Julius-Maximilians-
UNIVERSITÄT
WÜRZBURG

Chair for
INFORMATICS I
Efficient Algorithms and
Knowledge-Based Systems

Institute for Informatics

Faster Force-Directed Graph Drawing with the Well-Separated Pair Decomposition

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Julius-Maximilians-Universität Würzburg

September 24, Graph Drawing 2015

Introduction

Classical force-directed algorithm

- [Eades, 1984, Fruchterman & Reingold, 1991]
- Attracting forces: $O(m)$ time with $m = \#$edges
- Repulsive forces: $\Theta(n^2)$ time with $n = \#$vertices

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Many speed-up techniques for force-directed algorithms known:

- Hierarchical $O(n \log n)$ force-calculation [Barnes & Hut, 1986]
- Potential-field-based multilevel algorithm [Hachul & Jünger, 2005]
- Evaluation of multilevel layout methods [Bartel et al., 2011]

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Our approach

- Using well-separated pair decomposition to speed up repulsive force calculation to $O(n \log n)$ time.
- Can be combined with other speed-up techniques.

Well-Separated Pairs

Definition

Let $s > 0$. A and B in $\mathbb{R}^2$ are s-well-separated iff there are two balls C_A and C_B with

- $A \subseteq C_A$ and $B \subseteq C_B$,
- radius $r := r(C_A) = r(C_B)$, and
- distance $\geq s \cdot r$.

[Callahan & Kosaraju, 1995]

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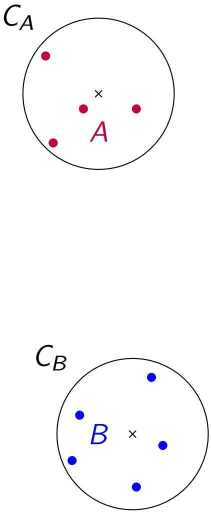

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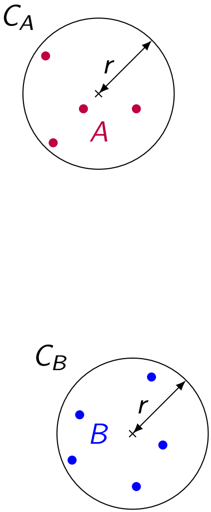

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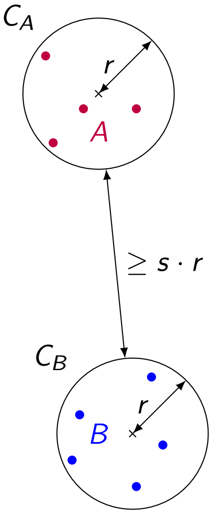

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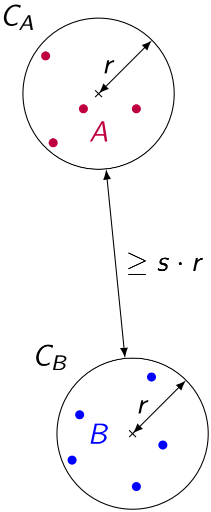

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Well-Separated Pair Decomposition (WSPD)

Definition

Let S be a finite set in $\mathbb{R}^2$. A WSPD for S with respect to $s > 0$ is a sequence

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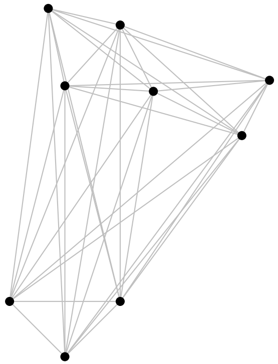

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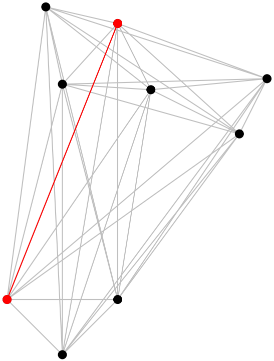

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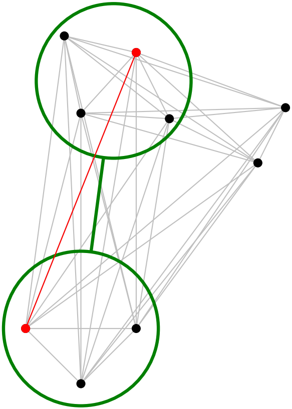

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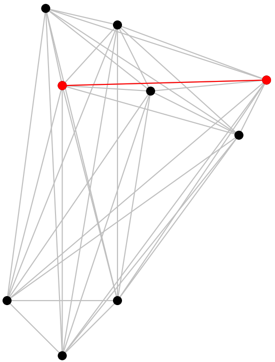

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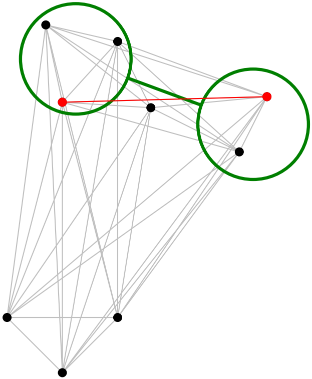

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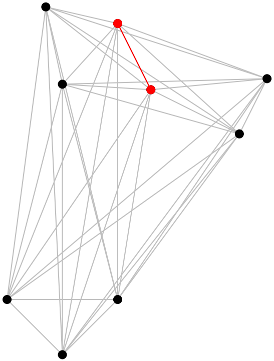

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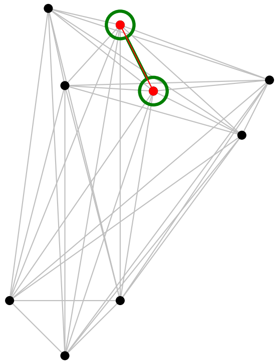

Computation of WSPD

Theorem

Let S be a set of n points in $\mathbb{R}^2$. For any $s > 0$, a WSPD consisting of $O(n)$ pairs can be computed in $O(n \log n)$ time.

[Callahan & Kosaraju, 1995]

Repulsive Force Calculation

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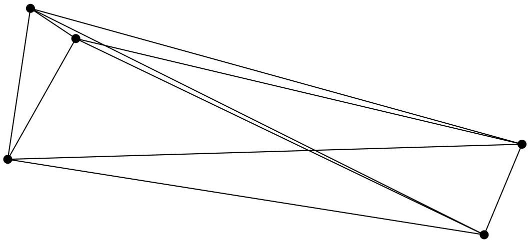

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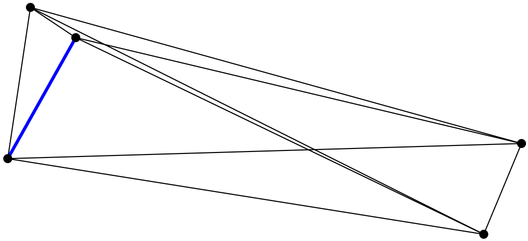

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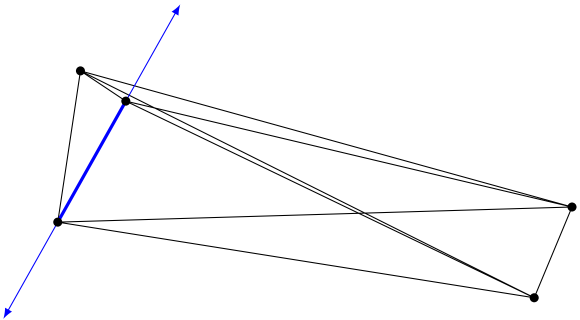

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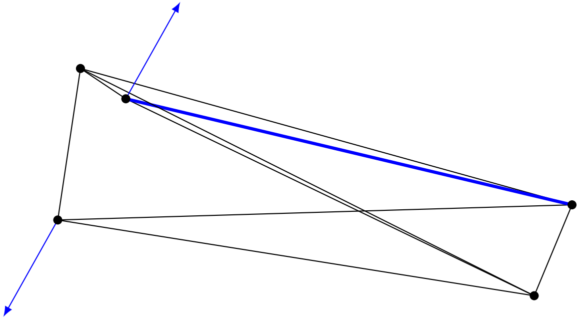

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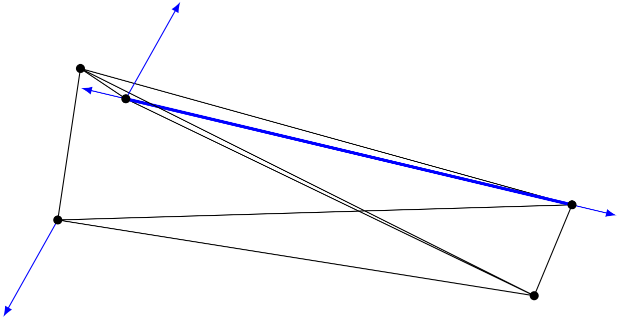

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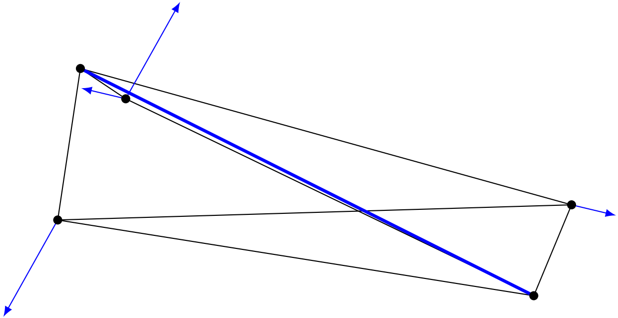

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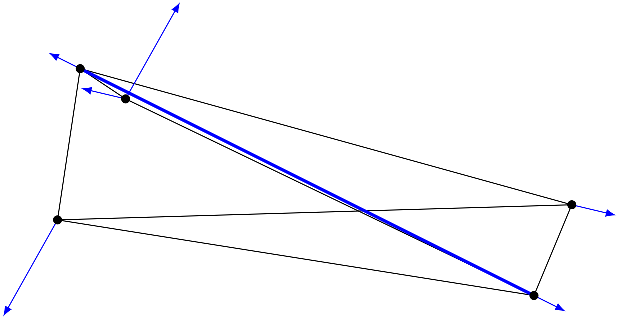

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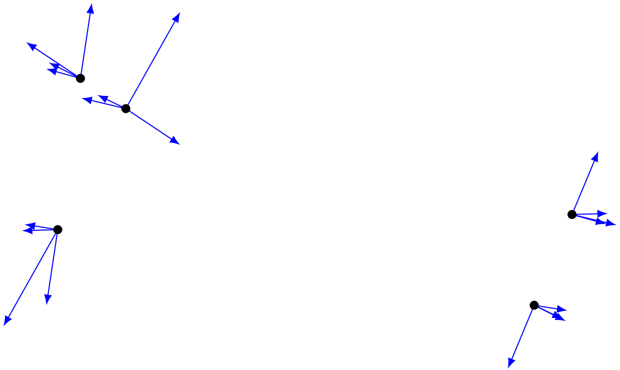
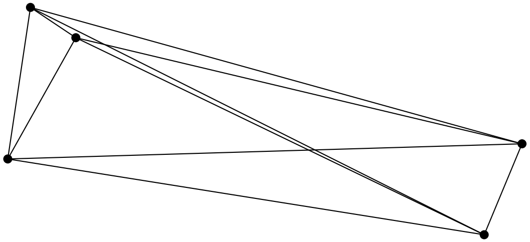


Compute forces for $\Theta(n^2)$ pairs of vertices

Repulsive Force Calculation



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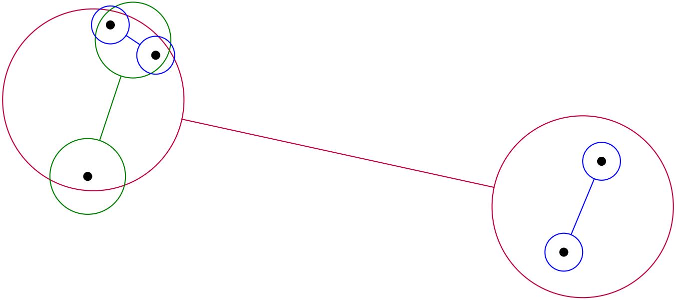

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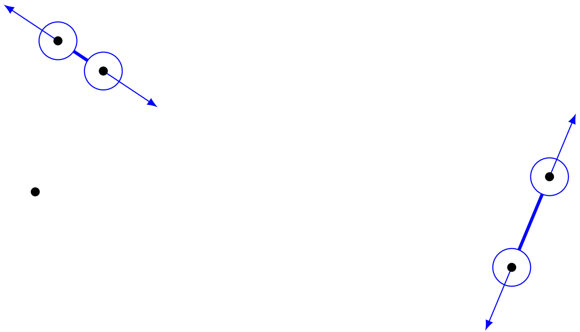

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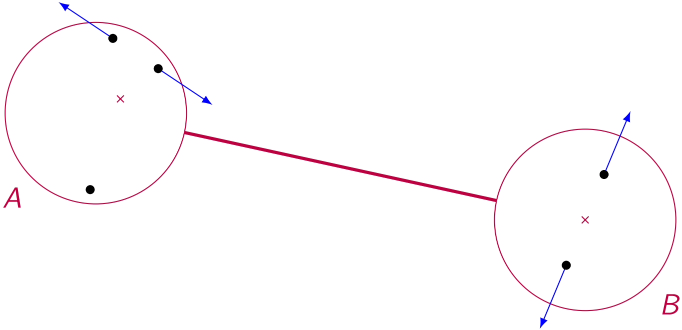

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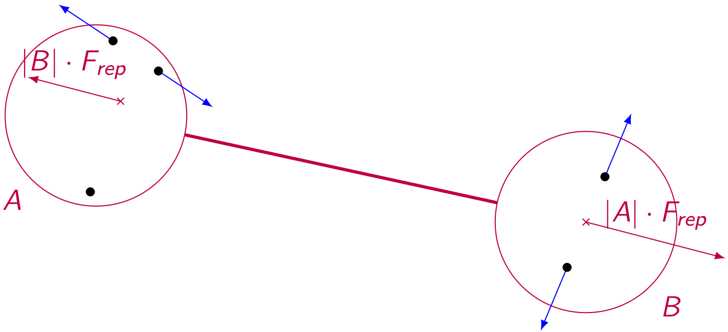

Repulsive Force Calculation

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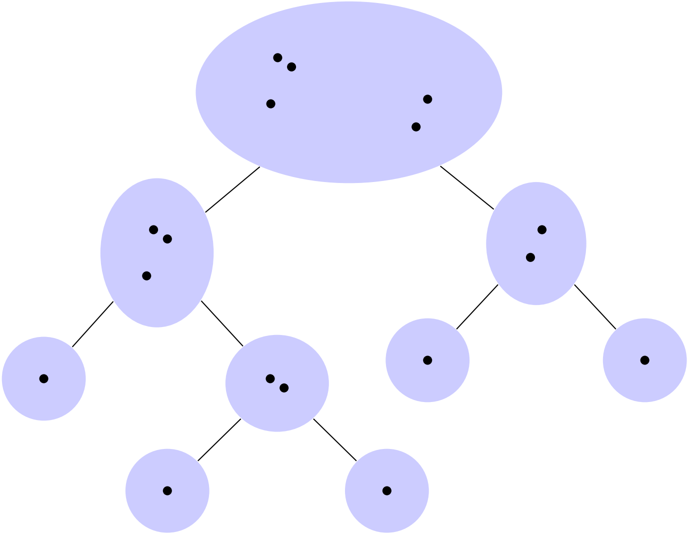
Compute forces for $k \in O(n)$ well-separated pairs

Using the Split Tree



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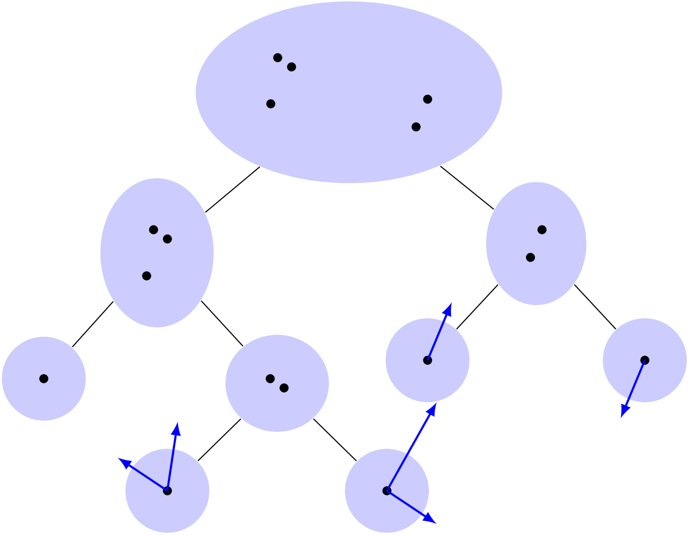

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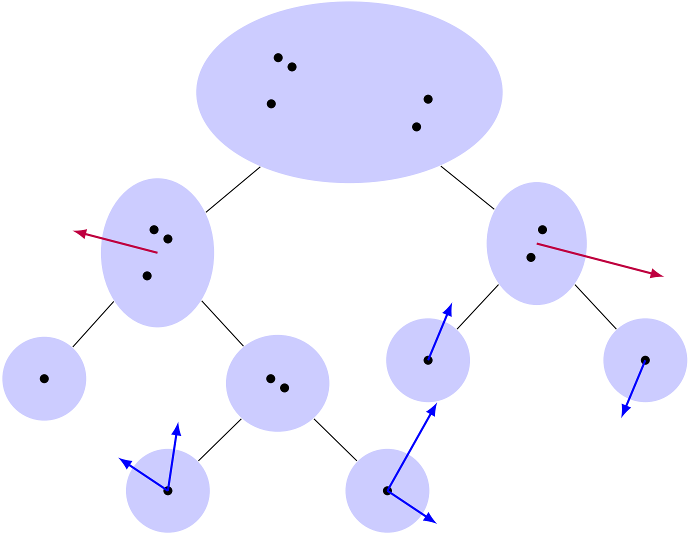

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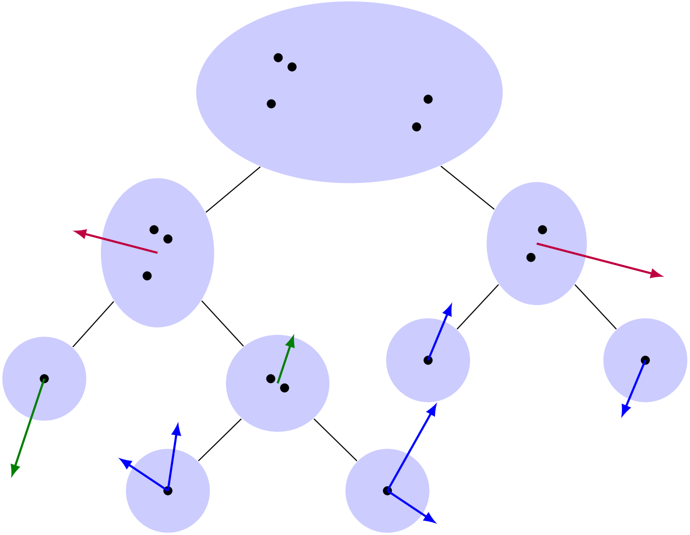

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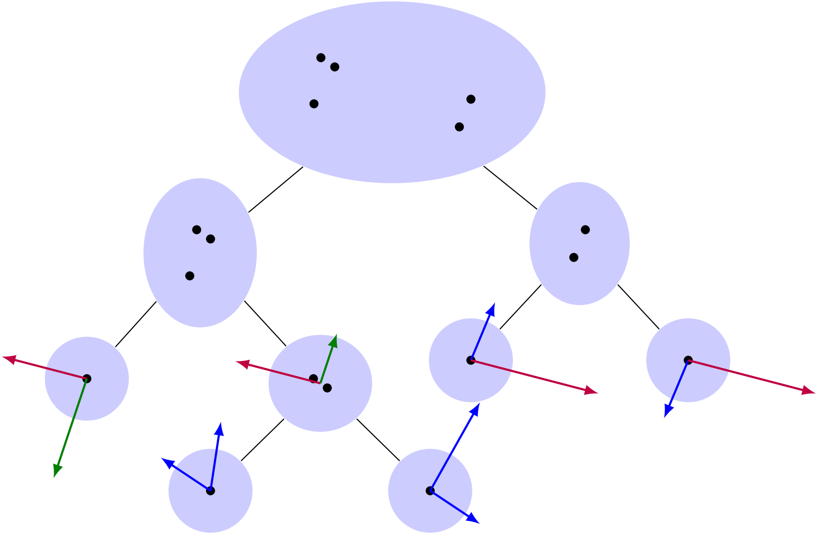

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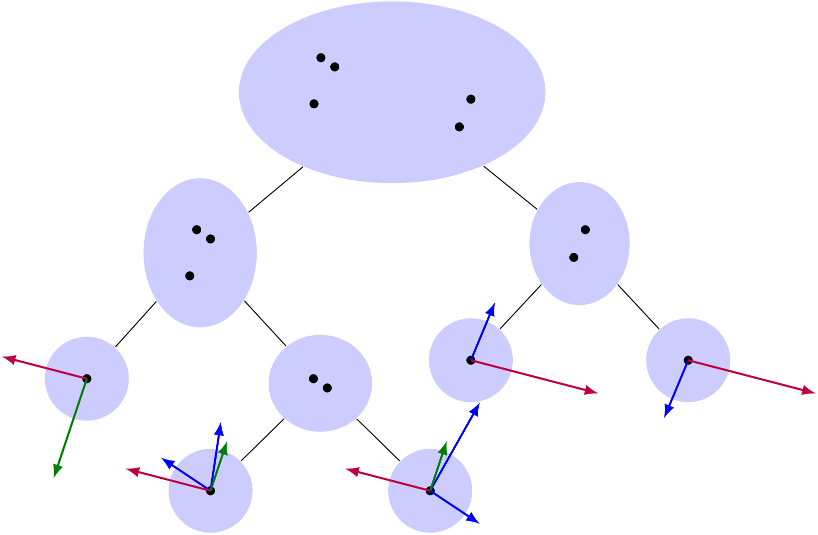


Running time

Compute the WSPD using a split tree	$O(n \log n)$
Compute forces for every pair	$O(n)$
Propagate forces in the split tree	$O(n)$
Total	$O(n \log n)$

Experiments

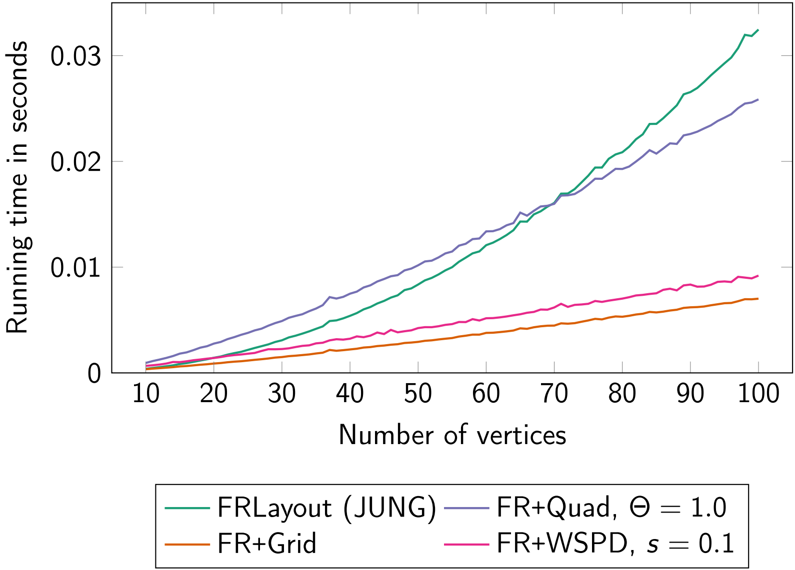
Implementation

- in Java
- based on FRLayout in Jung (Java Universal Network/Graph Framework)

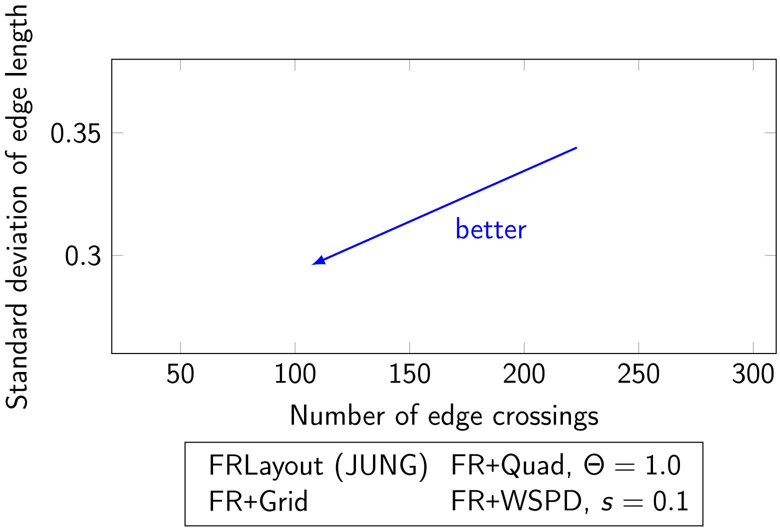
Comparison to similar speed-up techniques:

- Quadtree based [Barnes & Hut, 1986]
- Grid variant [Fruchterman & Reingold, 1991]

Experiments: Running time



Experiments: Quality



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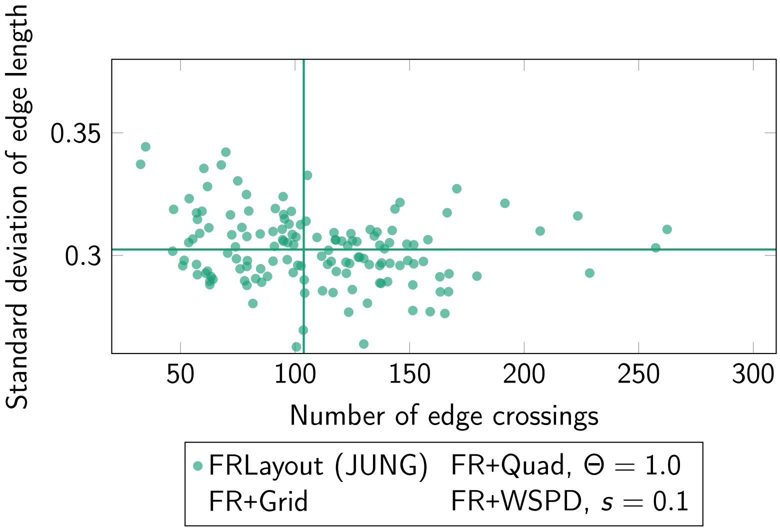

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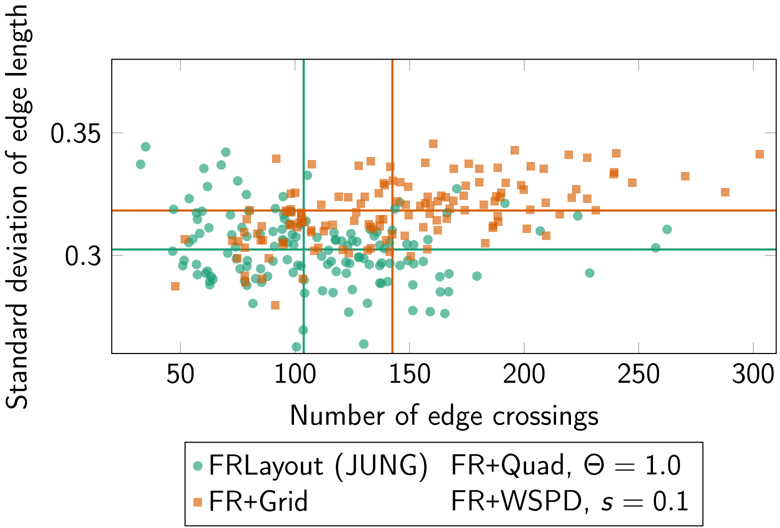

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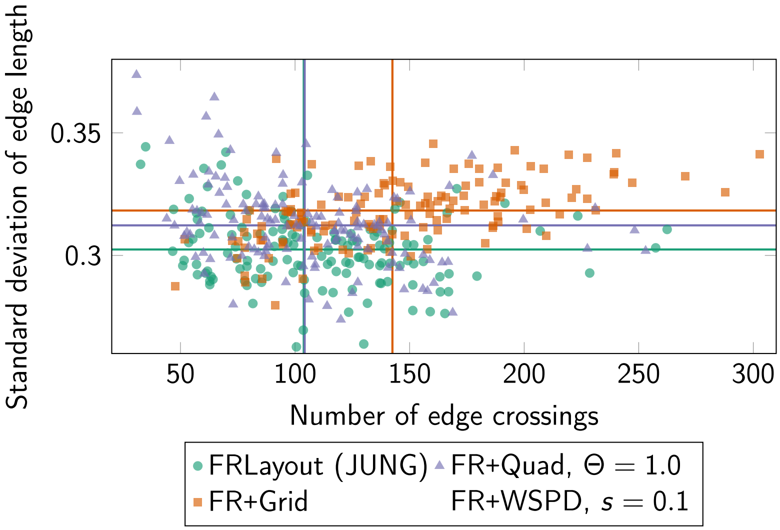

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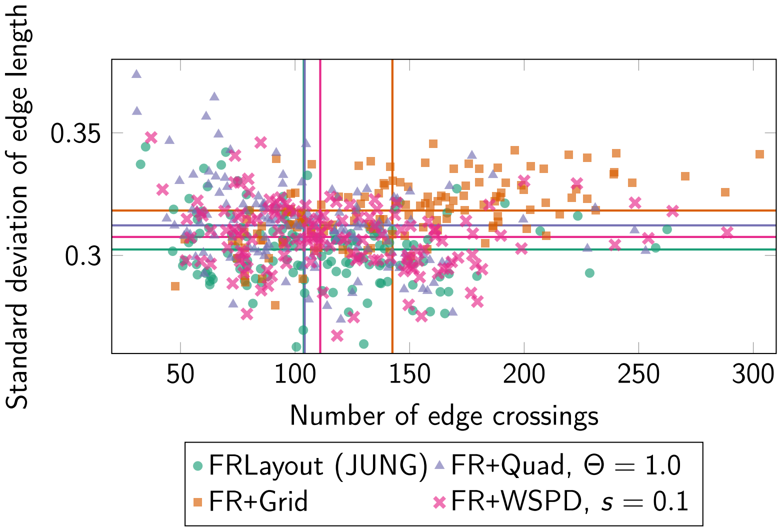

Summary

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- Notably faster than unmodified algorithm
- Similar to unmodfied algorithm in terms of quality
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APPENDIX

Example drawings

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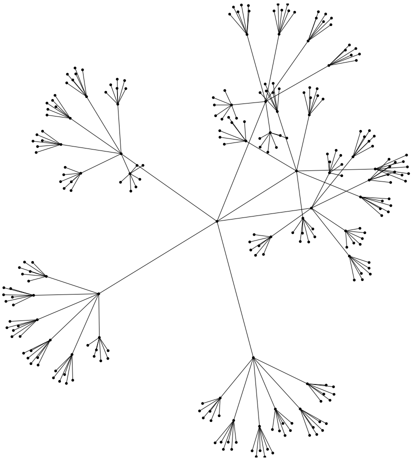
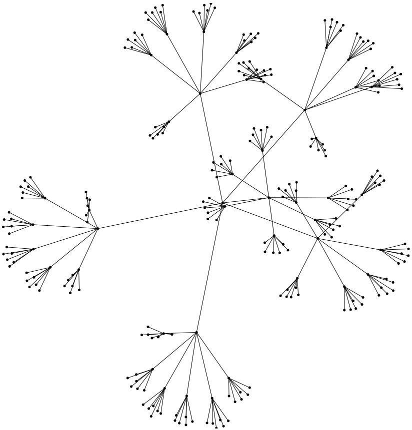


FR+WSPD, $s = 0.1$



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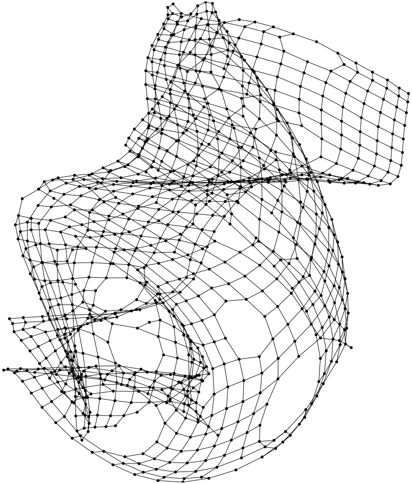

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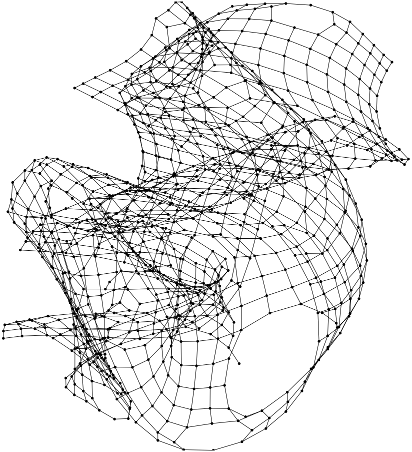
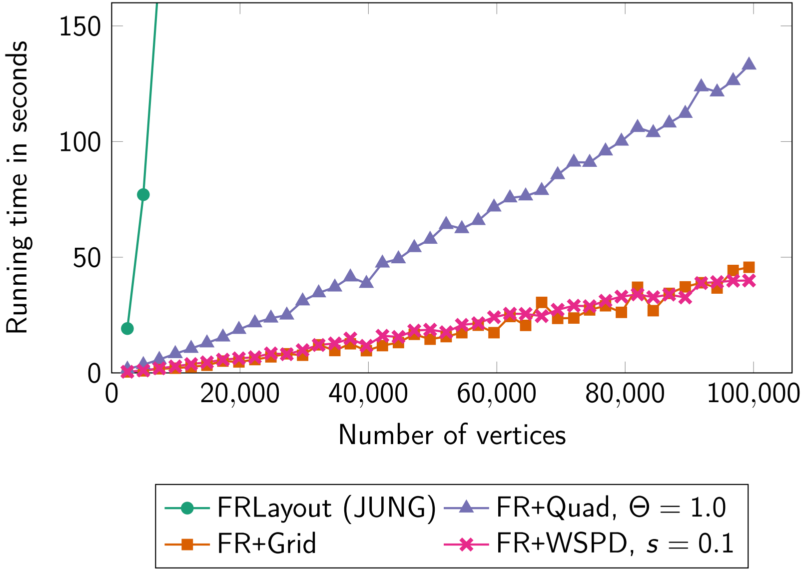
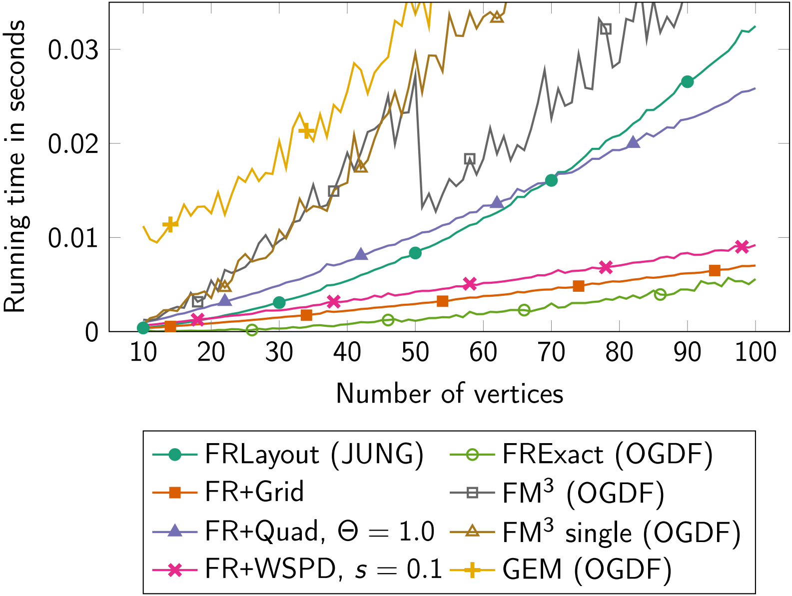


Experiments: Running time on large graphs



Experiments: Running times compared to OGDF



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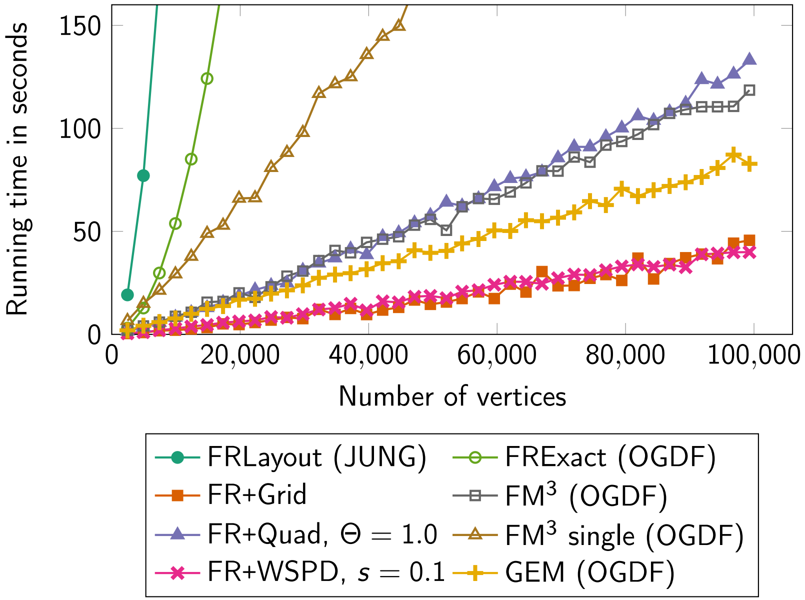

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