On the Relationship between Map Graphs and Clique Planar Graphs

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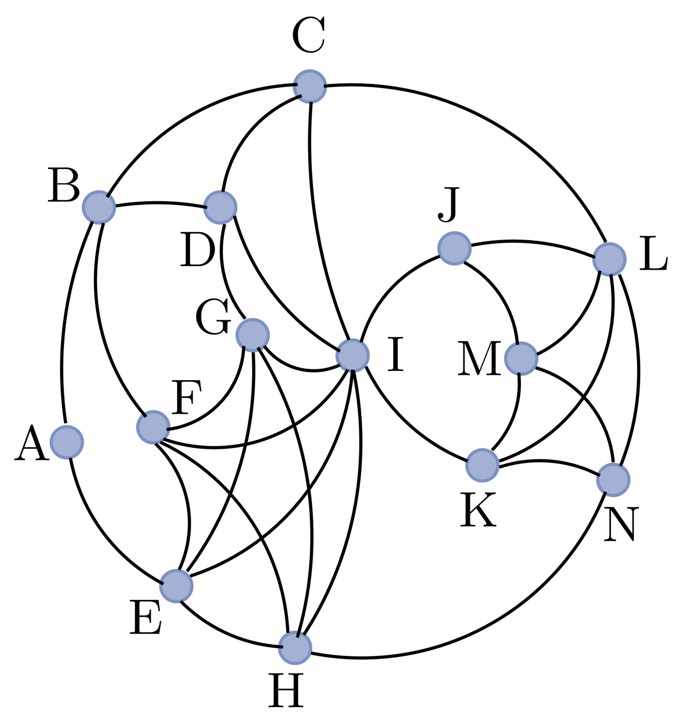
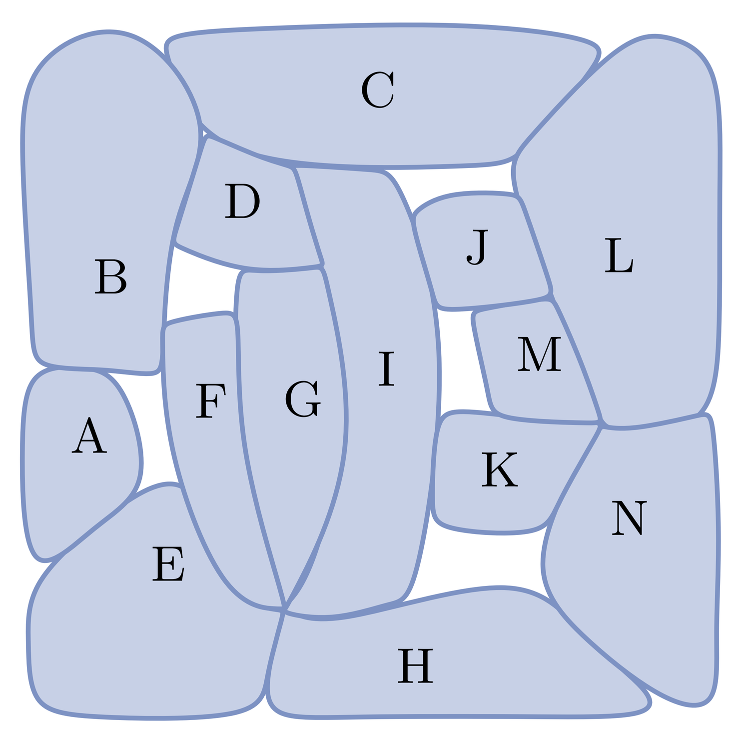
A **map graph** [2] is a contact graph of internally-disjoint regions of the plane, where the contact can be even a point

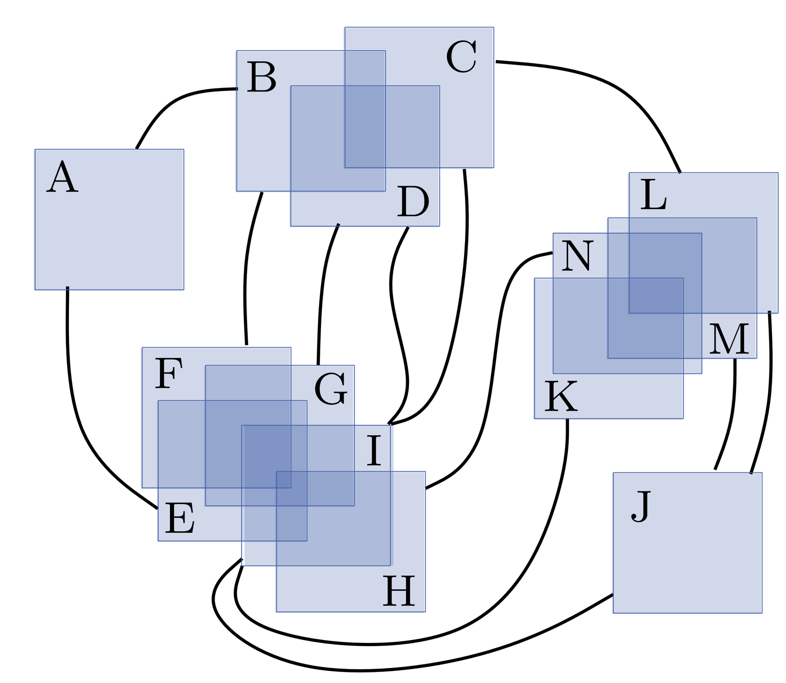
A **clique planar graph** [1] is a graph $G = (V, E)$ that admits a representation where each vertex $u \in V$ is represented by an axis-aligned unit square $R(u)$ and where, for some partition of V into vertex-disjoint cliques $S = \{c_1, \dots, c_k\}$, each edge (u, v) is represented by the intersection between $R(u)$ and $R(v)$ if u and v belong to the same clique (*intersection edges*) or by a non-intersected curve connecting the boundaries of $R(u)$ and $R(v)$ otherwise (*link edges*)



A non-planar graph...



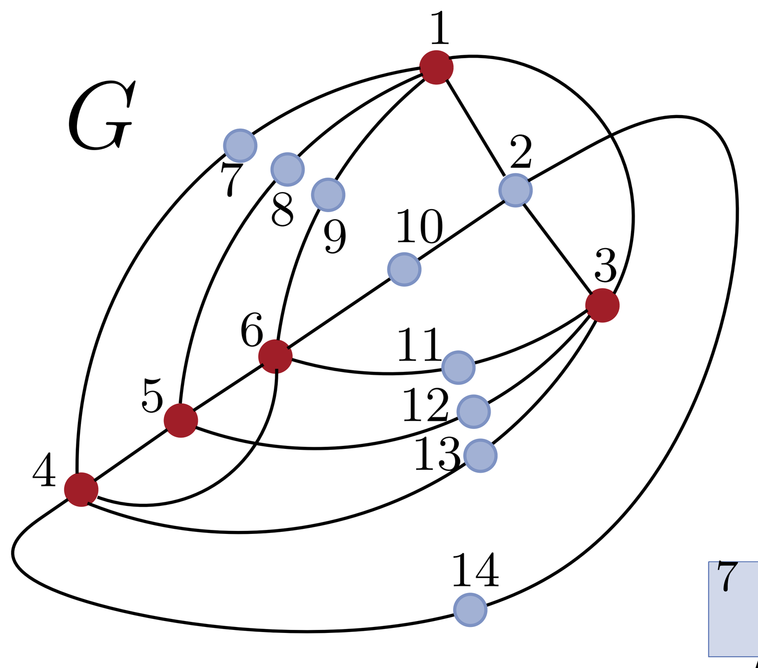
...which is a map graph...



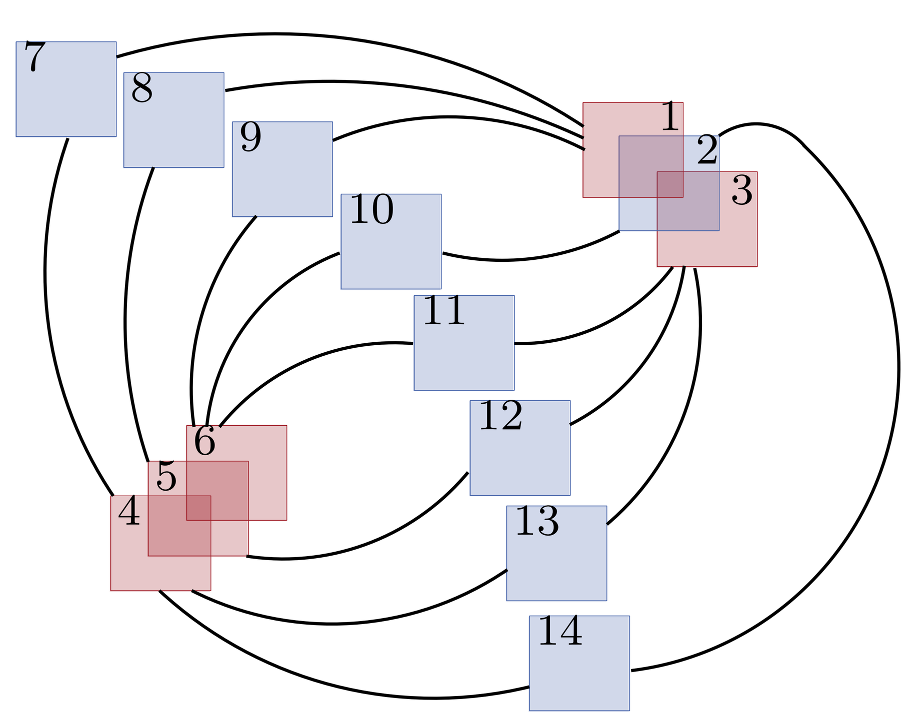
... and also a clique planar graph

Are map graphs always clique planar graphs and vice versa?

Clique planar but not map graph



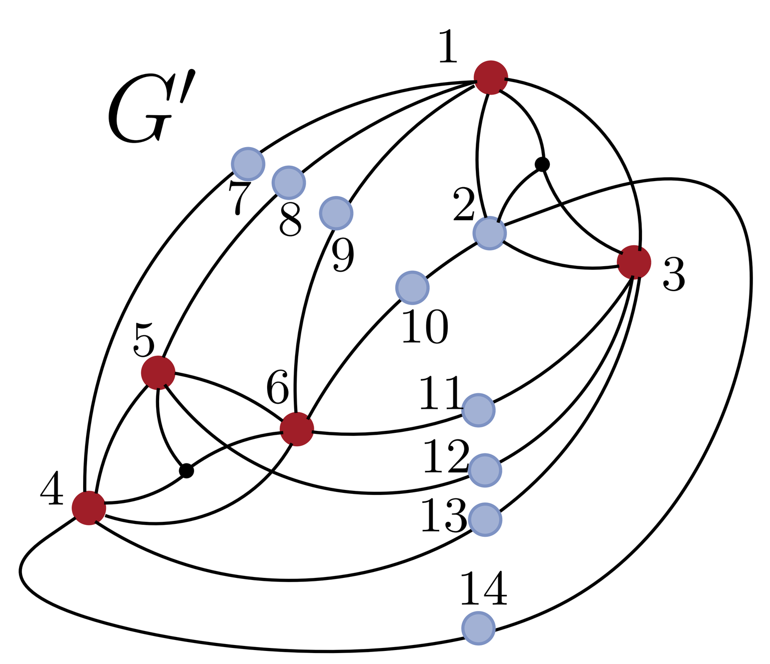
This graph...



...is clique planar

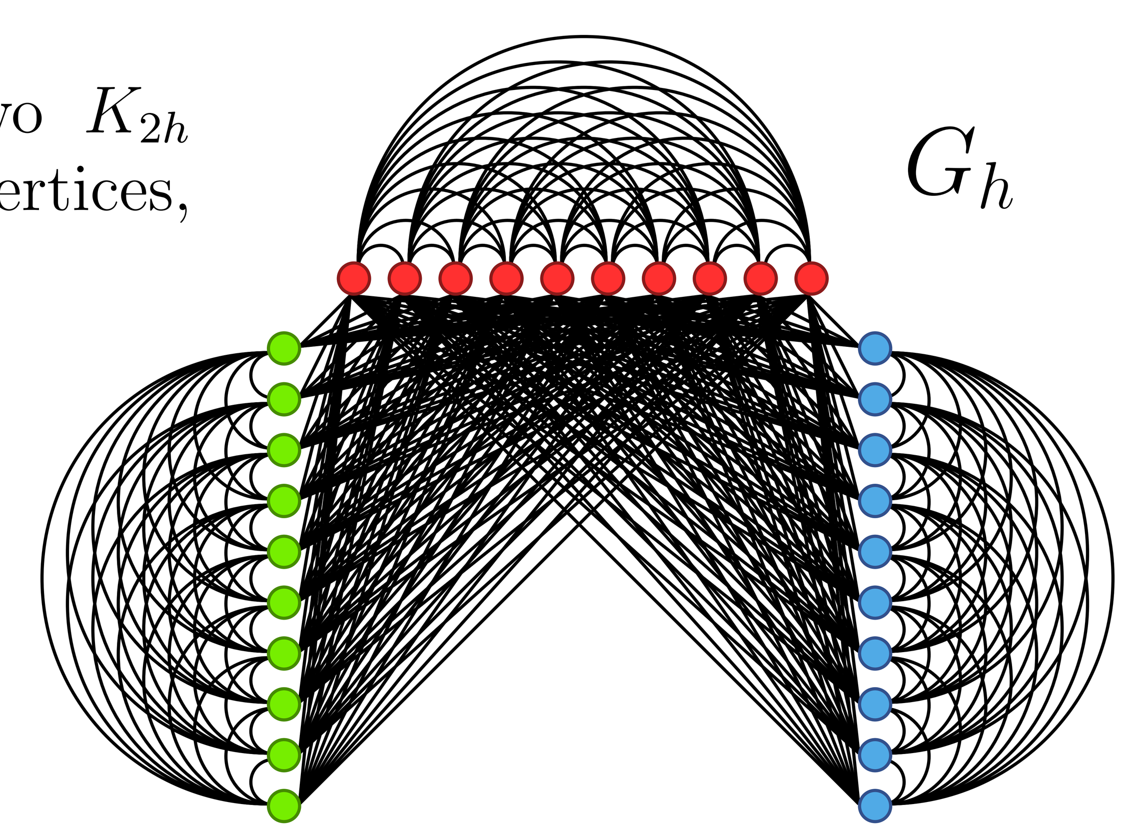
In a map graph representation either the triangles $\{1, 2, 3\}$ or $\{4, 5, 6\}$ (or both) could be represented by three regions sharing a point.

However, this would imply the planarity of a graph G' obtained from G by augmenting one or both (or none) of such triangles to wheels. G' is not planar as vertices $1, 3, 4, 5$ and 6 form a K_5 subdivision.

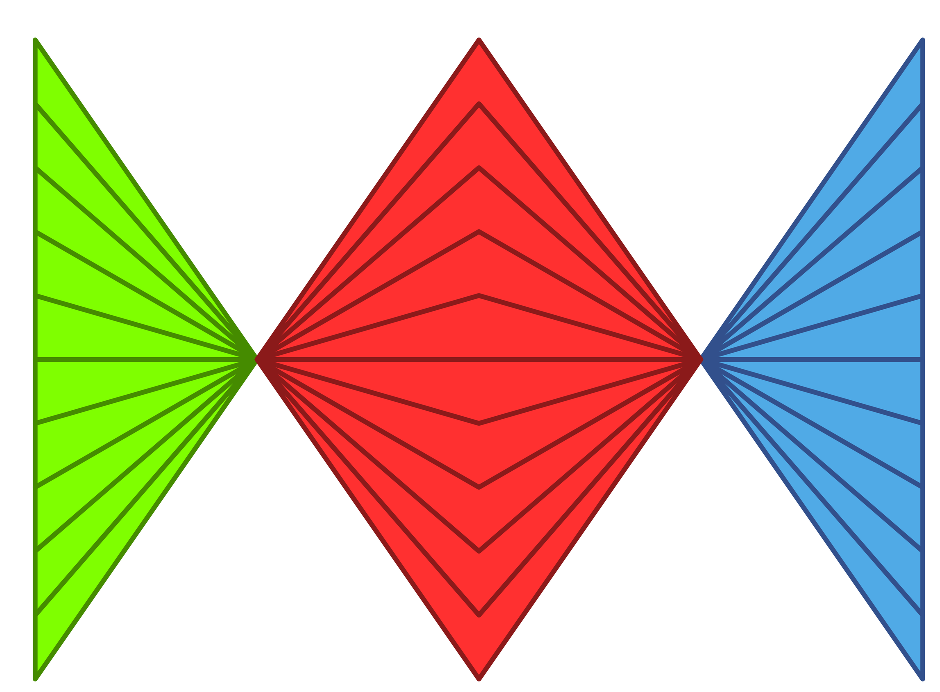


Map graph but not clique planar

This graph (two K_{2h} sharing h vertices, with $h = 10$)...



...is a map graph



However, it is not a clique planar graph. In fact, in any partition S of the vertices of G_h into vertex-disjoint cliques there are at least $h/2$ red vertices that do not fall into the same clique with the green or blue vertices. The link edges among such vertices induce a $K_{\frac{h}{2},h}$. Hence, for $h \geq 6$ the clique planarity of G_h would imply the planarity of $K_{3,3}$.

1. Angelini, P., Da Lozzo, G., Di Battista, G., Frati, F., Patrignani, M., Rutter, I.: *Intersection-link representations of graphs.* In: Di Giacomo, E., Lubiw, A. (eds.) Graph Drawing (GD 15). LNCS (2015), to appear
2. Chen, Z., Grigni, M., Papadimitriou, C. H.: *Map graphs.* J. ACM 49(2), 127–138 (2002)