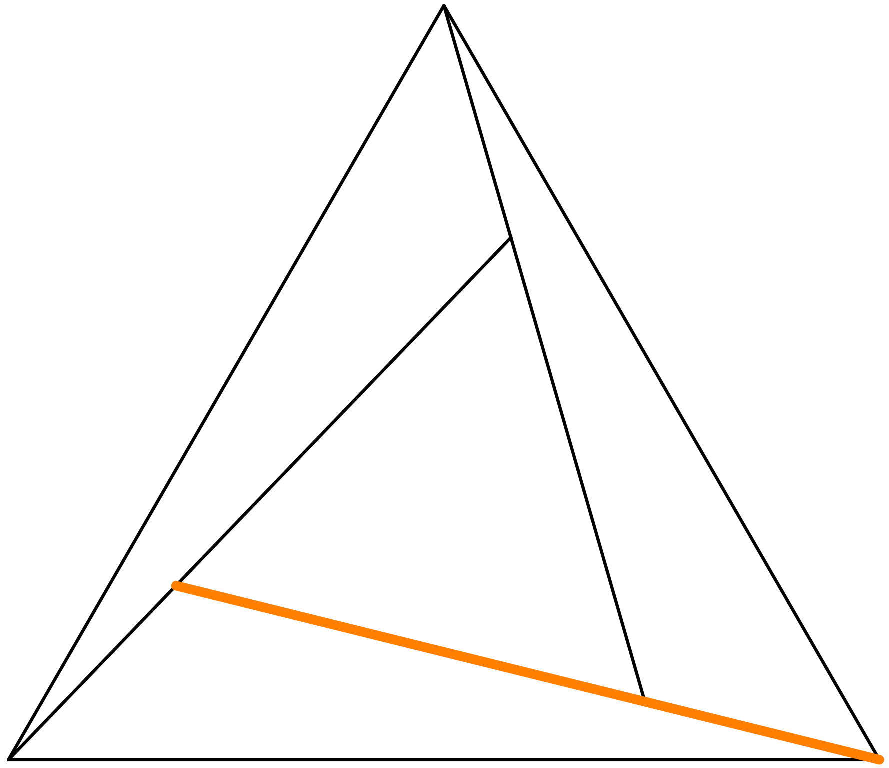

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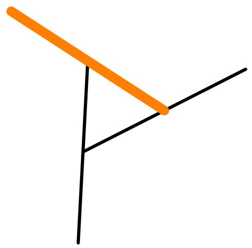

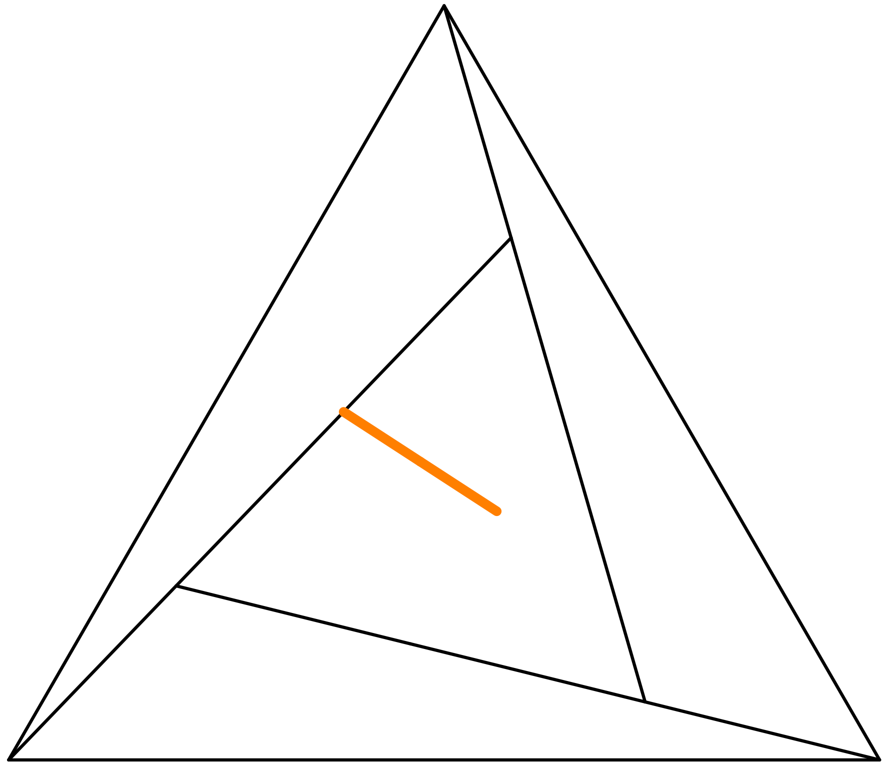

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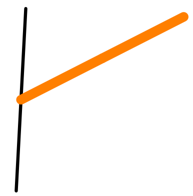

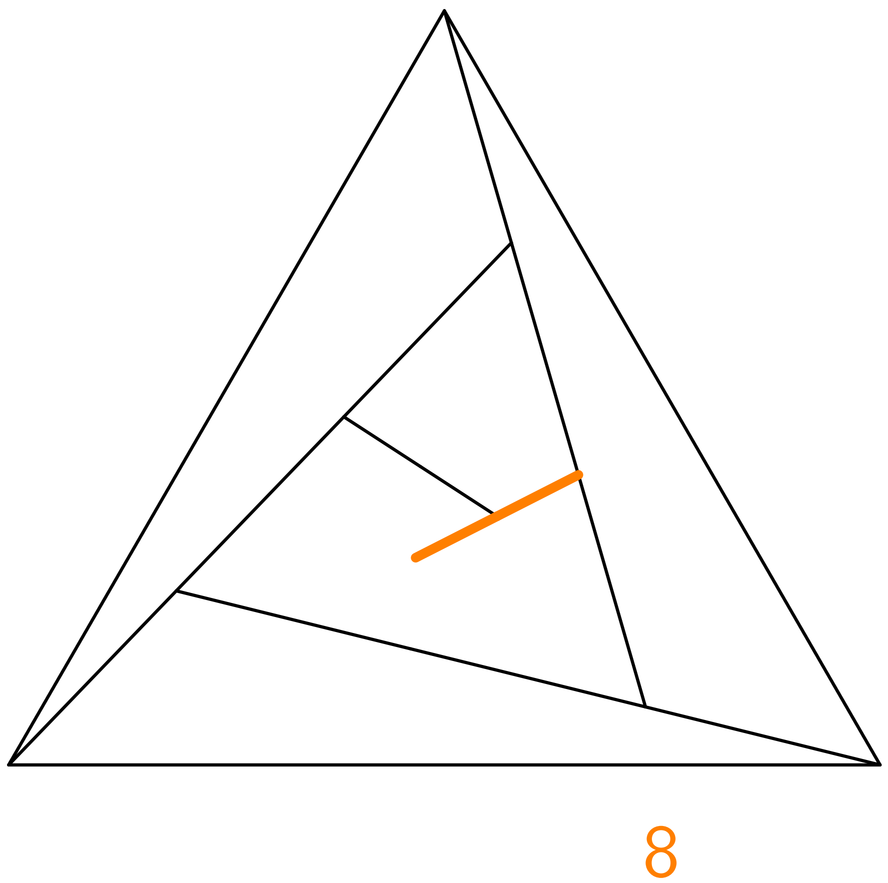

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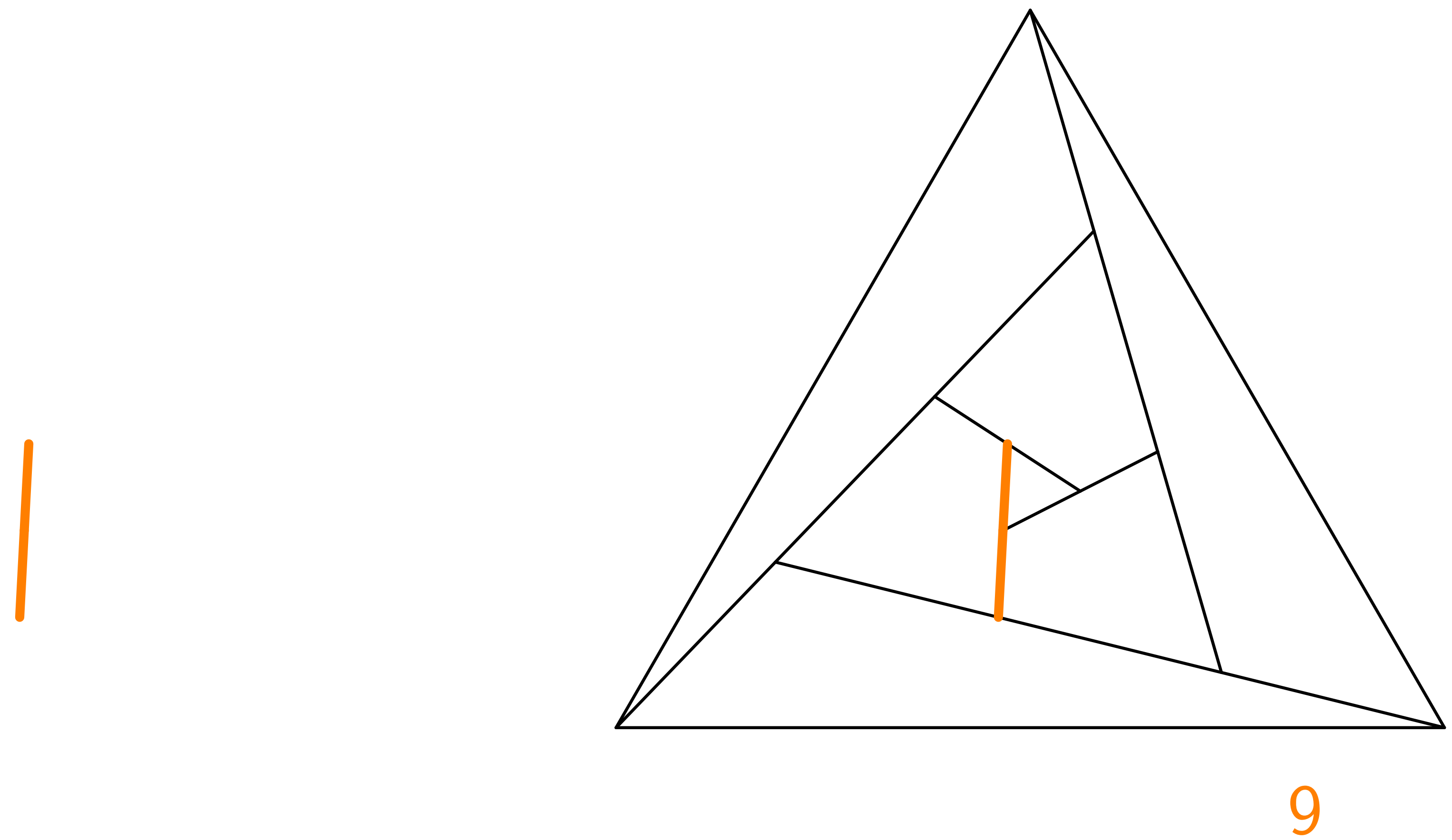

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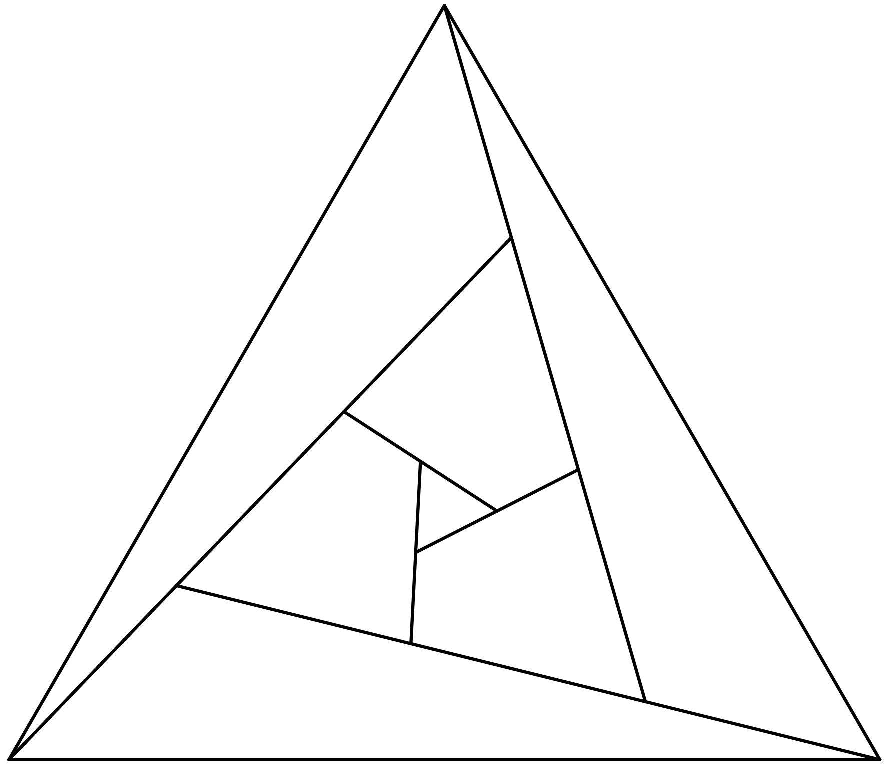

$9$ line segments for $18$ edges

# Known results

| | Class | Lower | Upper | |
|---|---|---|---|---|
| Segments | Tree | $K/2$ | $K/2$ | [Durocher et al, 2013] |
| | 2- and 3-trees | $2V$ | $2V$ | [Dujmović et al, 2007] |
| | 3-connected | $2V$ | $5V/2$ | [Dujmović et al, 2007] |
| | Triangulation | $2V$ | $7V/3$ | [Durocher, Mondal, 2014] |
| | Planar | $2V$ | $16V/3 - E$ | [Durocher, Mondal, 2014] |

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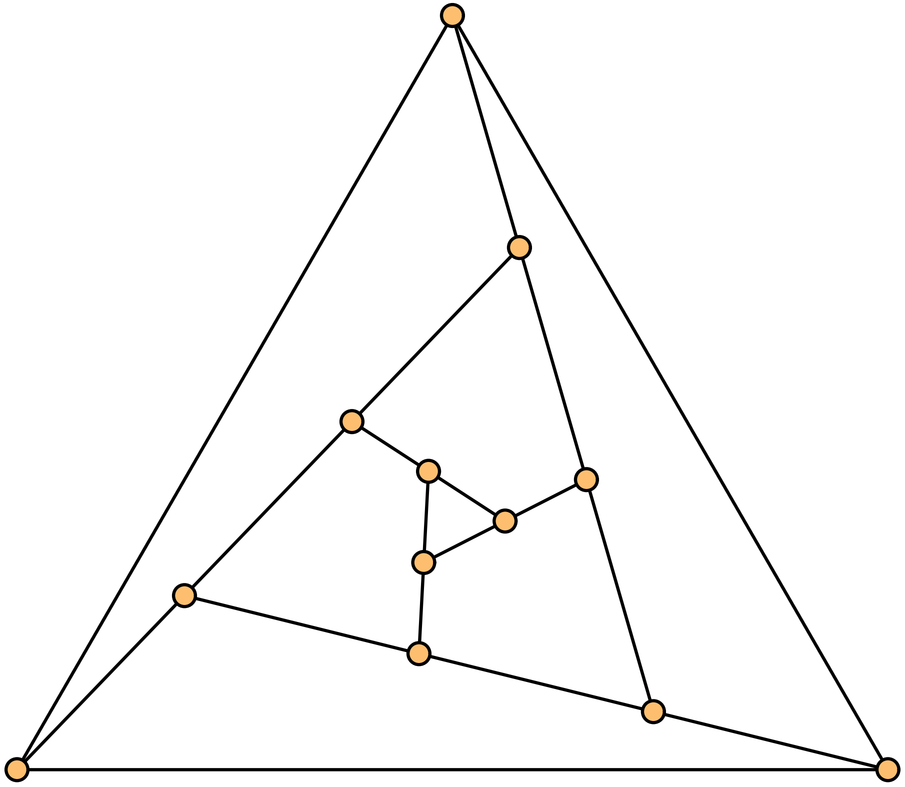
| Circ. arcs | 3-trees | $E/6$ | $11E/18$ | [Schulz, 2013] |
| | 3-connected | $E/6$ | $2E/3$ | [Schulz, 2013] |

# Our results

Line-segment drawings

Planar cubic 3-connected graphs



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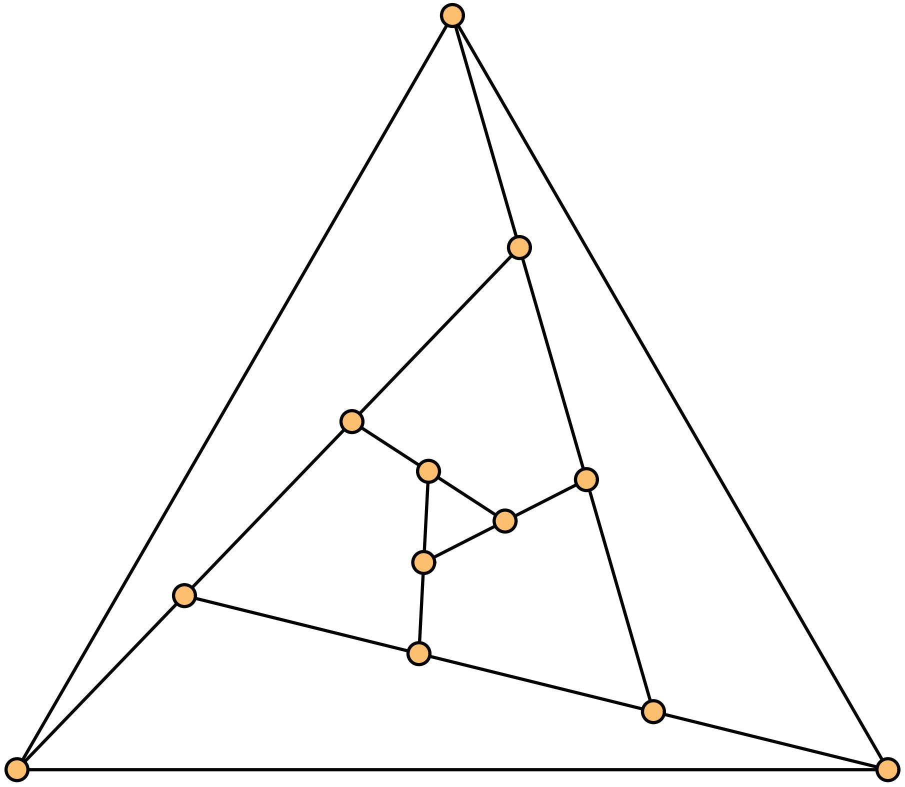
Two new algorithms

$n/2 + 3$ segments

[Mondal et al, 2013]

Resolve flaw & improved



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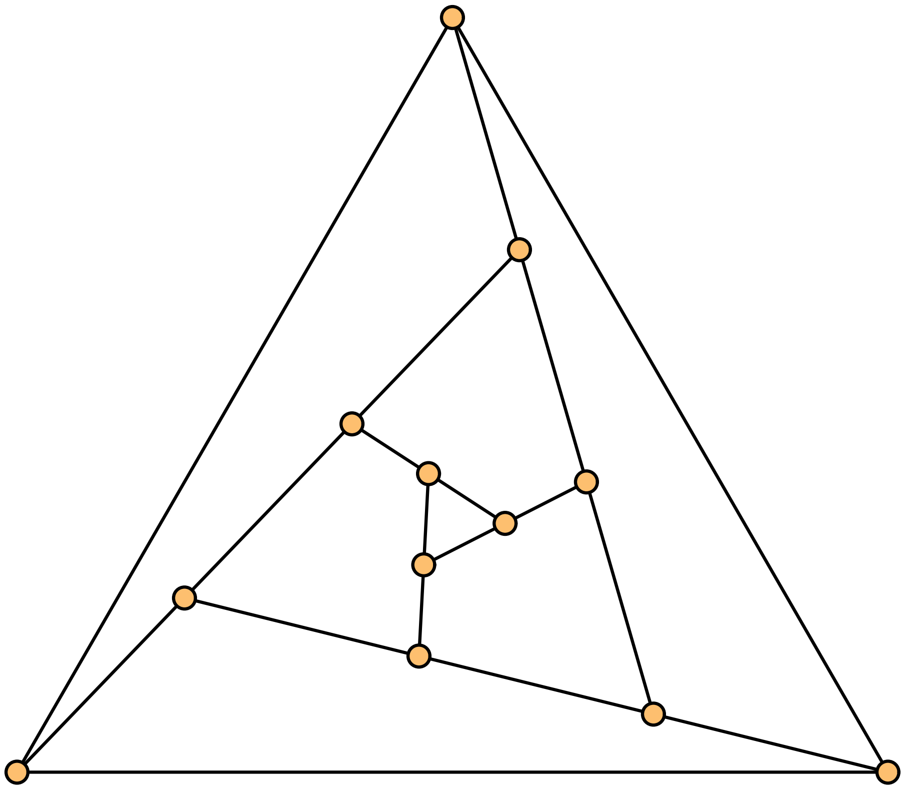
Experimental comparison



# Deconstruction algorithm

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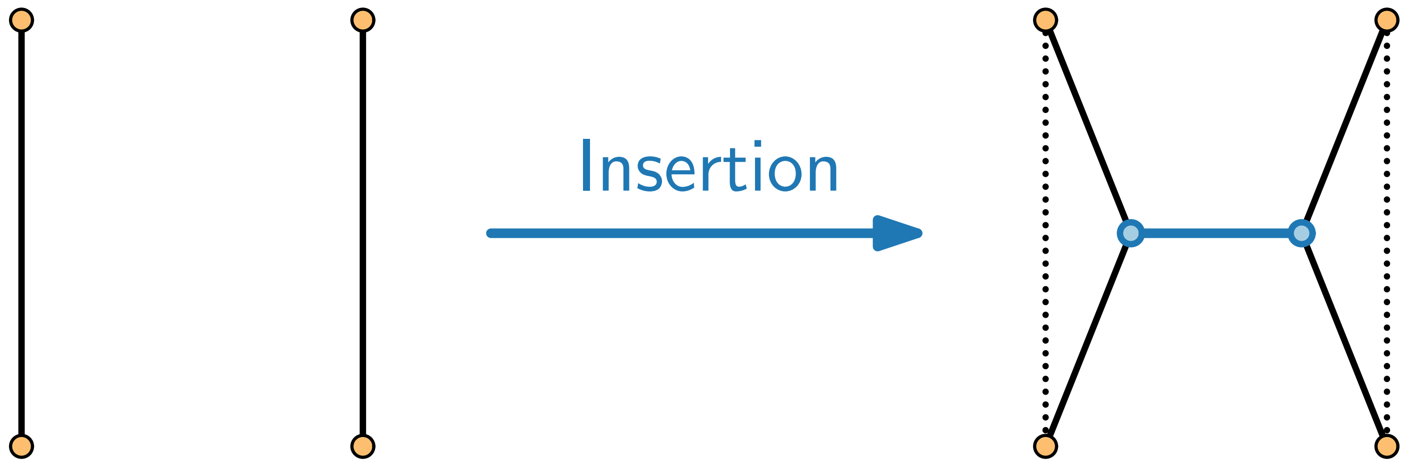
**Theorem.**

Every graph can be constructed

from the triangular prism

with insertions

maintaining a given outer face.



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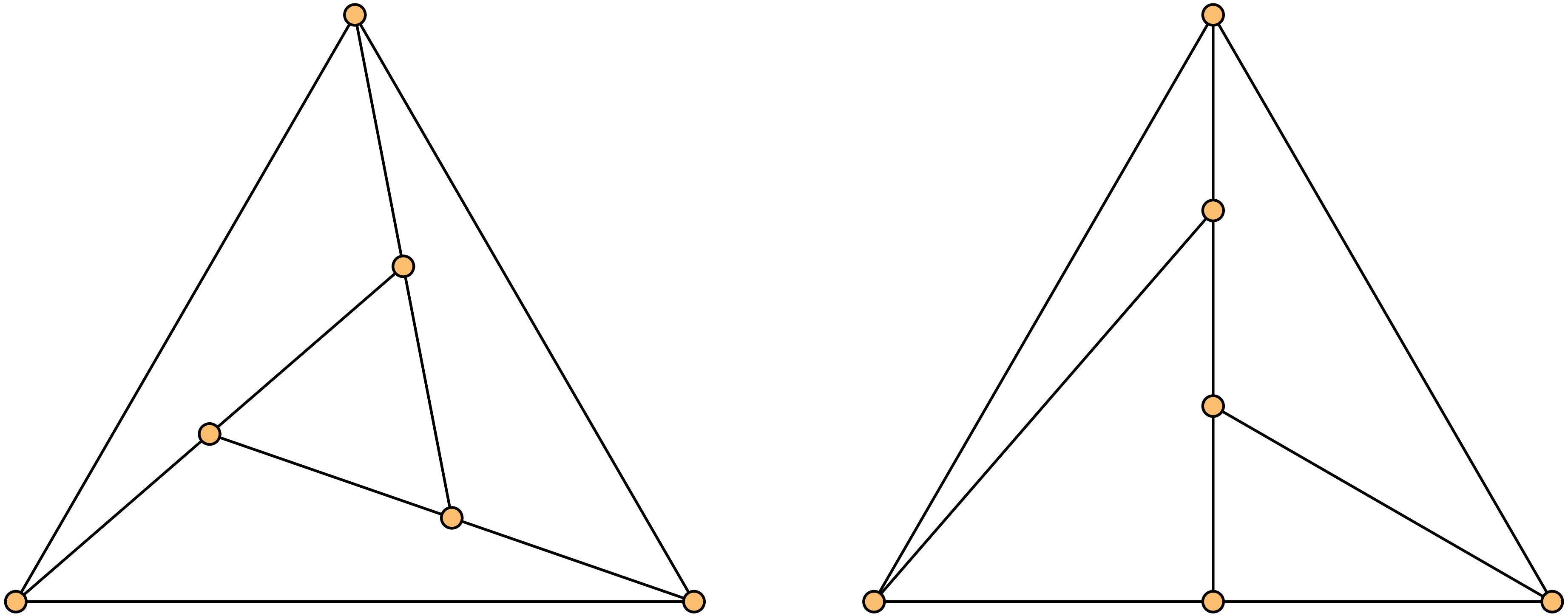
**Algorithm**

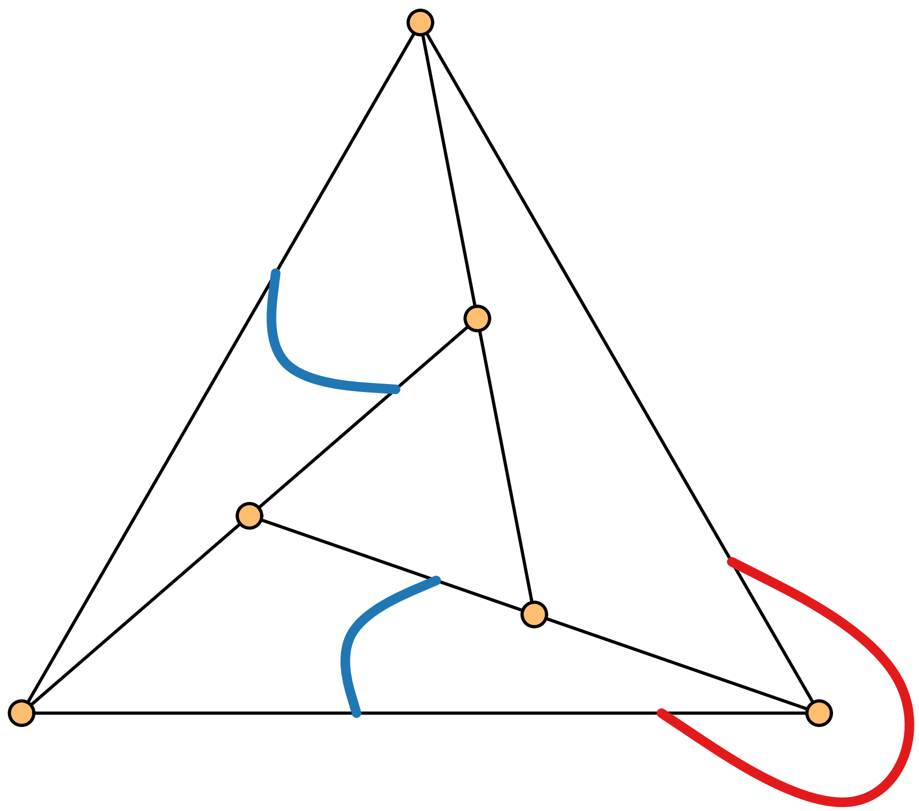
1. Draw triangular prism



# Deconstruction algorithm

**Algorithm**

1. Draw triangular prism
2. Construct graph, maintaining drawing



Inner faces are convex

No insertions on outer face

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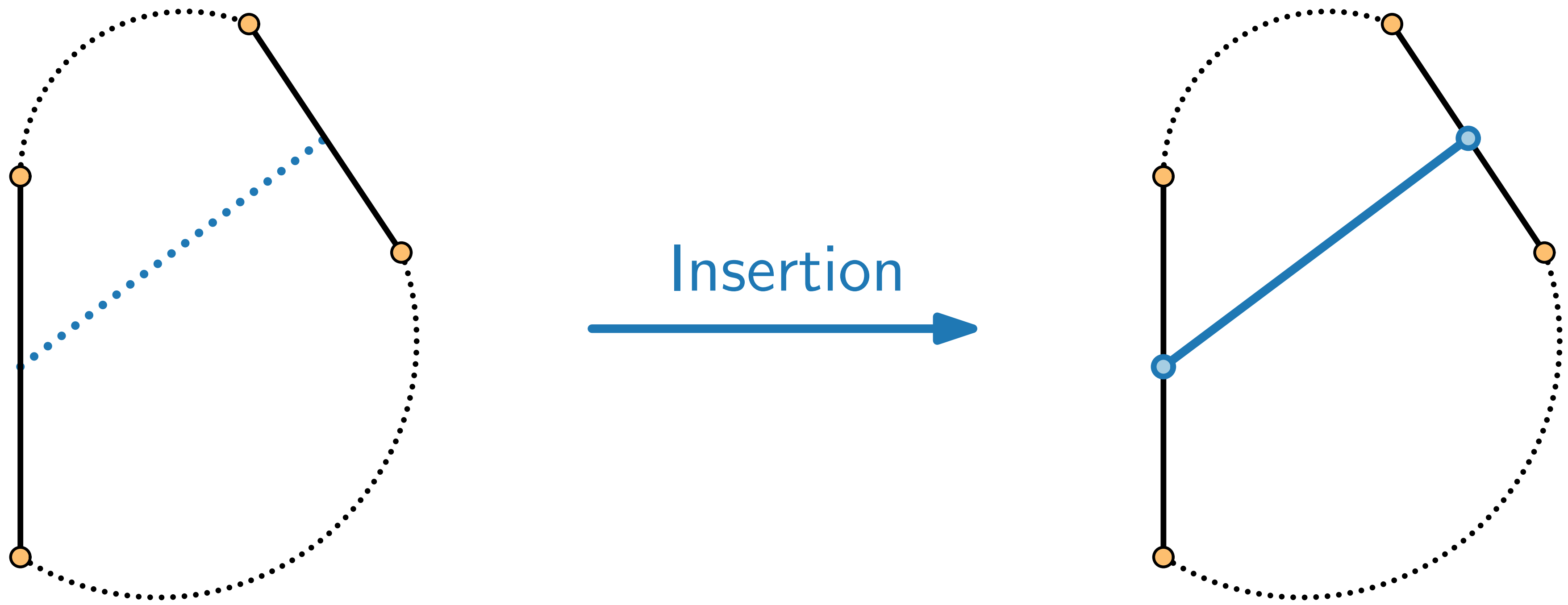

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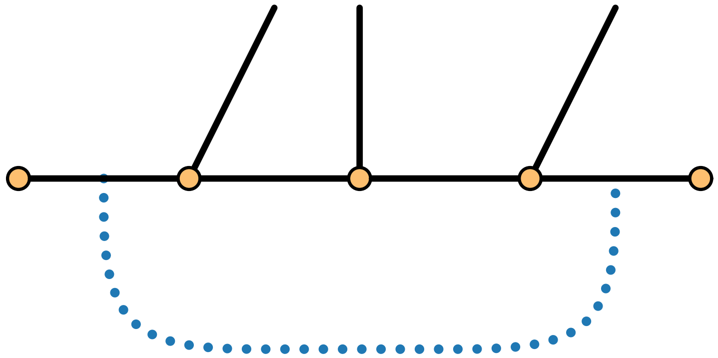

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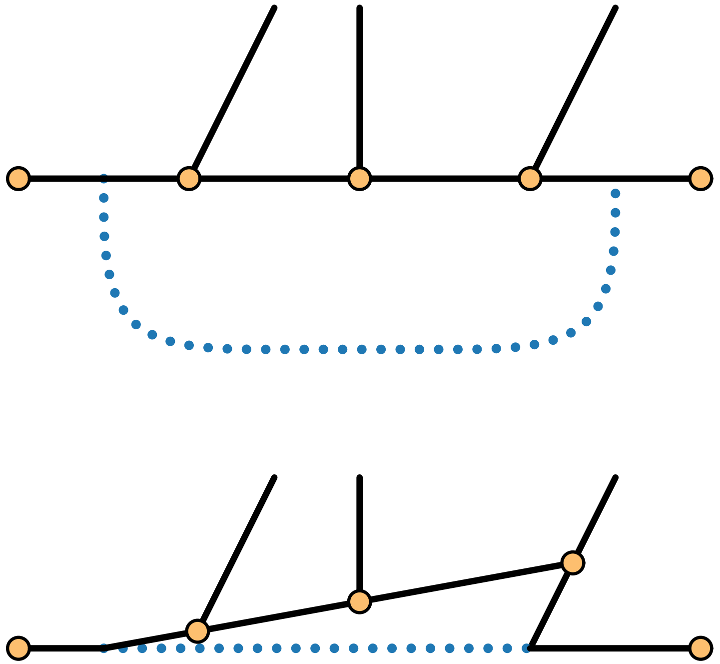

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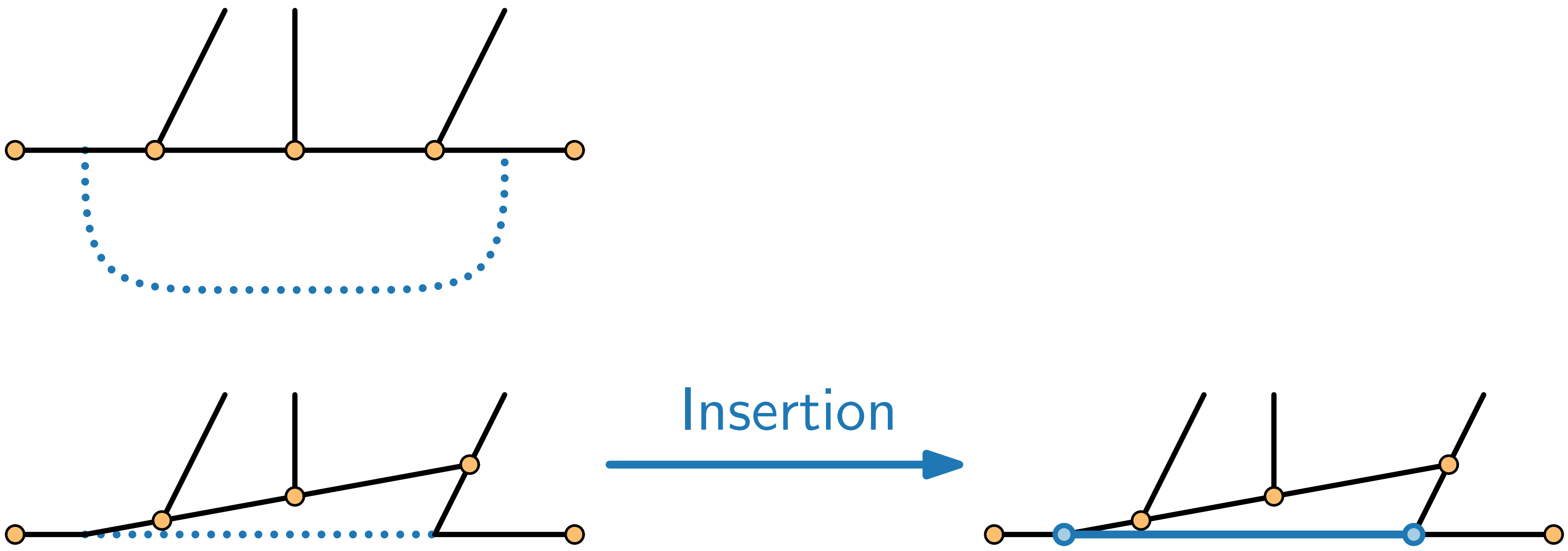

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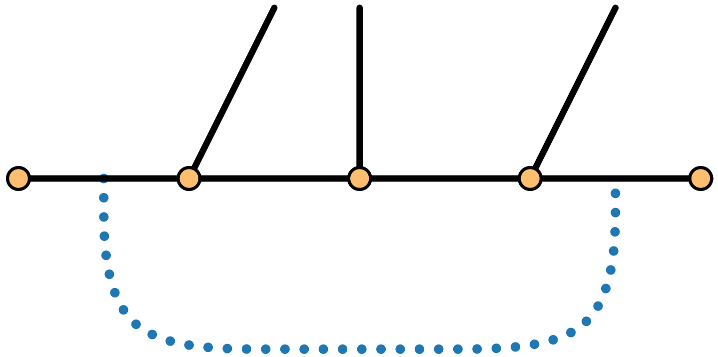

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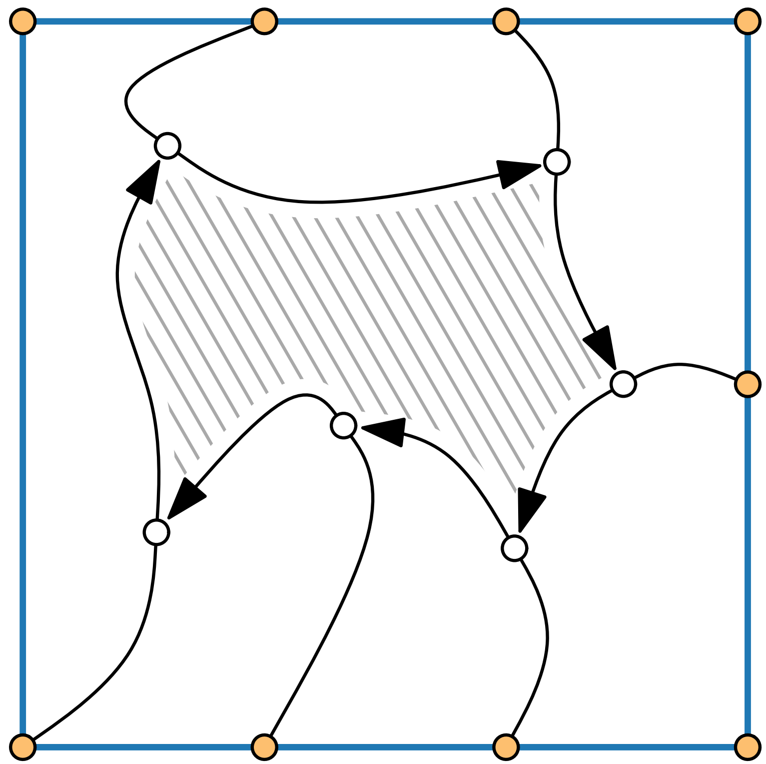
*Pre:* cycle $C$ drawn convex

*Post:* inside of $C$ drawn



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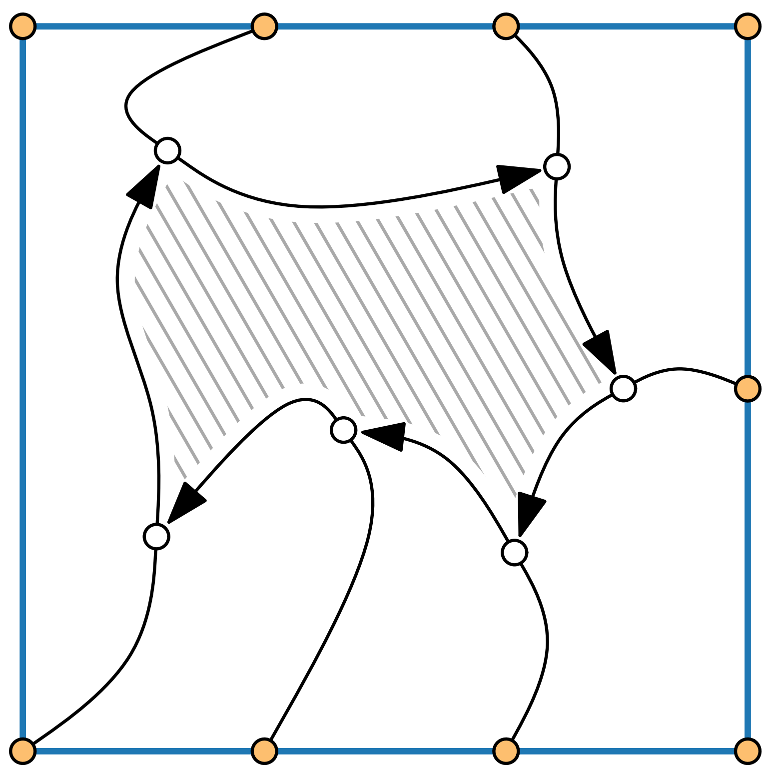

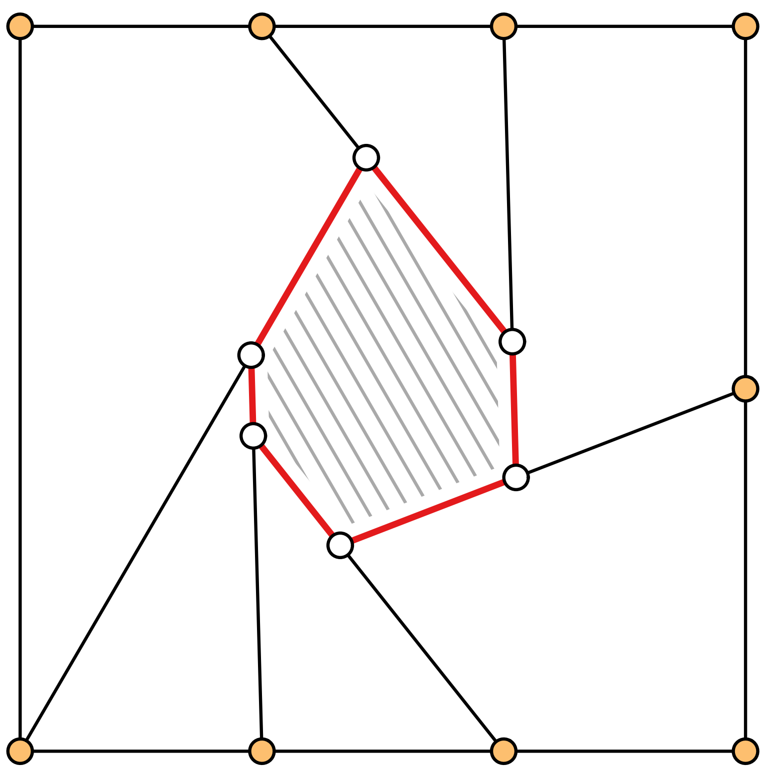

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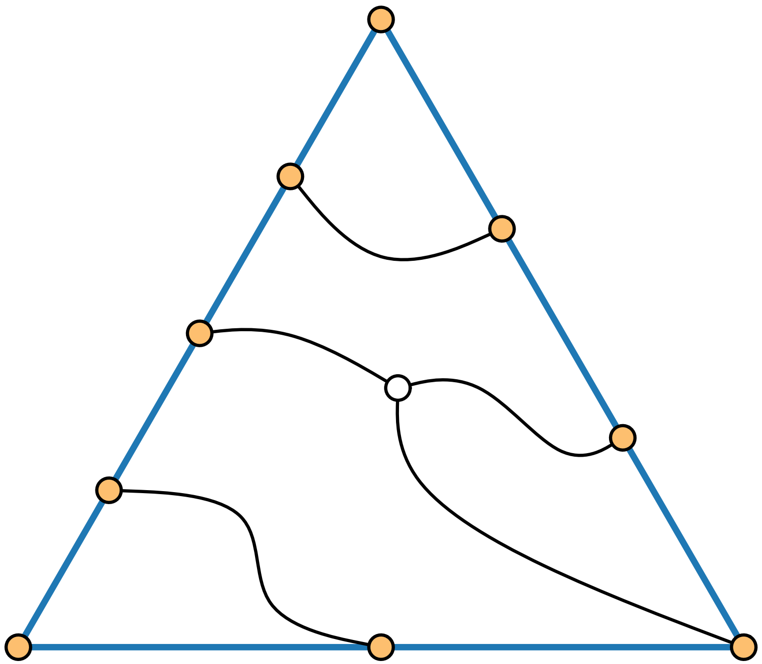

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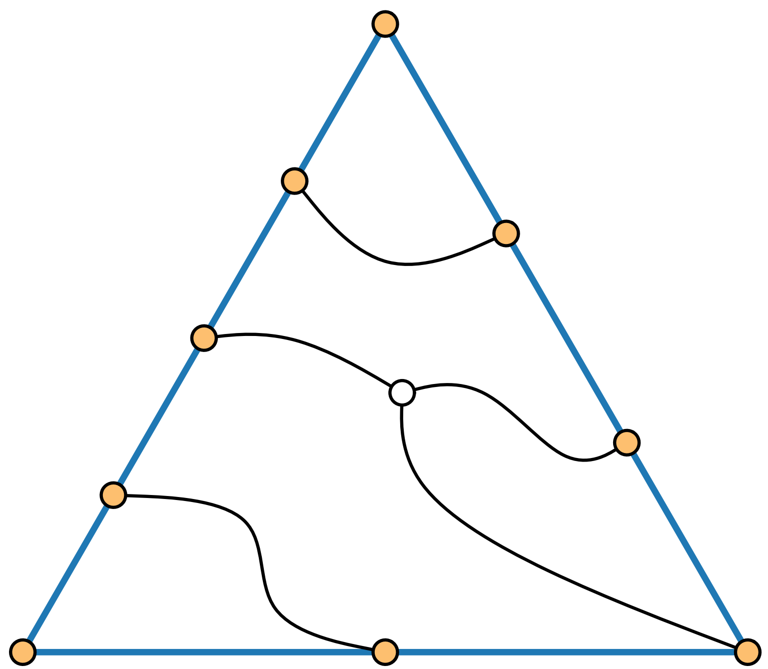

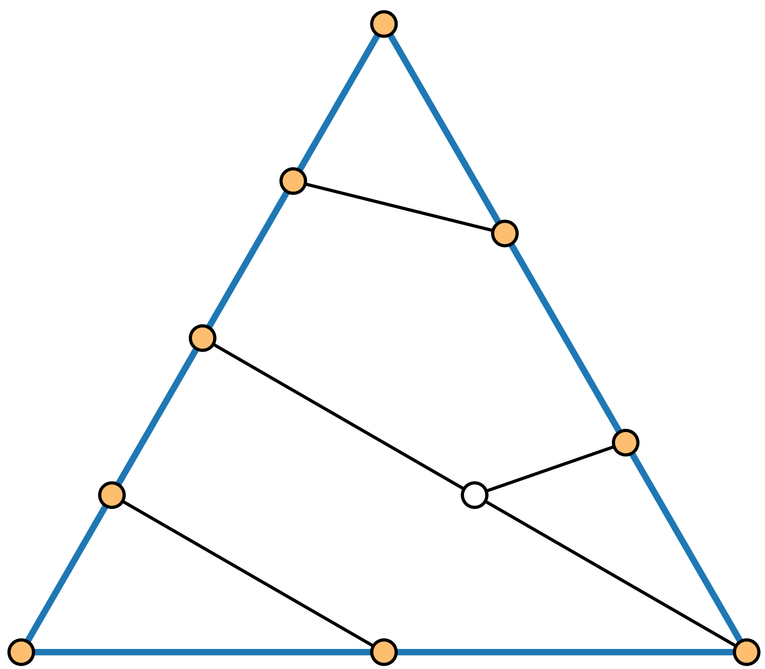

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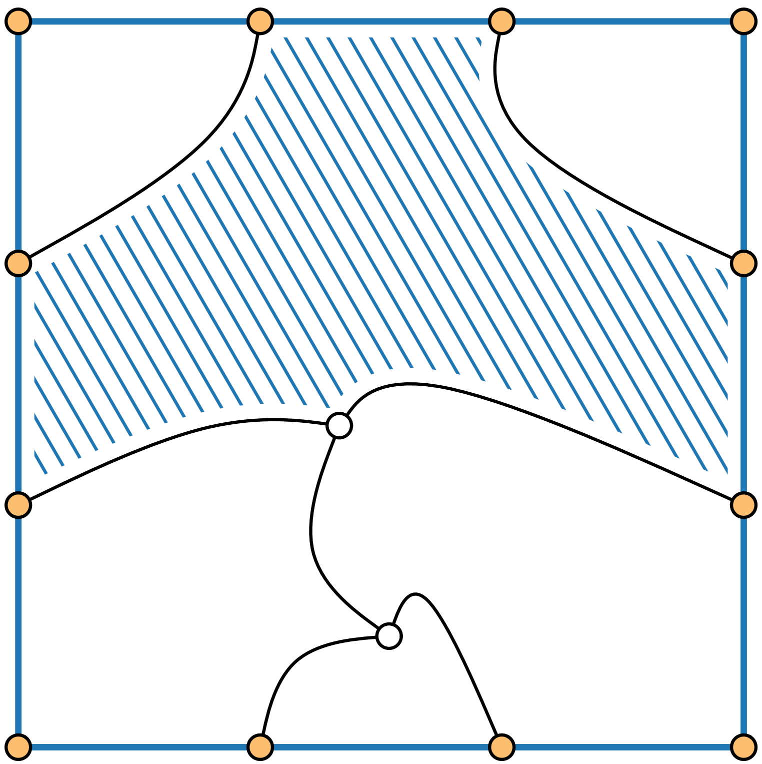

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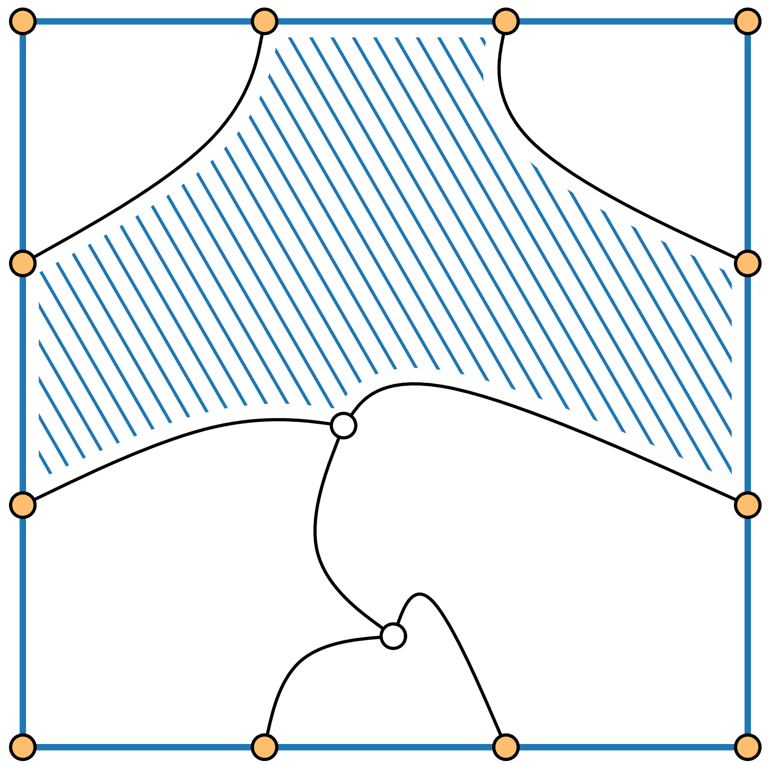

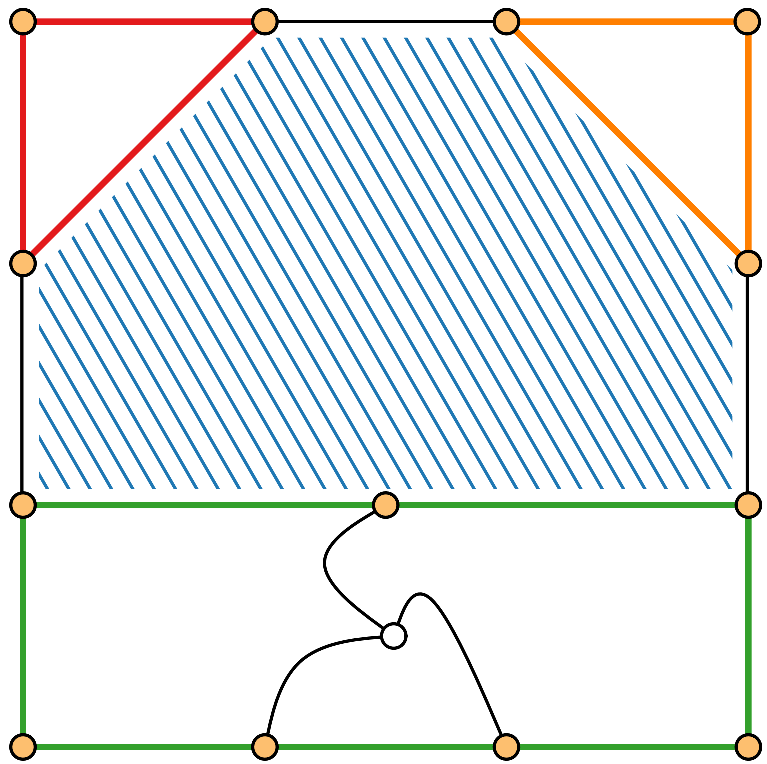

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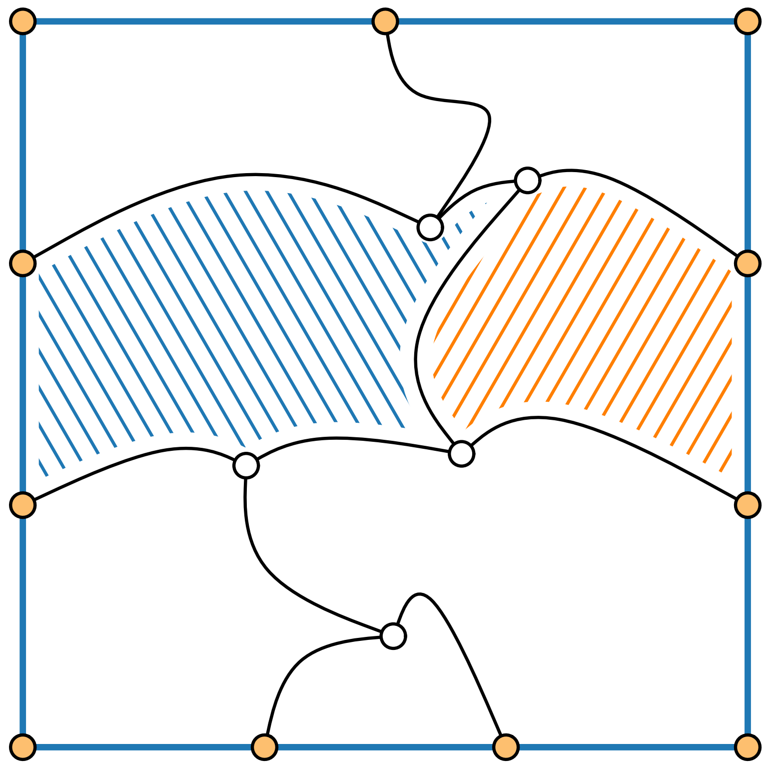

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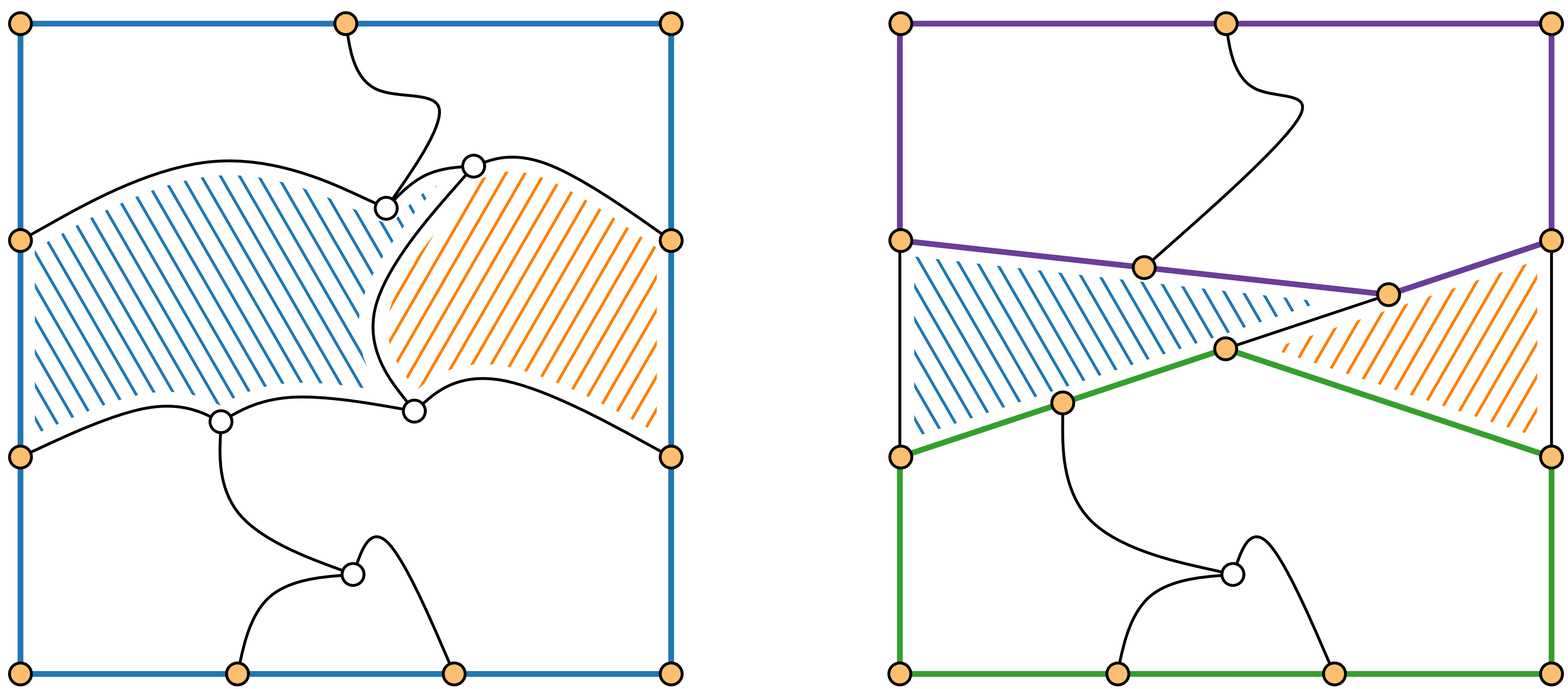
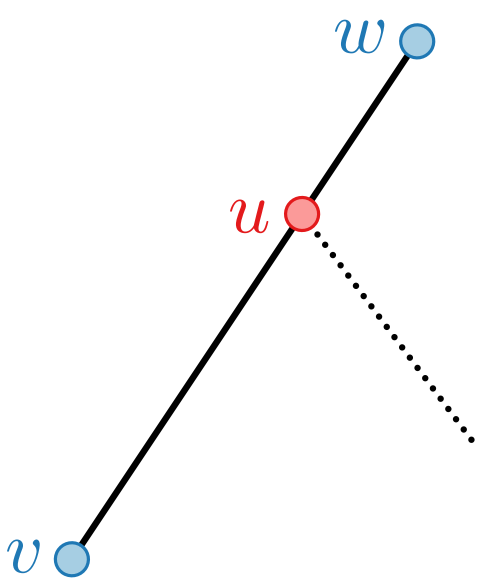


# Postprocessing

Set of harmonic equations [Aerts & Felsner, 2013 ]

$u = \lambda v + (1 - \lambda) w$, for $\lambda \in (0, 1)$



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Set of harmonic equations [Aerts & Felsner, 2013 ]

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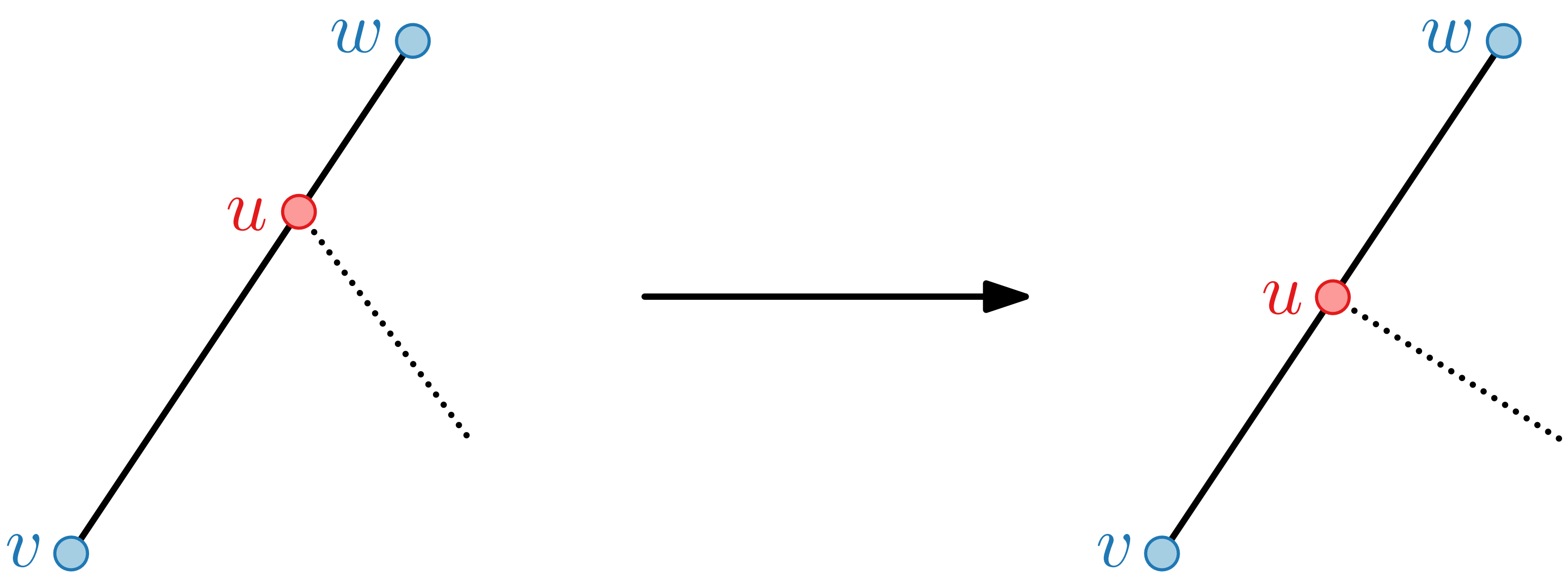
Solve for uniform edge length, i.e. $\lambda = 1/2$



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**"Grid"**

$n/2 + 4$ segments

$6$ slopes

$(n/2+1)^2$ grid

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Resolved flaw in algorithm

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Not on a grid

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$n/2 + 4$ segments

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$(n/2 + 1)^2$ grid

Resolved flaw in algorithm

**"Min"**

$n/2 + 3$ segments

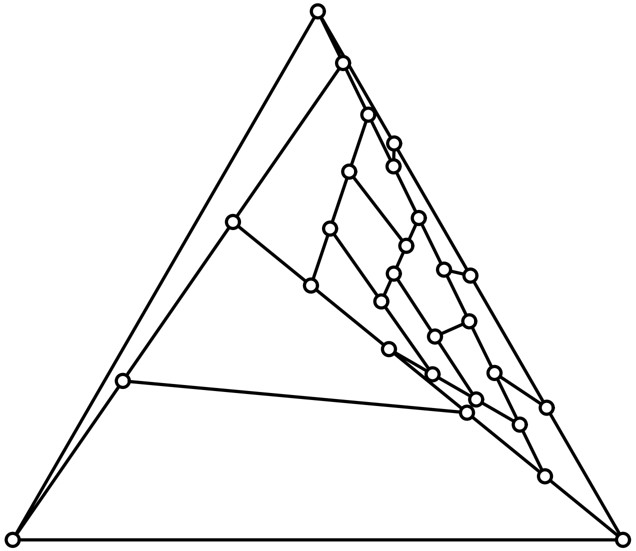
$7$ slopes

Not on a grid

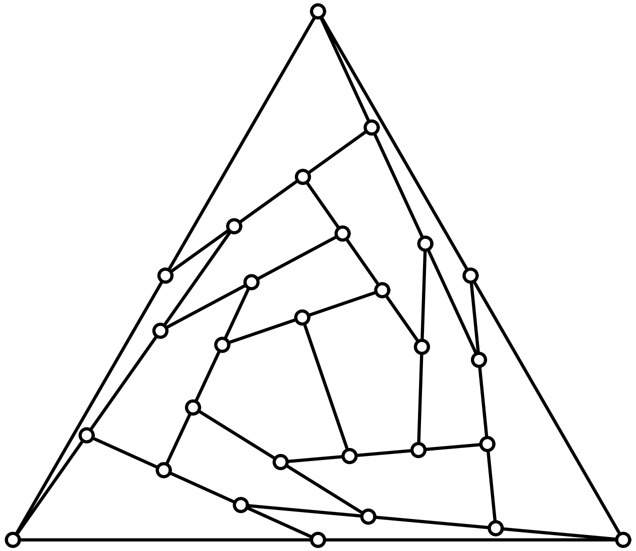
Reduced to $6$ slopes

On a grid

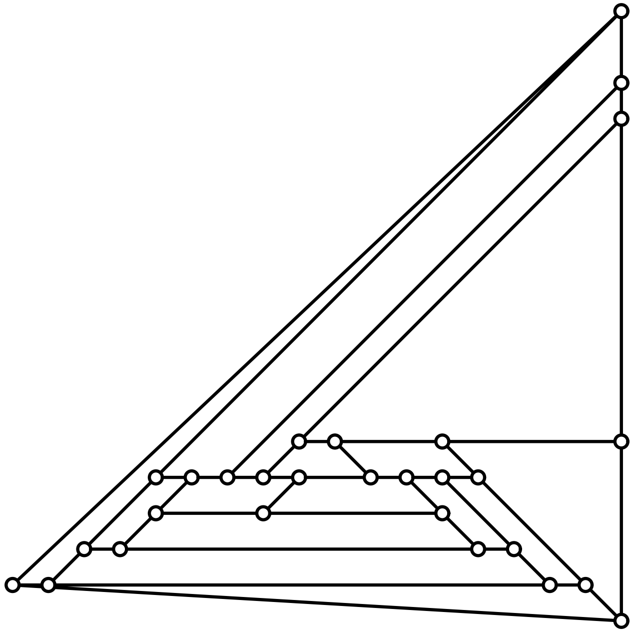
# Three algorithms



Deconstruction



Windmill



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# Measuring layout quality

$2000$ graphs with $24 \ldots 30$ vertices
using plantri

Six measures for each graph-algorithm pair

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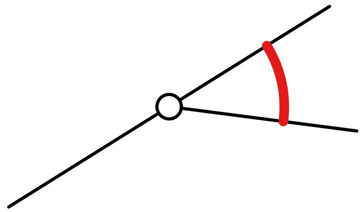
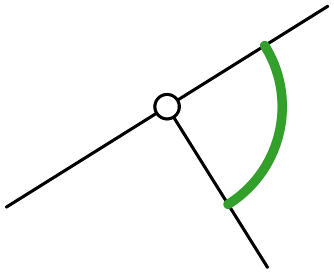
Angular resolution





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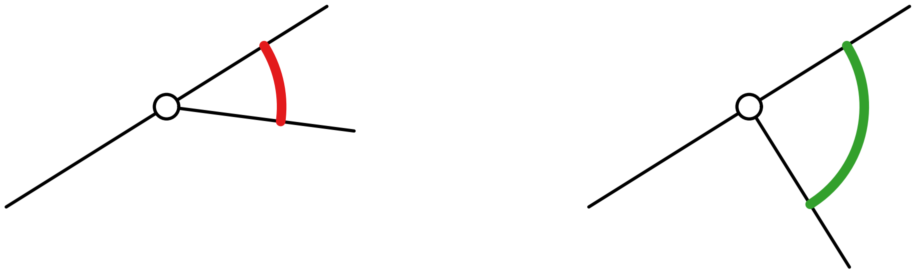
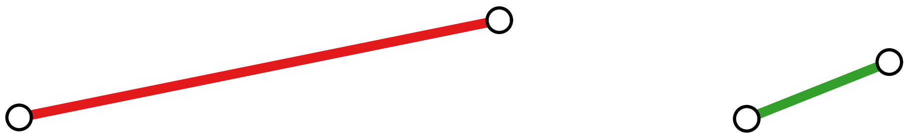


Edge length



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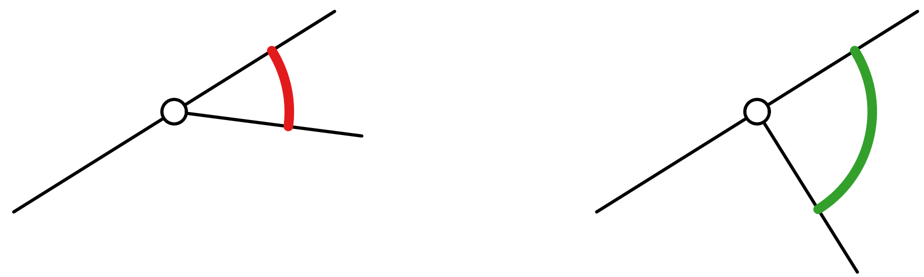

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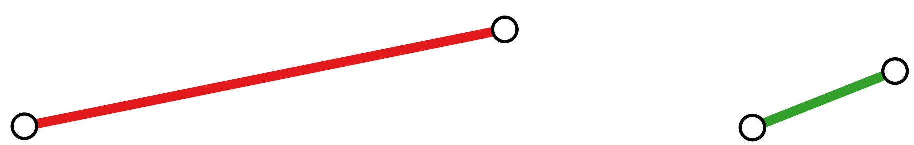
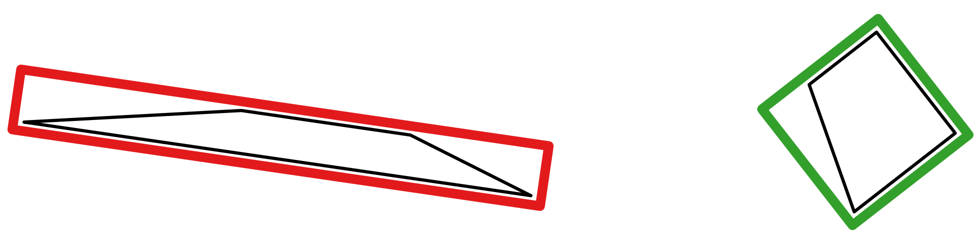


Face aspect ratio



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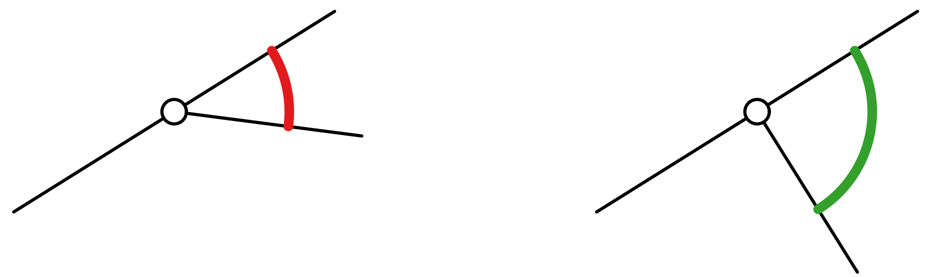

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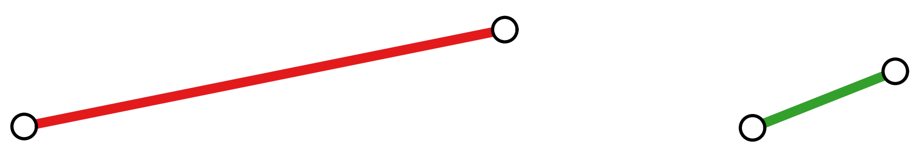

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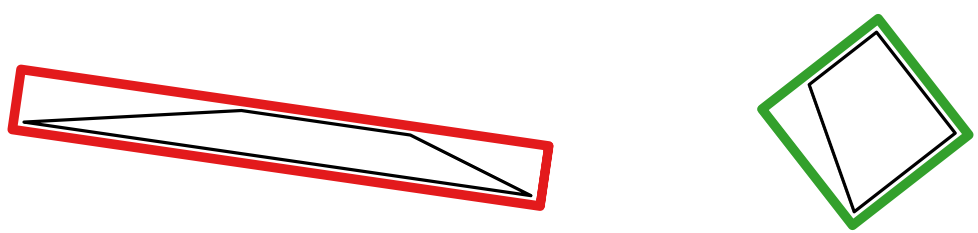
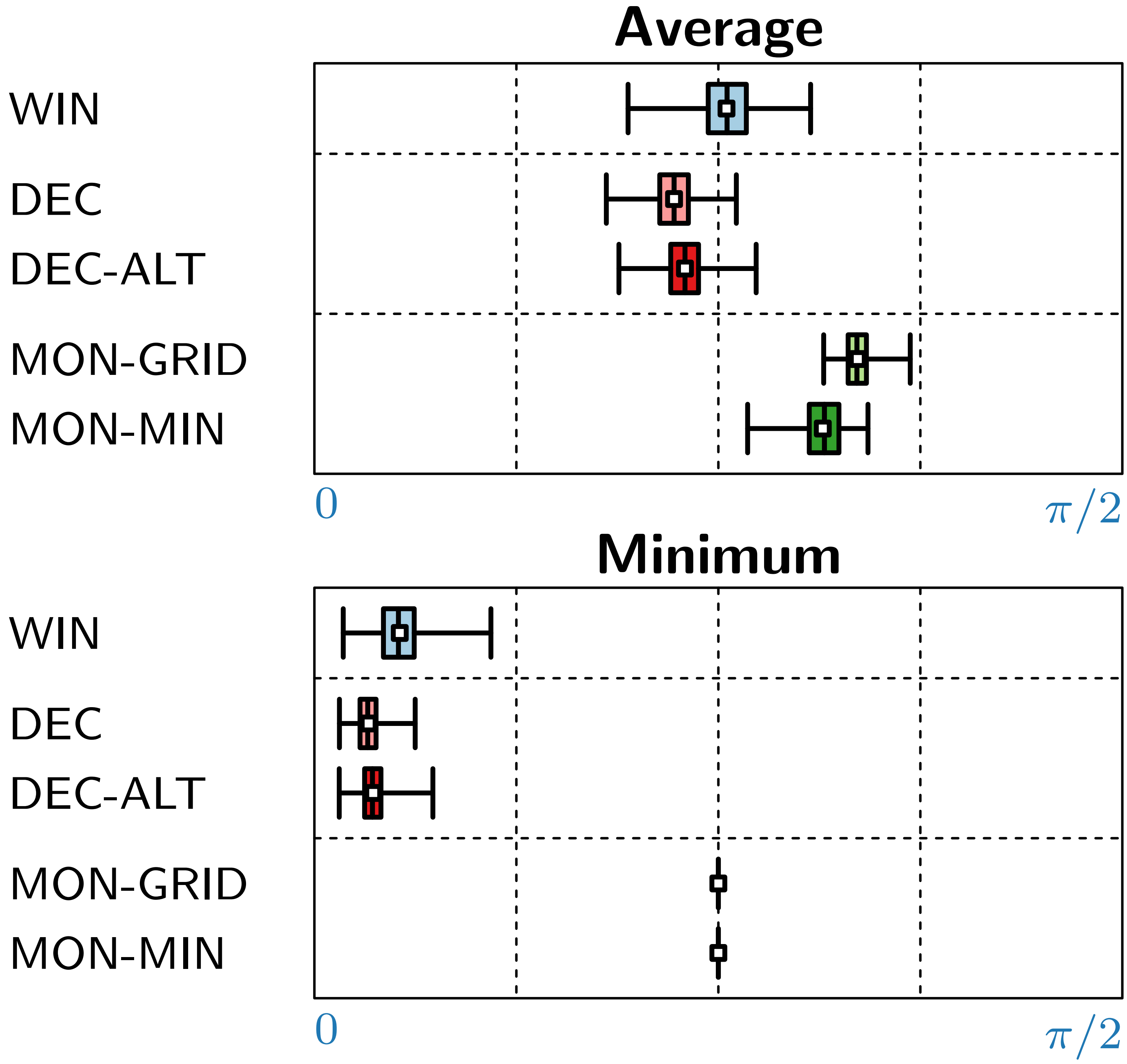
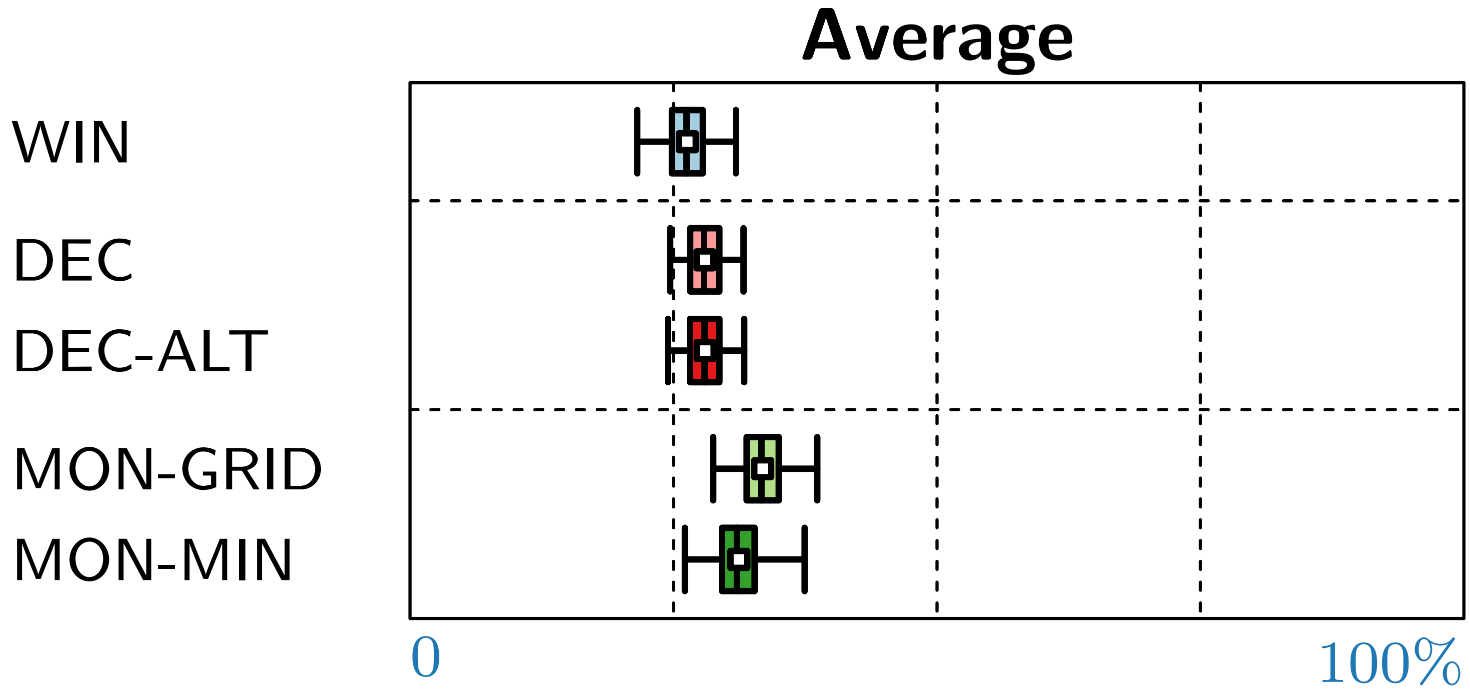


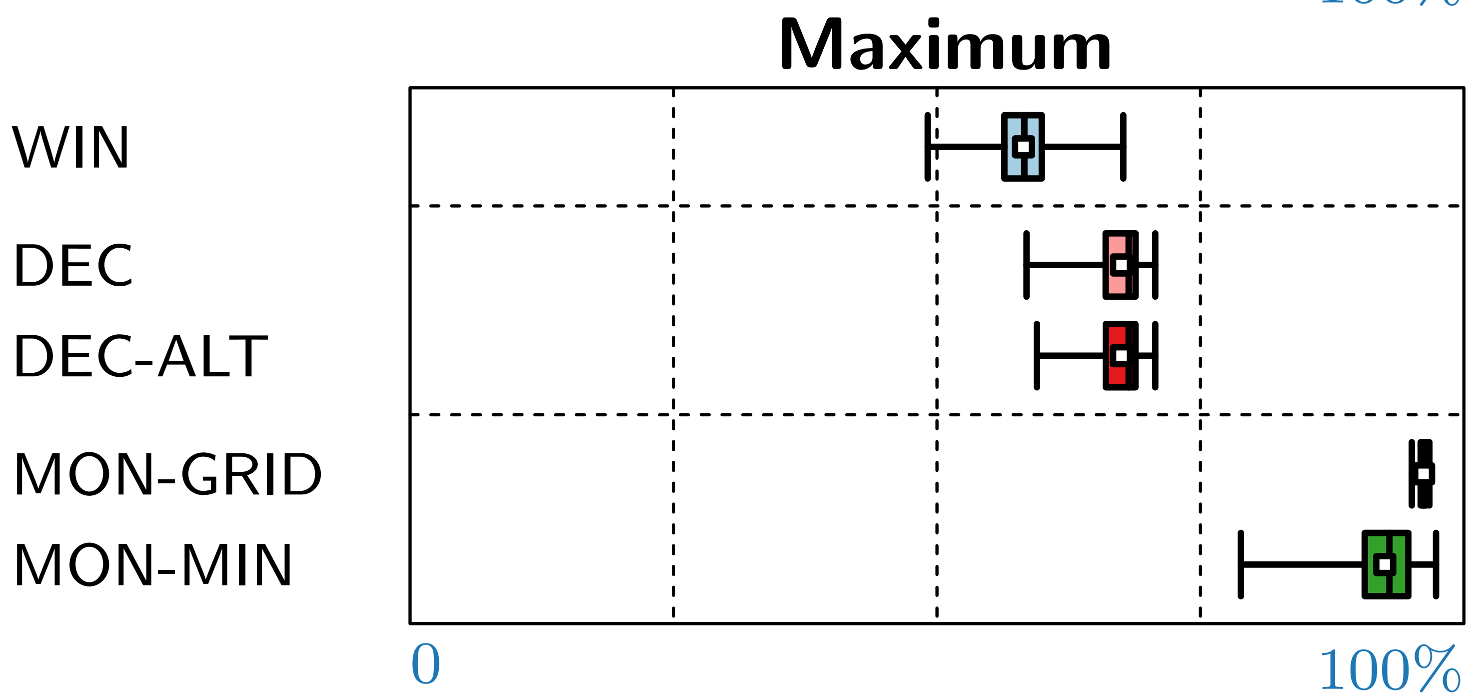
Average and worst-case

# Angular resolution

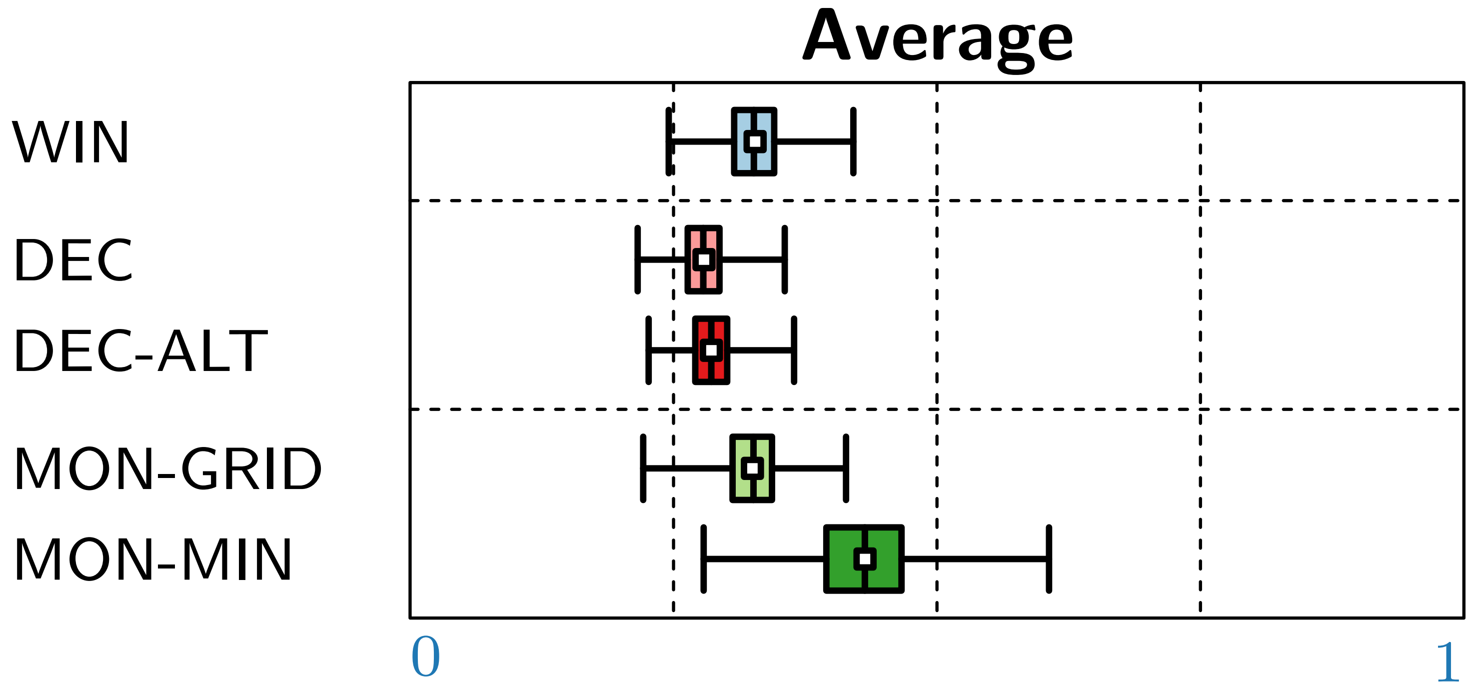


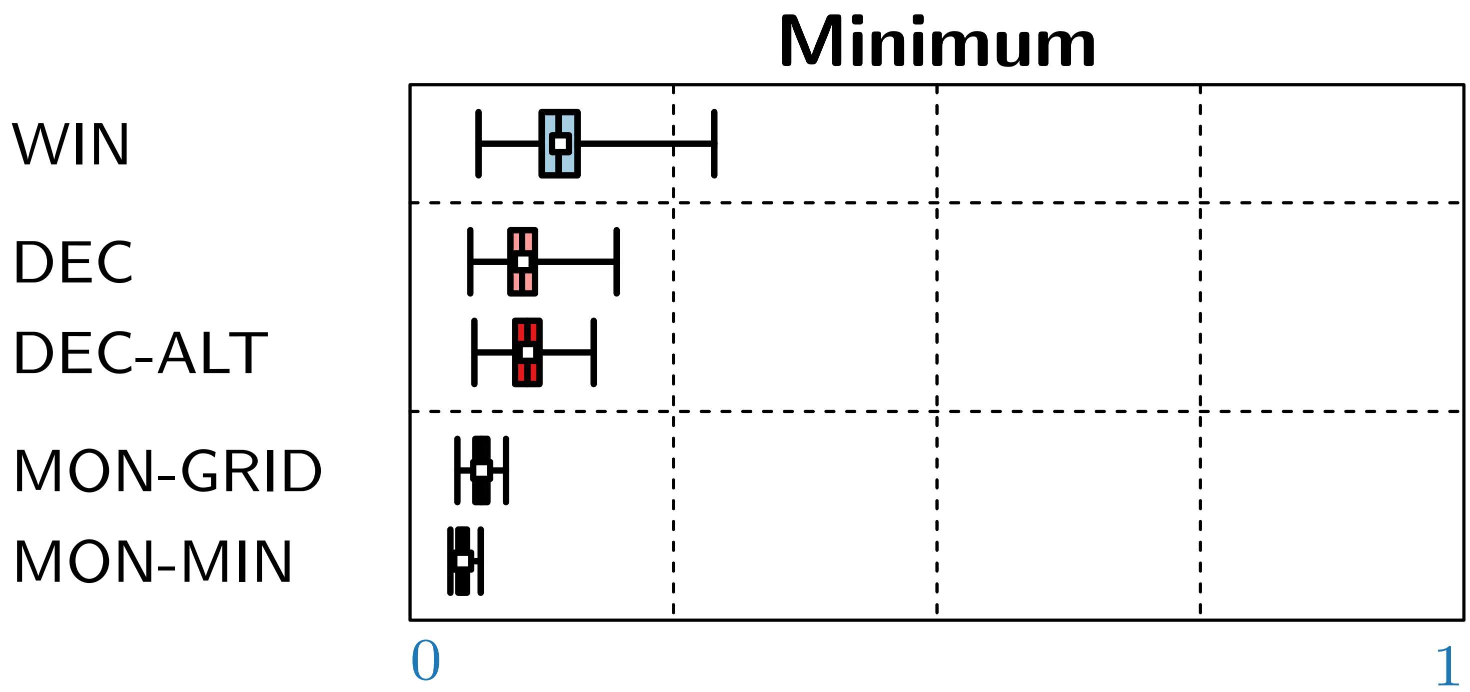
# Edge length


Average
WIN
DEC
DEC-ALT
MON-GRID
MON-MIN
0
100%

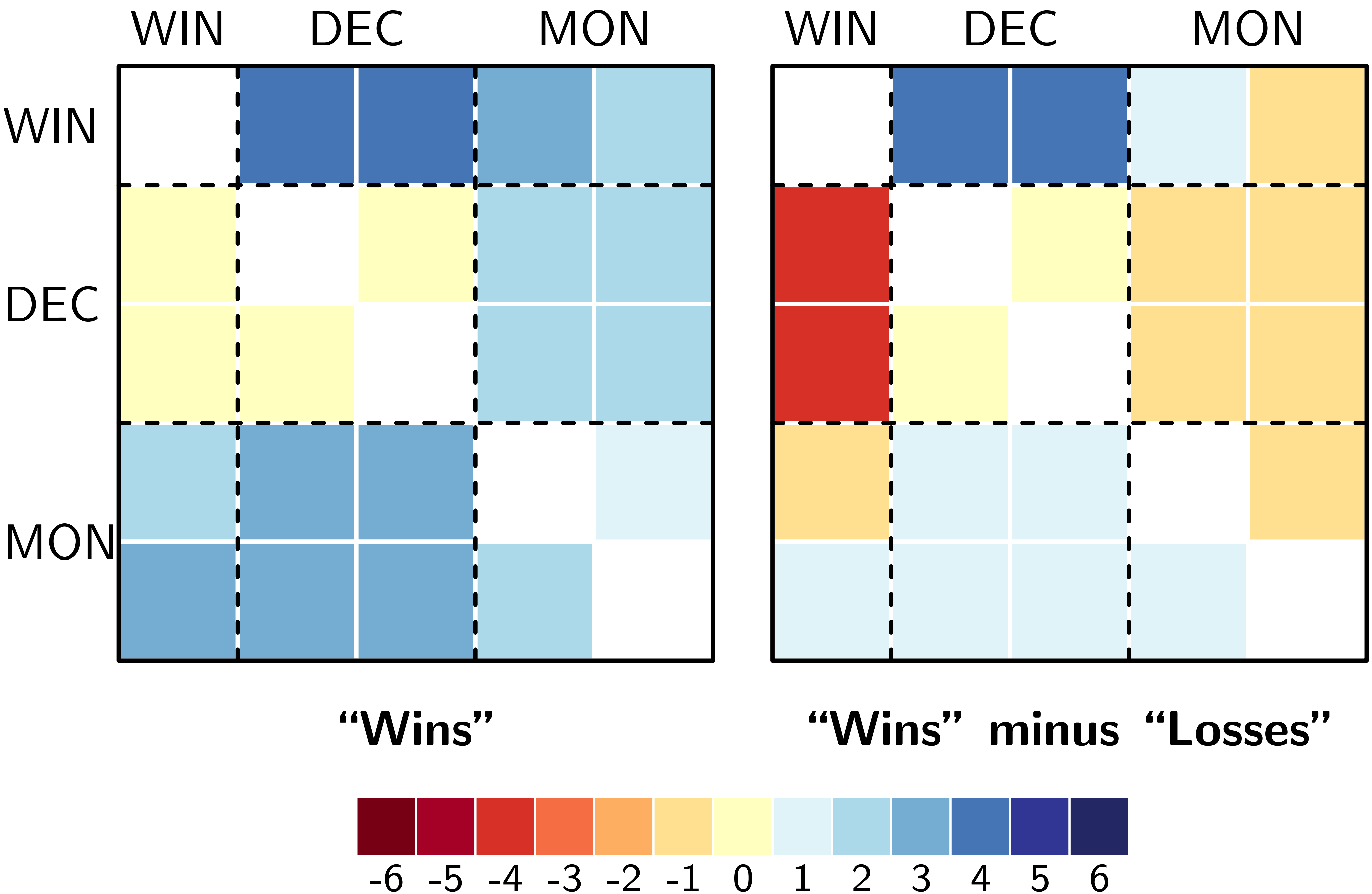

Maximum
WIN
DEC
DEC-ALT
MON-GRID
MON-MIN
0
100%

# Face aspect ratio



# Experiment summary



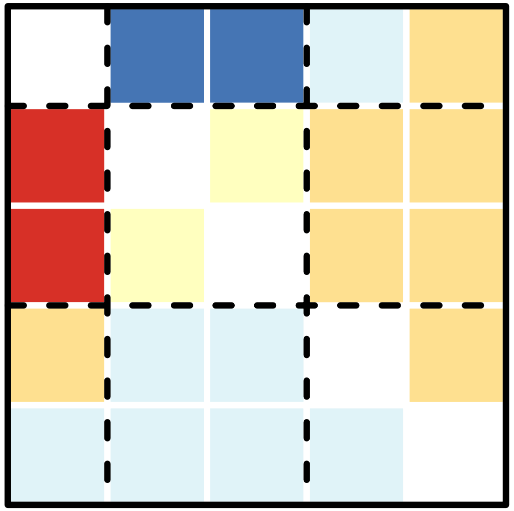
# Conclusion

## Minimal visual complexity

- Two new algorithms
- Fixed and improved [Mondal et al, 2013]

## Experiments

- Best depends on measure

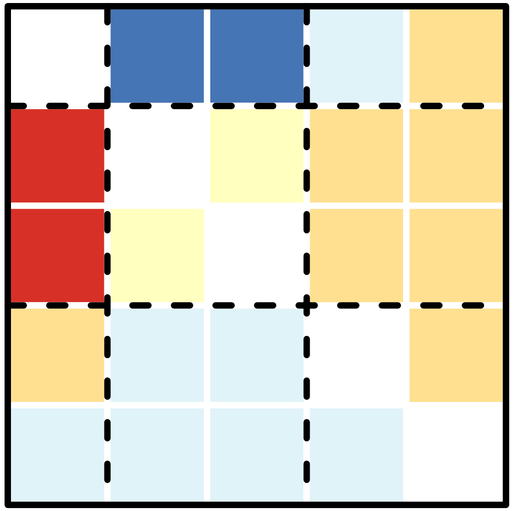


# Conclusion

## Minimal visual complexity

Two new algorithms

Fixed and improved [Mondal et al, 2013]



## Experiments

Best depends on measure

## Future work

Closing gap for other classes

Circular arcs

Visual complexity $\sim$ observer's assessment?

Visual complexity $\sim$ cognitive load?