Julius-Maximilians-
UNIVERSITÄT
WÜRZBURG

Chair for
INFORMATICS I
Efficient Algorithms and
Knowledge-Based Systems

Institute for Informatics

Simultaneous Drawing of Planar Graphs with Right-Angle Crossings and Few Bends

Alexander Wolff
Chair of Computer Science I
Universität Würzburg

Joint work with
Michael A. Bekos · Thomas C. van Dijk · *Philipp Kindermann*

Simultaneous Embedding

Given: Two planar graphs $G_1 = (V, E_1)$ and $G_2 = (V, E_2)$ on same vertex set

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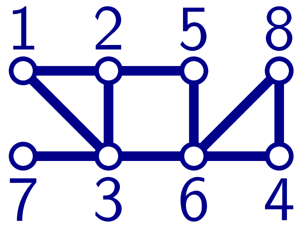

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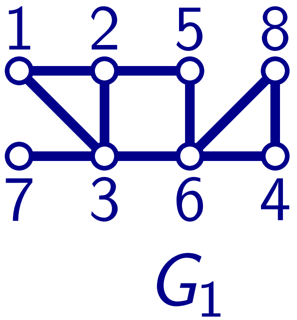

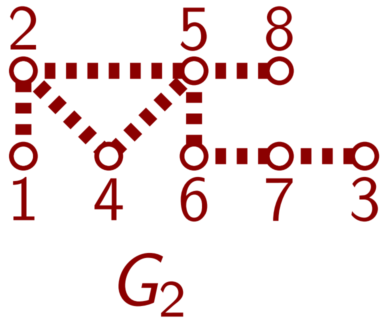

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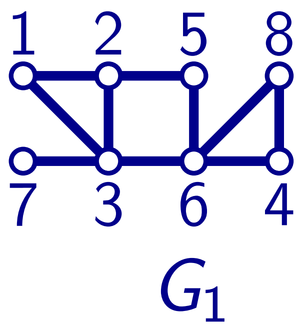

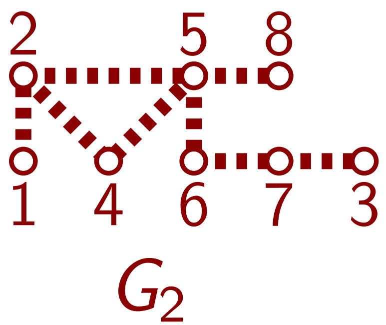

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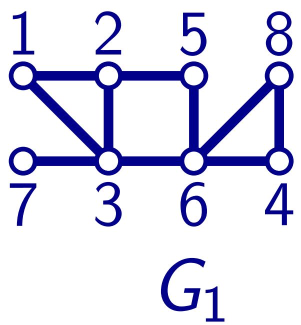

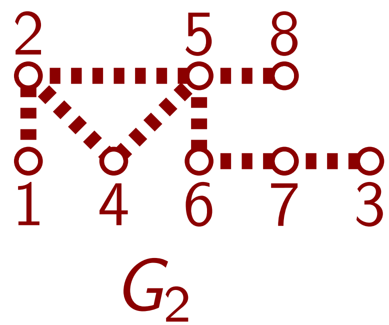

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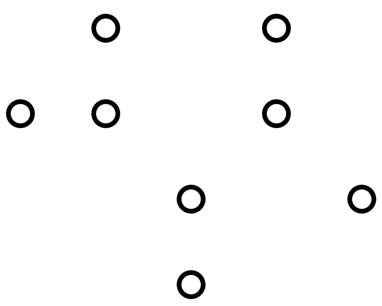

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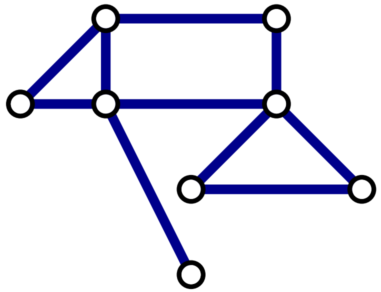

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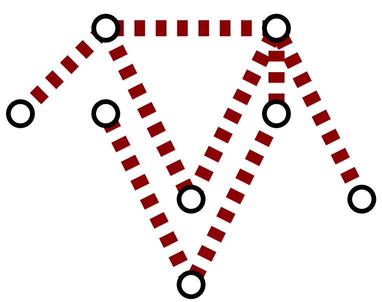

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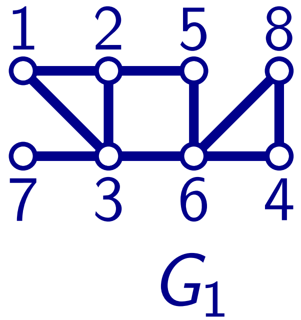

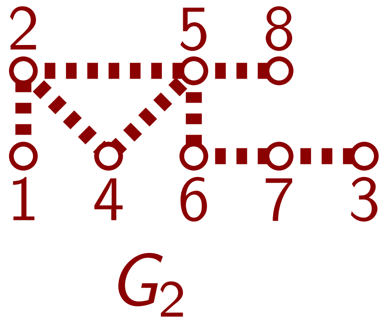

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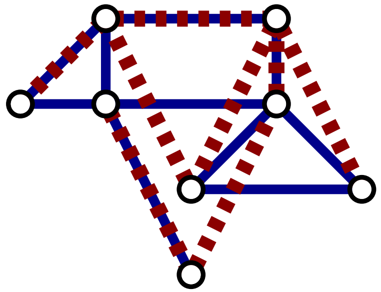

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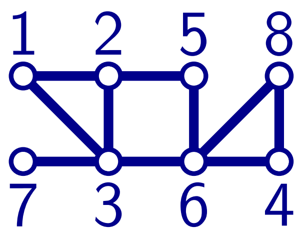

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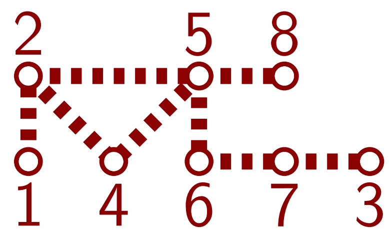

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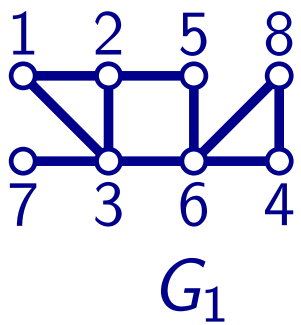

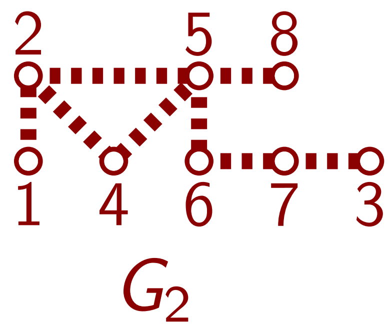

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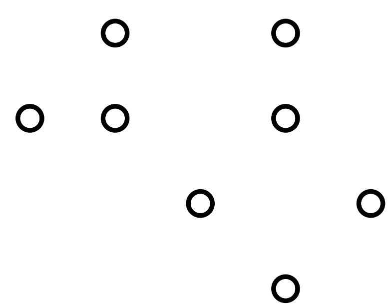

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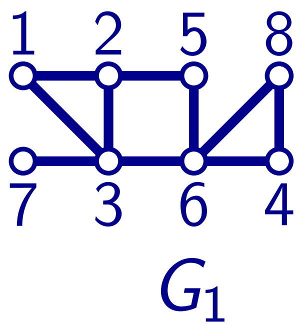

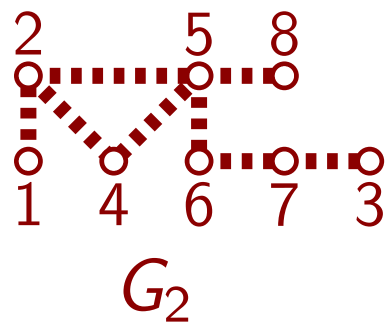

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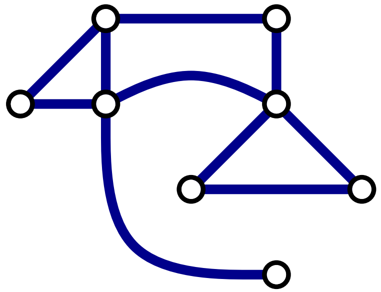

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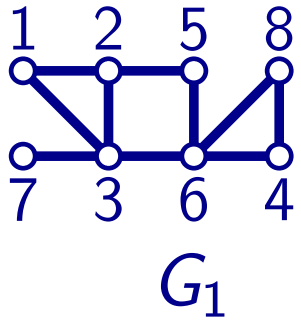

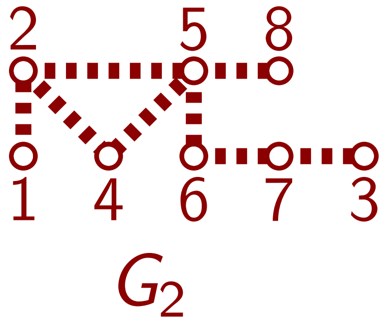

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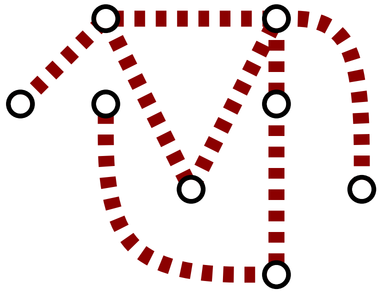

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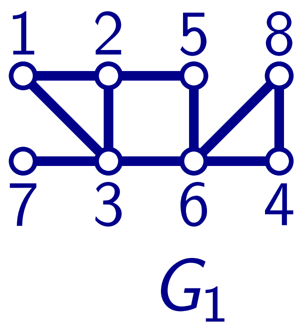

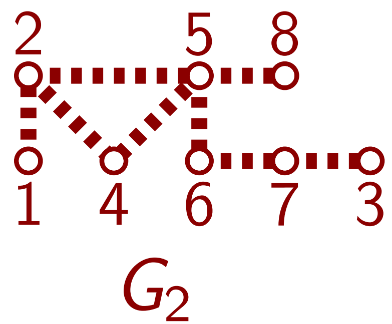

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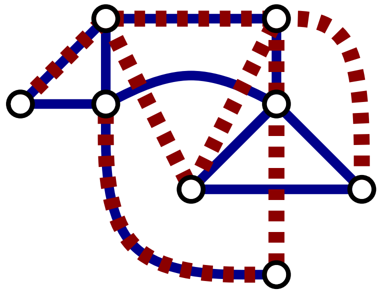

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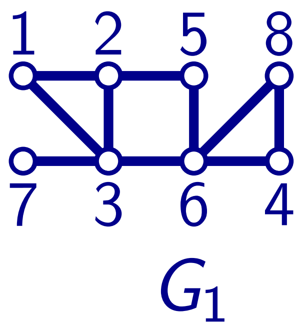

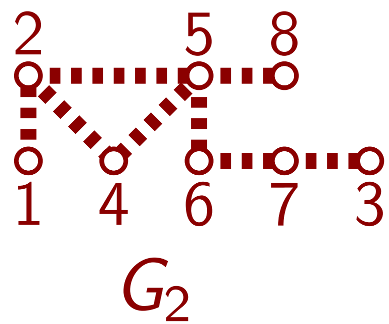

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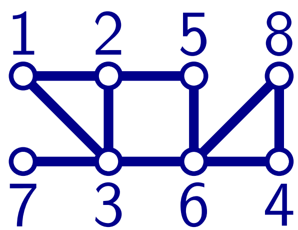

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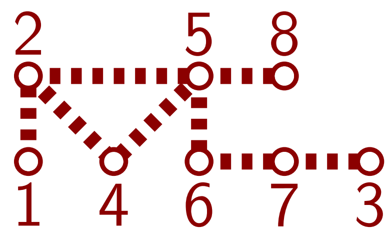

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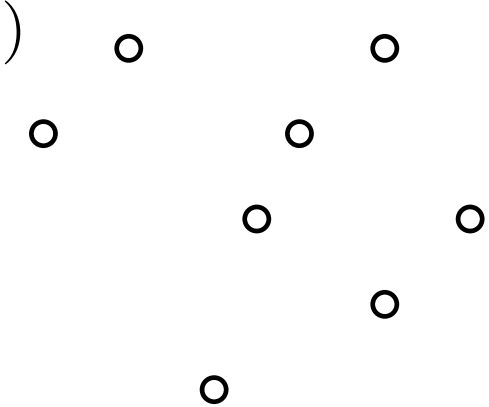

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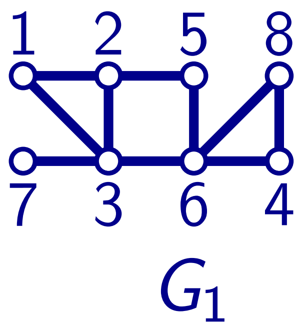

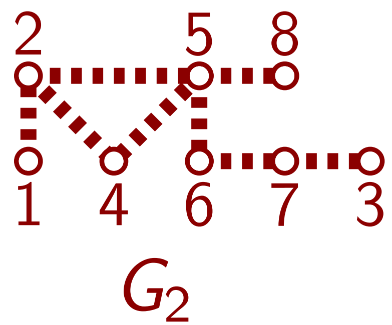

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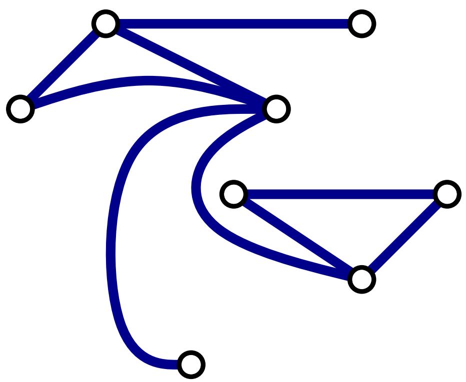

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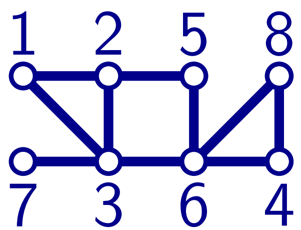

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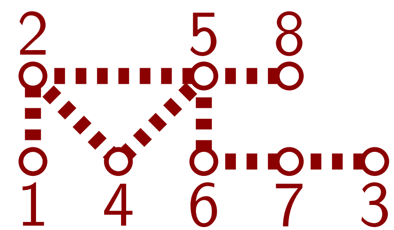

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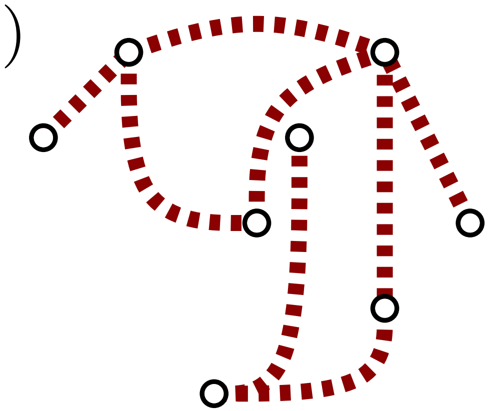

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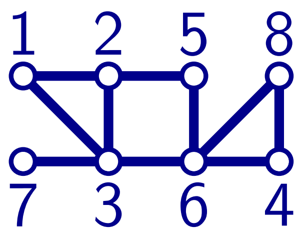

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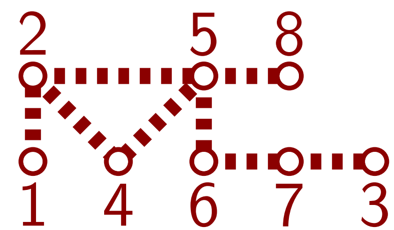

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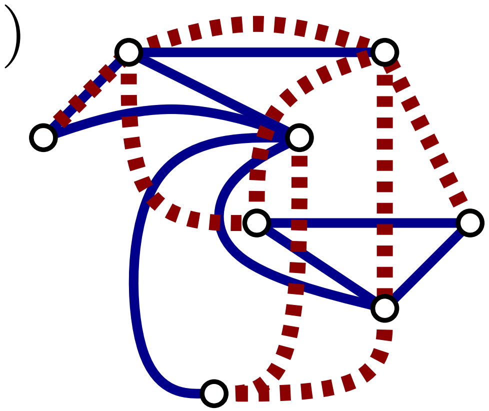

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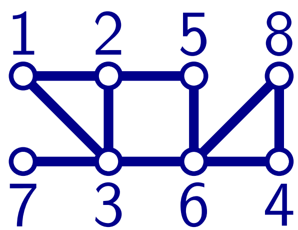

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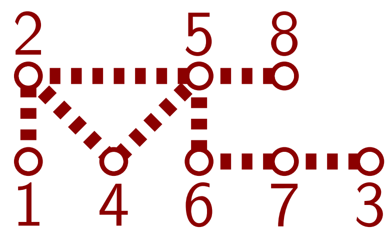

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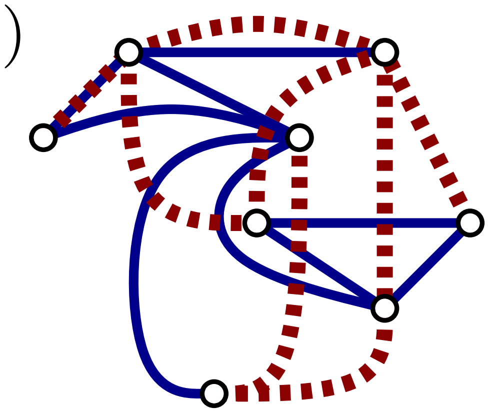

Large body of literature; see the survey
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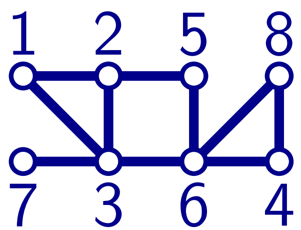

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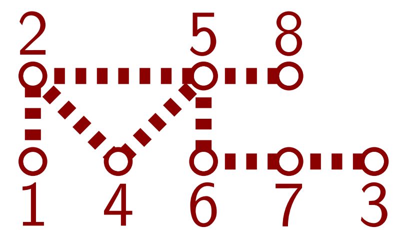

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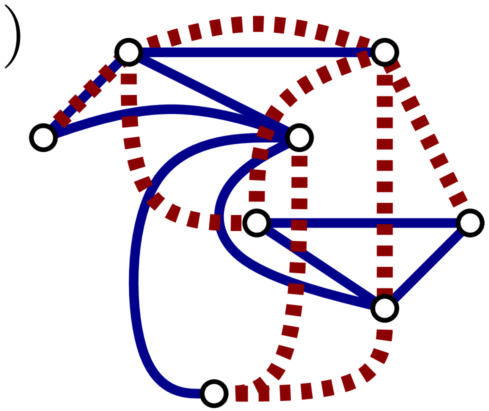

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RacSim Drawings

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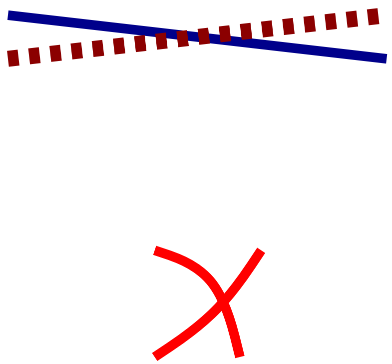

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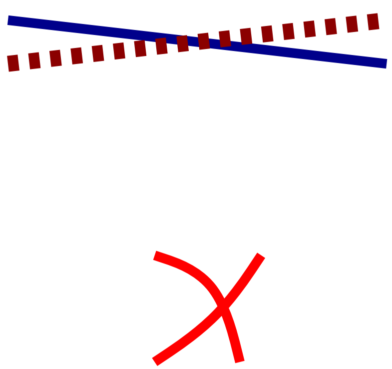

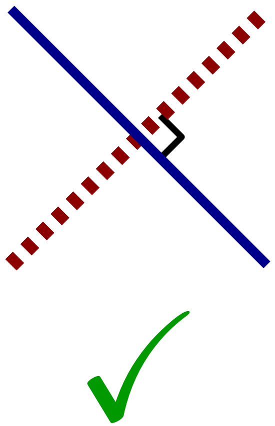

Geometric RacSim: RacSim with straight-line edges

A Few Known Results

[Angelini et al. JGAA'12]
There exist a tree and a path
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[Frati, Hoffmann, Kusters GD'15]

New!

A Few Known Results

[Argyrious et al. JGAA'13]

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- Two planar graphs admit a $(6, 16)$-Sefe.

Our Results for SE

Graph classes			Number of bends
Cycle	$\times$	Cycle	1×1
Caterpillar	$\times$	Cycle	1×1
Four Matchings			$1 \times 1 \times 1 \times 1$
Tree	$\times$	Matching	1×0
Wheel	$\times$	Matching	2×0
Outerpath	$\times$	Matching	2×1
Outerplanar	$\times$	Outerplanar	3×3
2-page book emb.	$\times$	2-page book emb.	4×4
Planar	$\times$	Planar	6×6

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Bend complexity can be seen as a measure that shows how difficult it is to simultaneously embed two graphs.

Path × Path

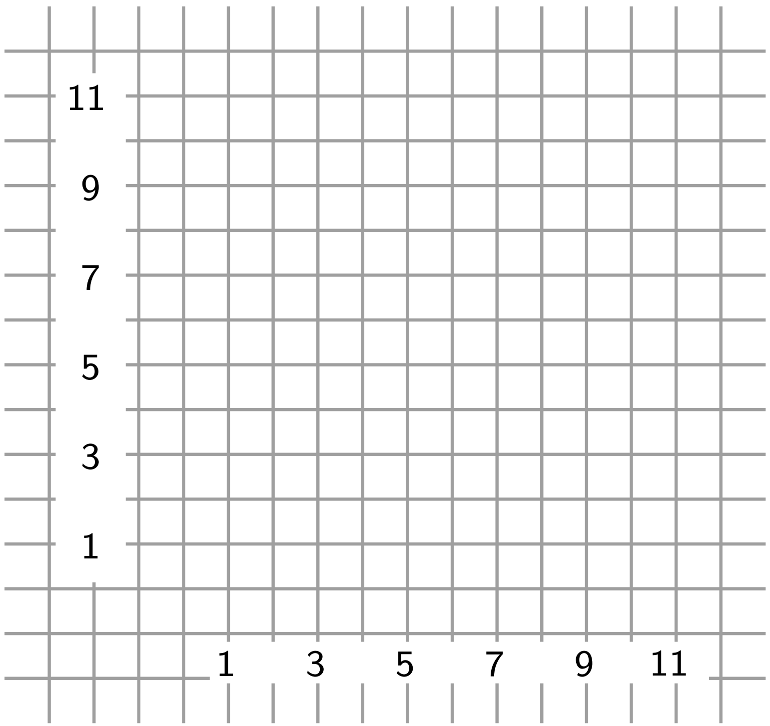
Path $\times$ Path

Path × Path

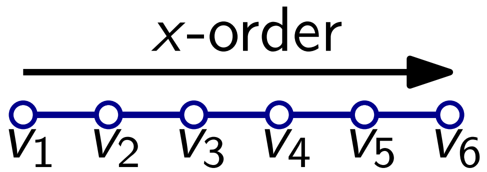
Path × Path

Path × Path

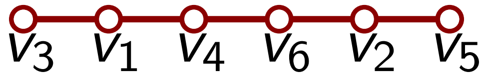
Path × Path



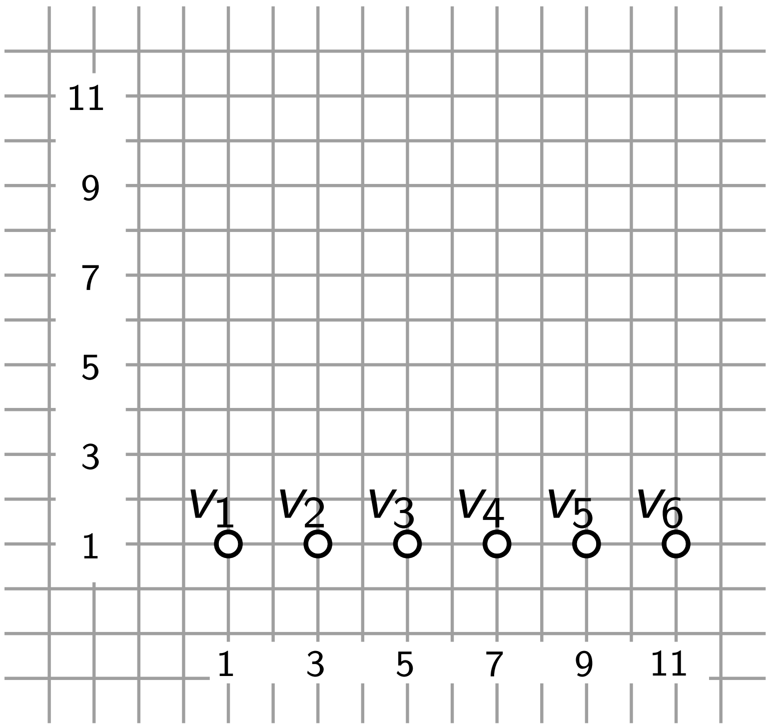
Path × Path



x-order
v_1 v_2 v_3 v_4 v_5 v_6

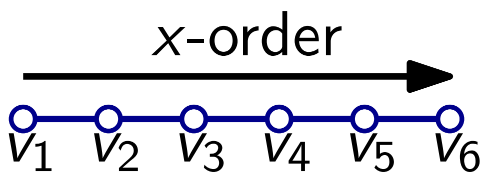


v_3 v_1 v_4 v_6 v_2 v_5



11
9
7
5
3
1
v_1 v_2 v_3 v_4 v_5 v_6
1 3 5 7 9 11

Path × Path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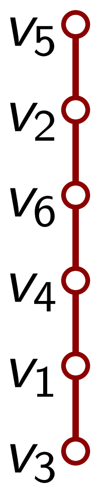
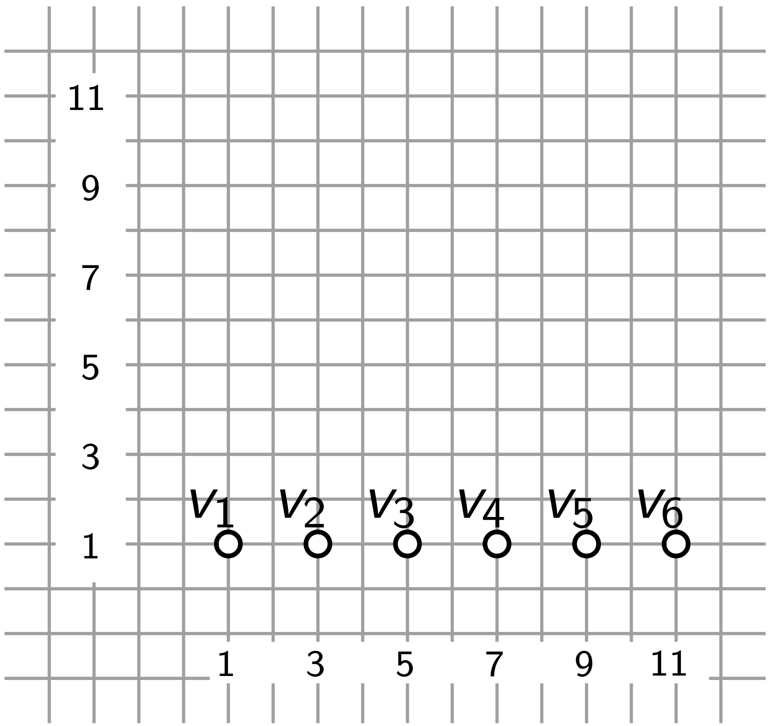

v5
v2
v6
v4
v1
v3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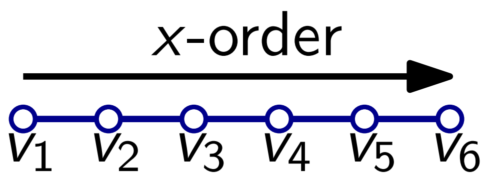

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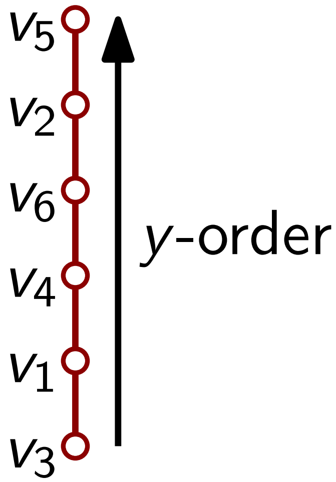
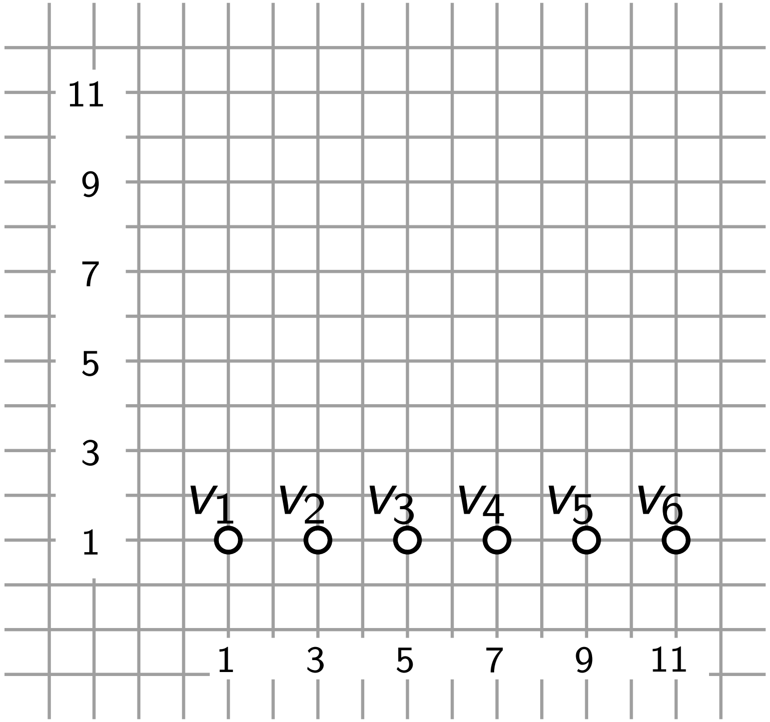


v5
v2
v6
v4
v1
v3
y-order



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v5
v6
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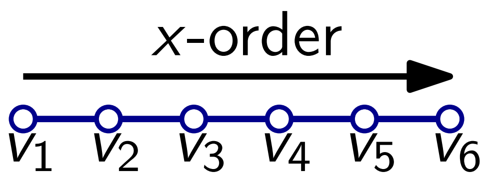
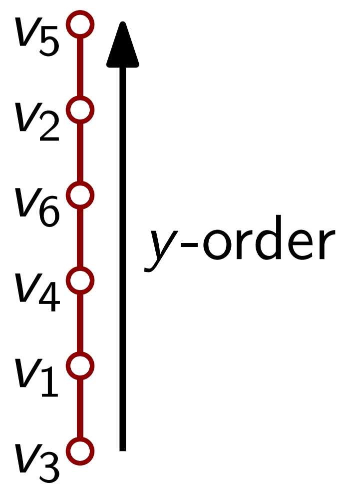
Path × Path


x-order
v_1 v_2 v_3 v_4 v_5 v_6


v_5
v_2
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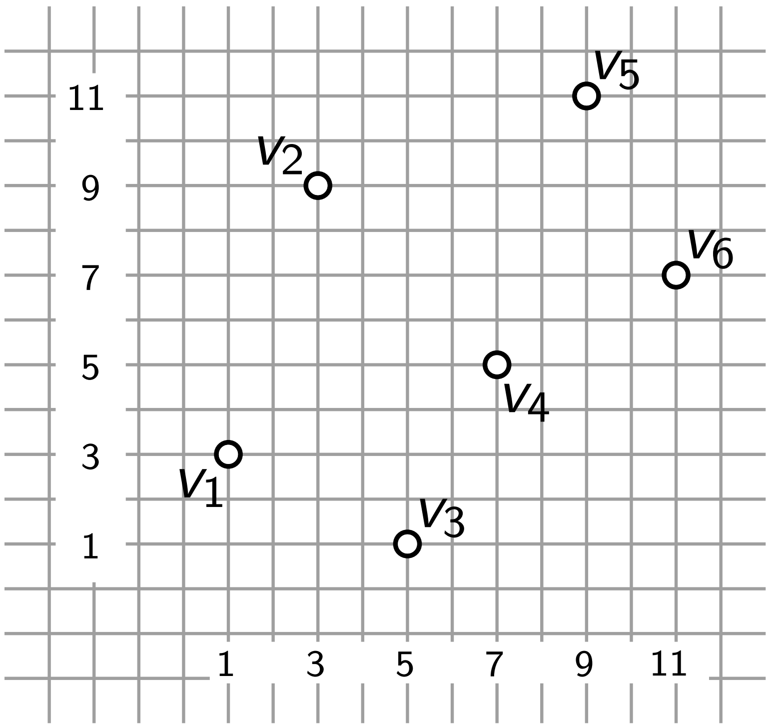
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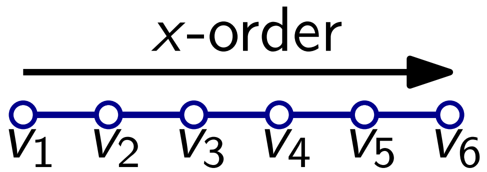
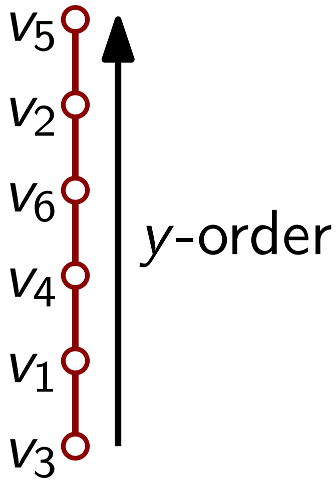
Path × Path





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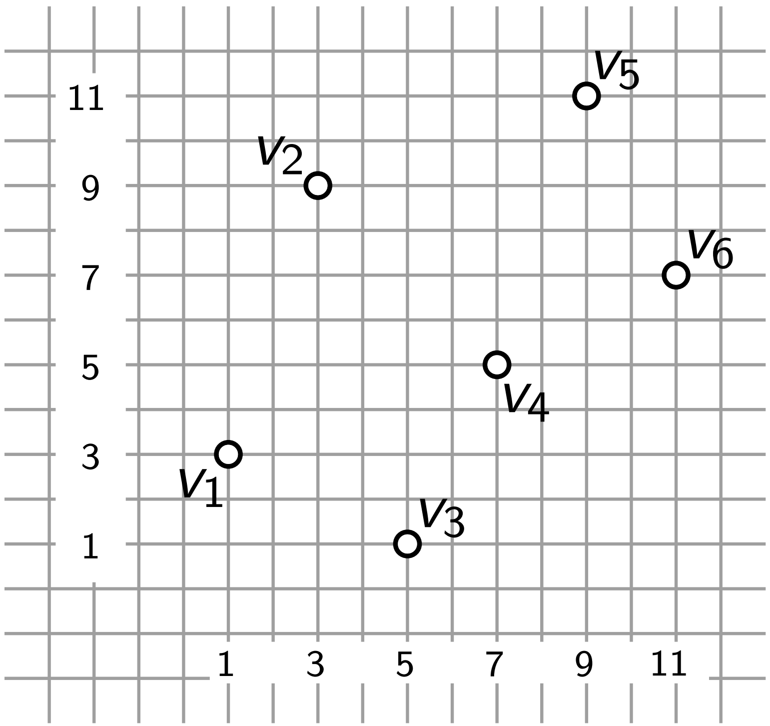

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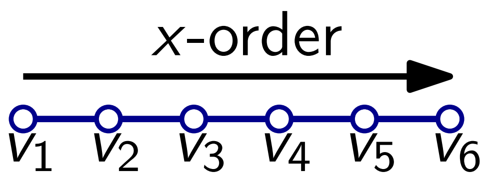

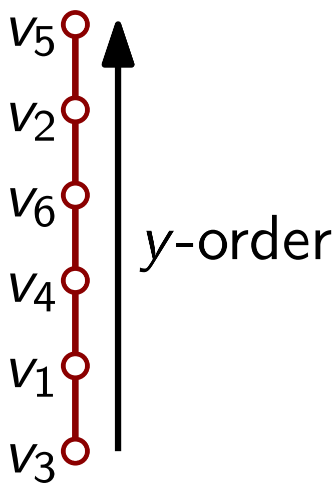

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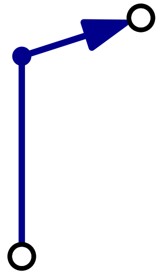

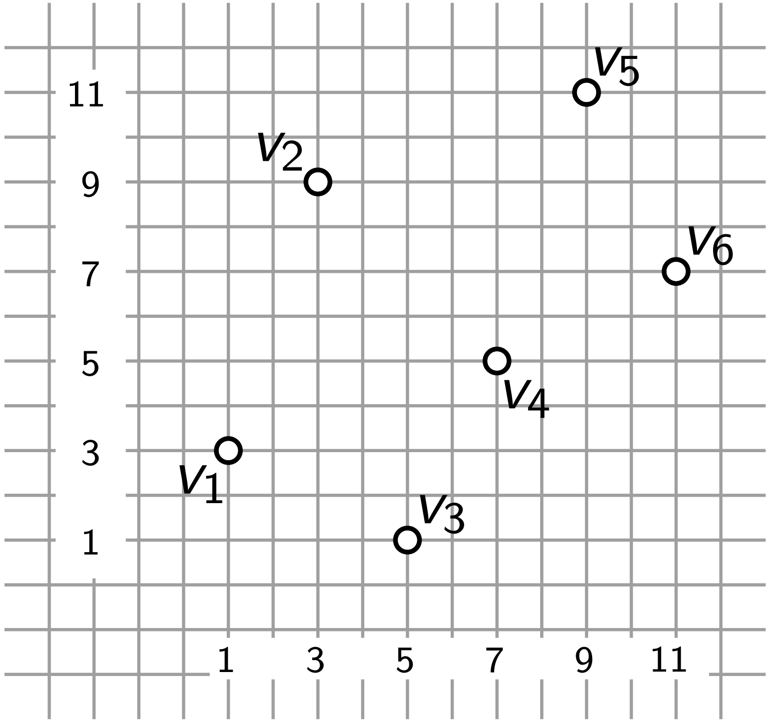

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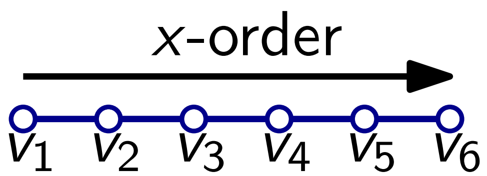

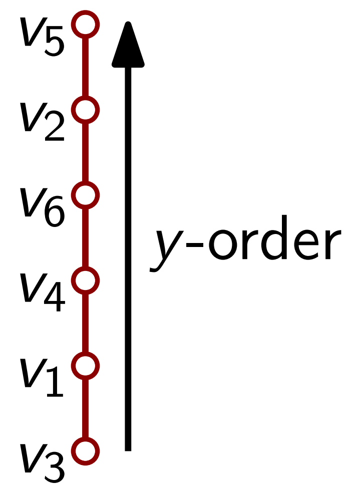

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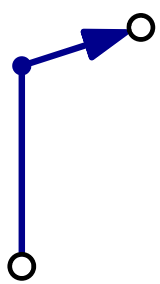

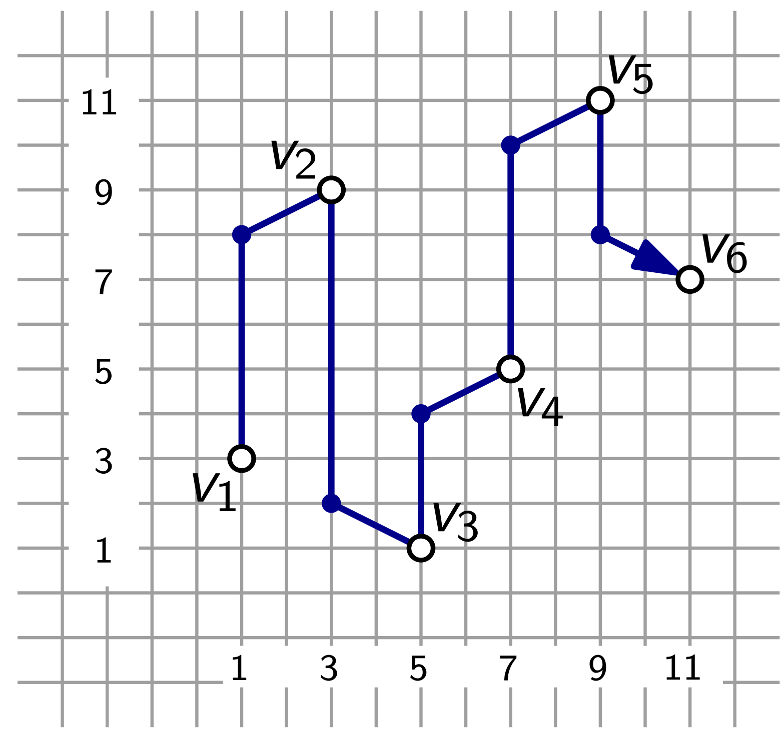

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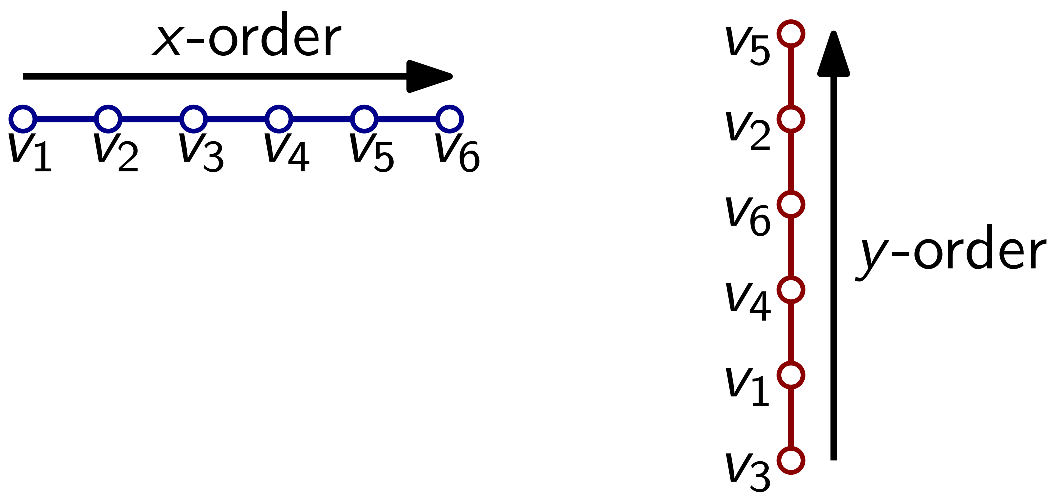

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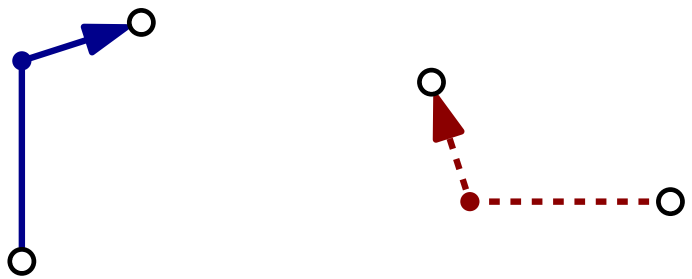

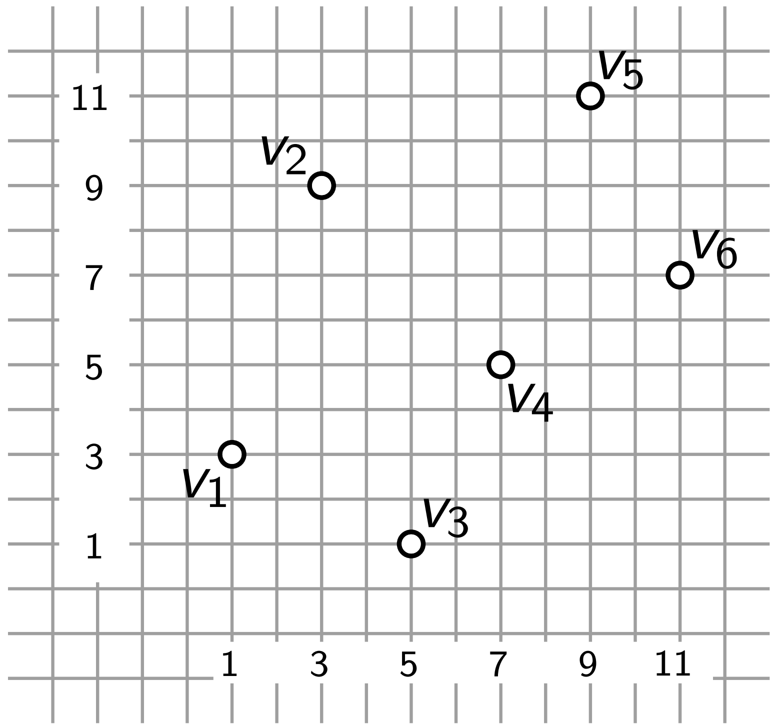

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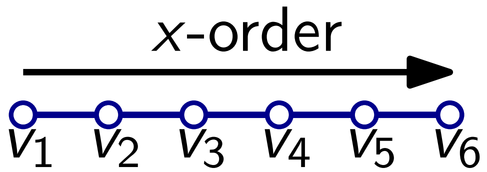

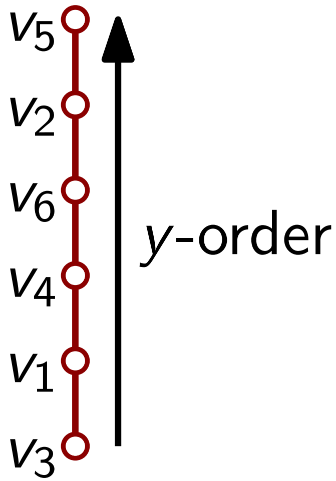

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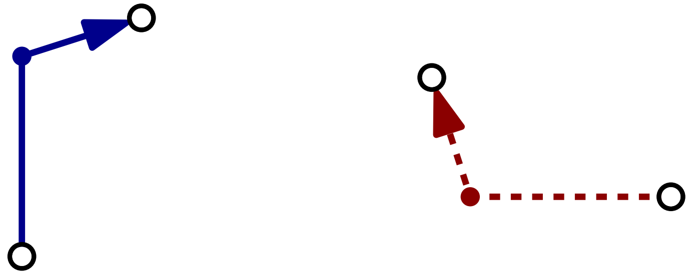

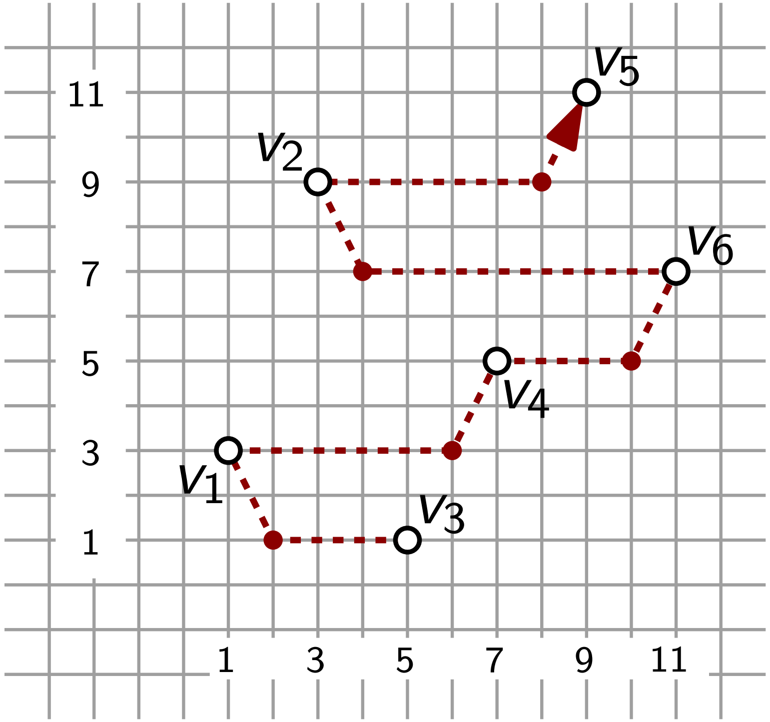

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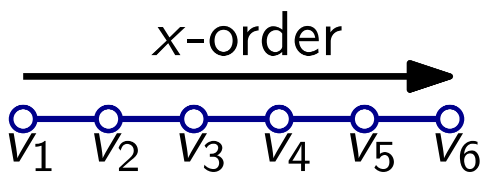

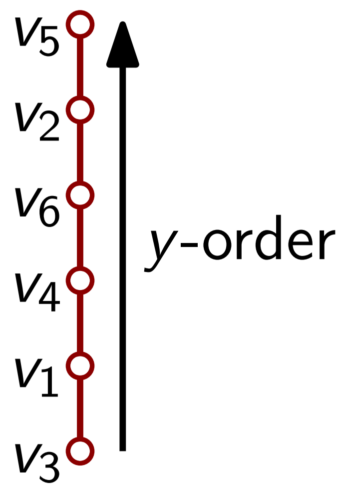

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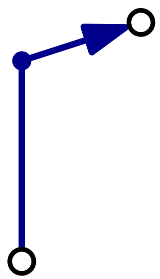

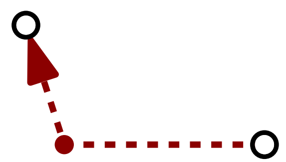

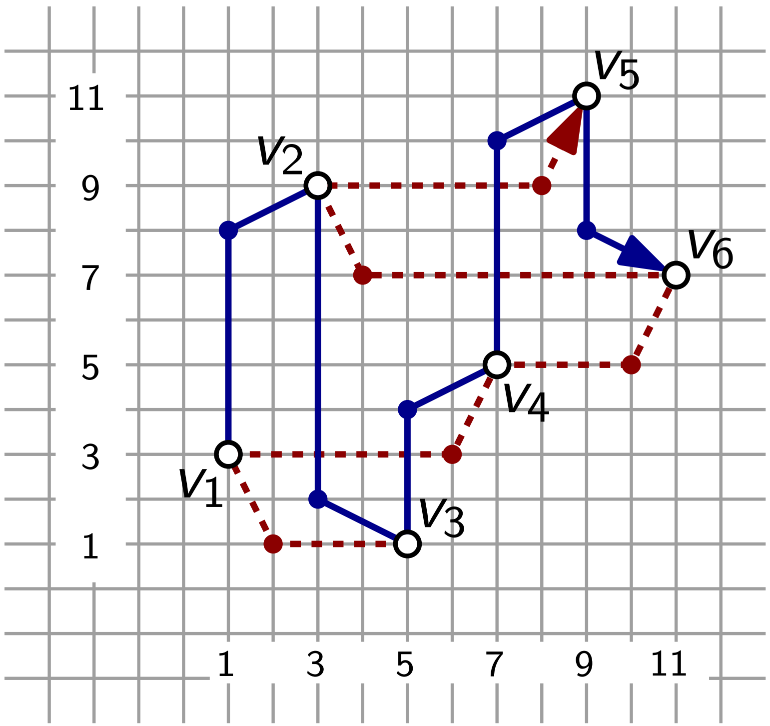

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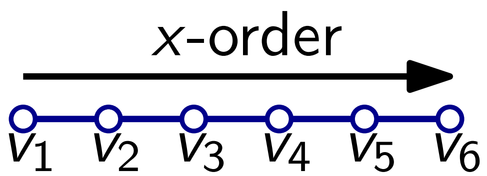

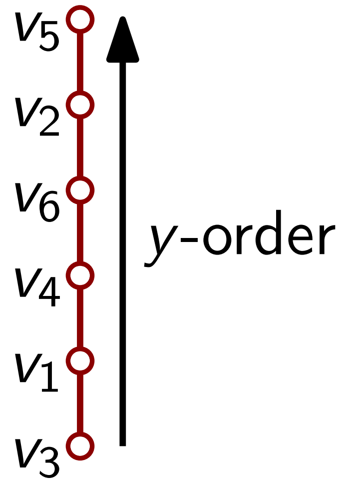

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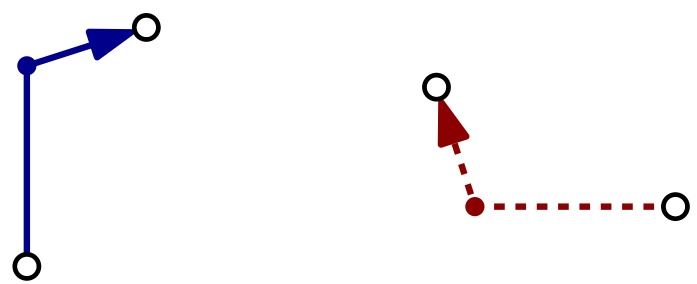
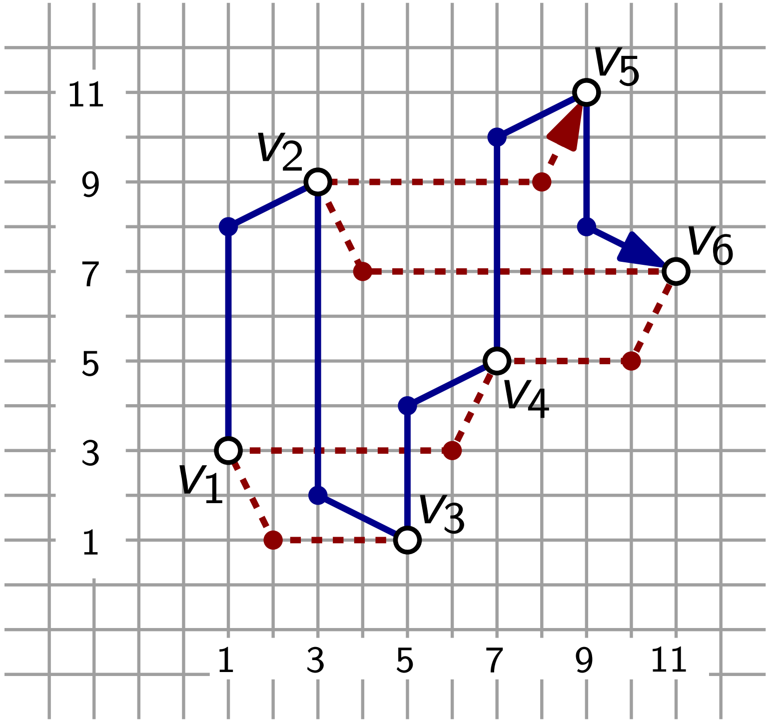
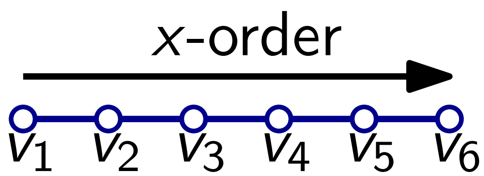


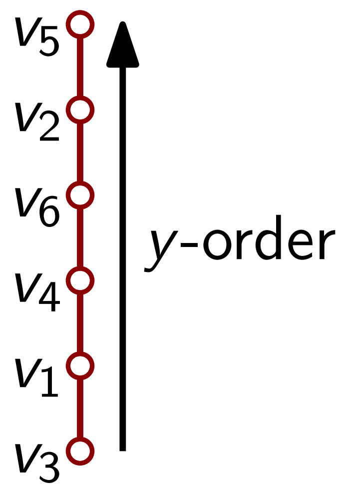


Main ideas:

- Combine x-order and y-order.

Path $\times$ Path





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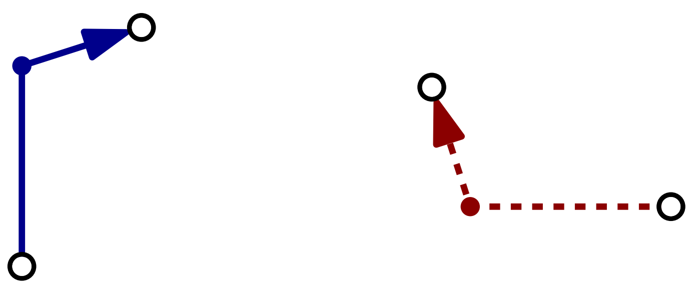

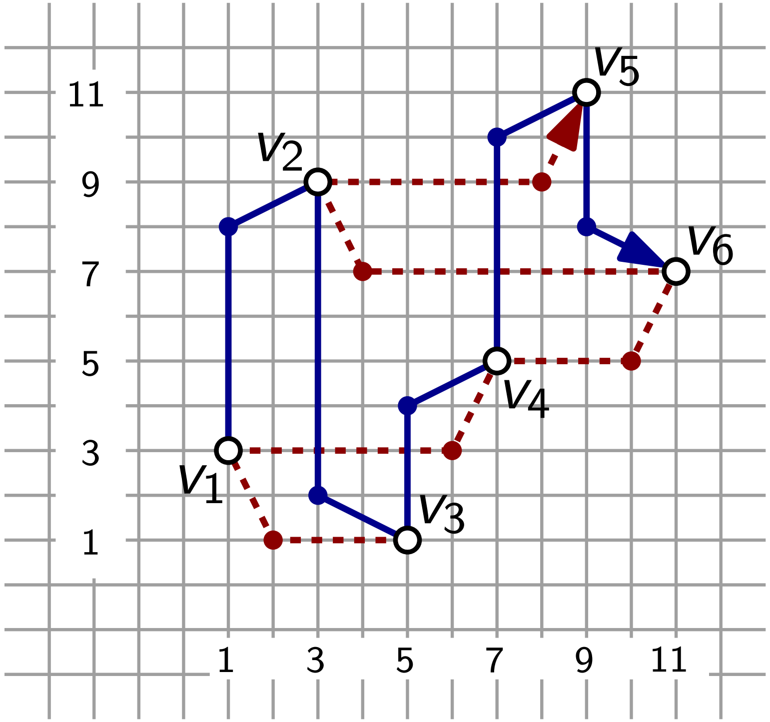

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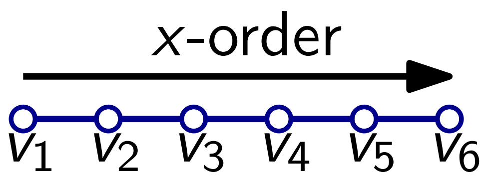

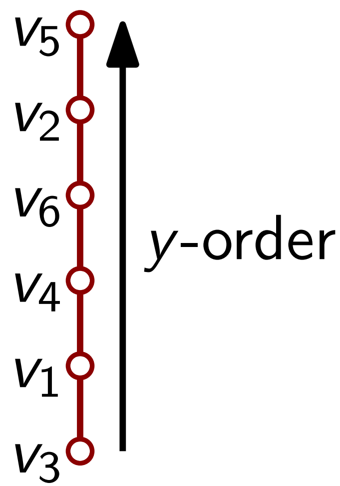

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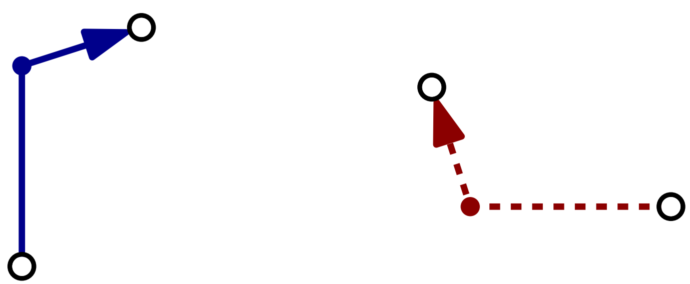

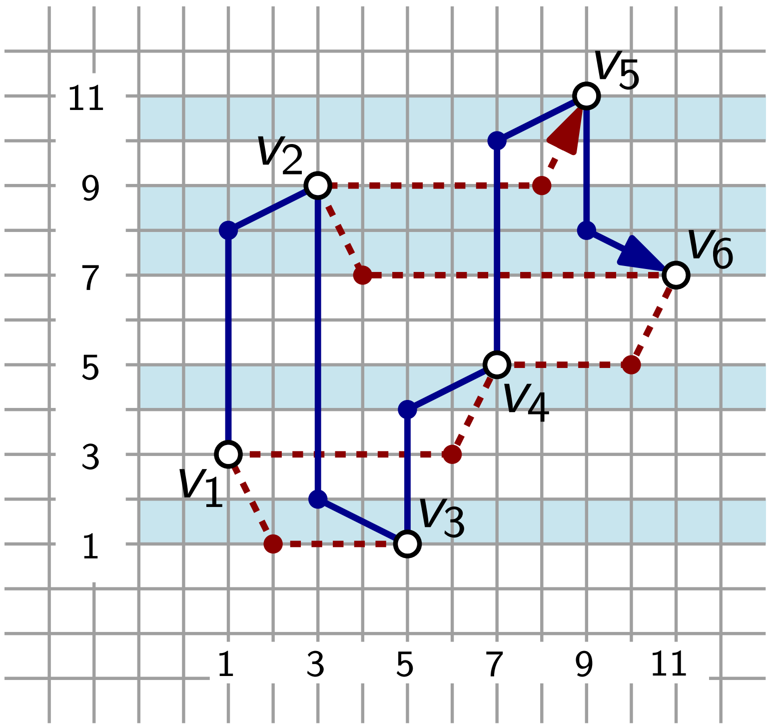

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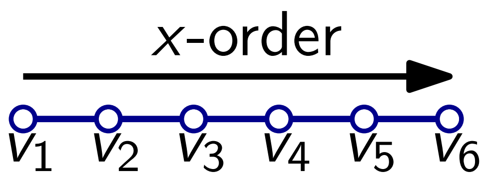

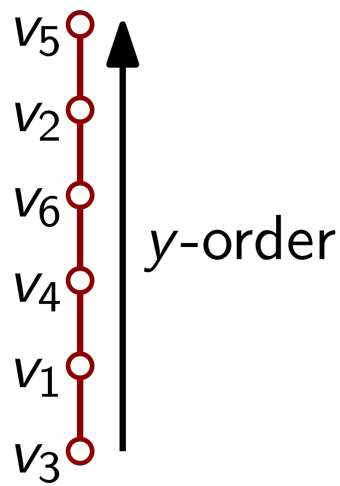

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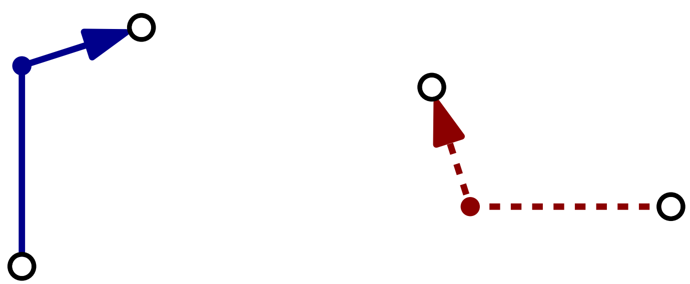

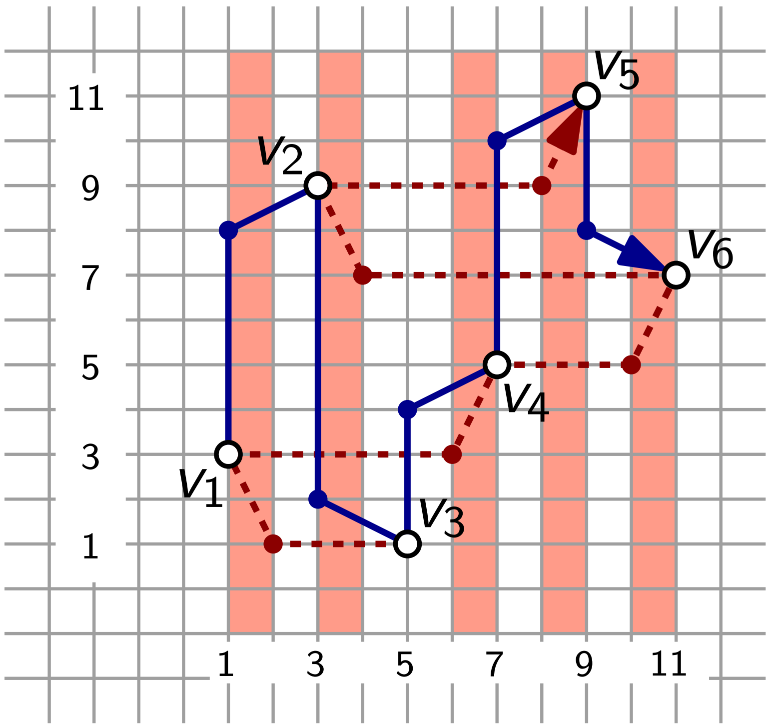

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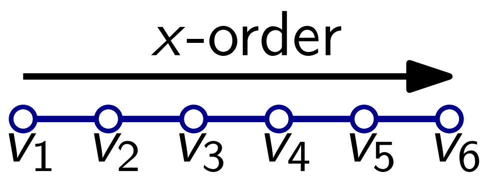

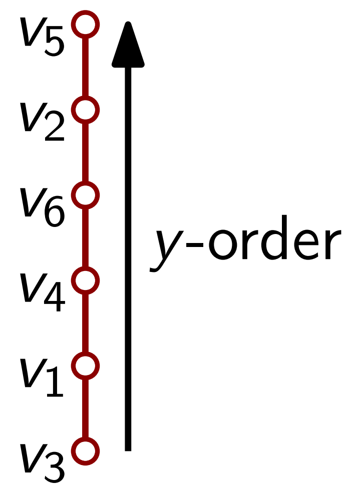

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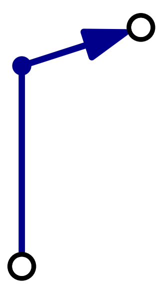

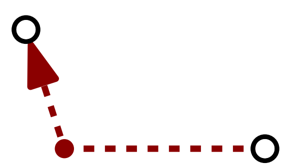

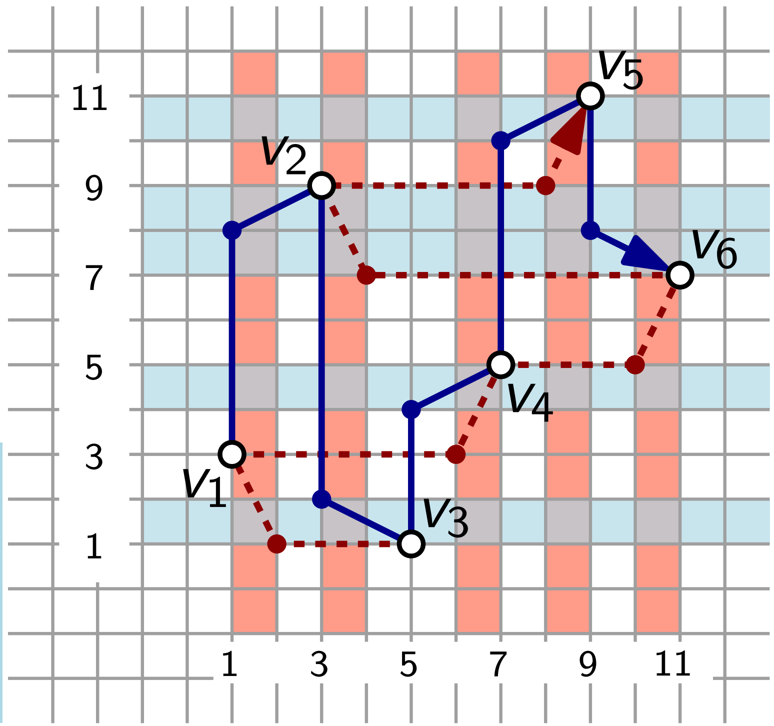

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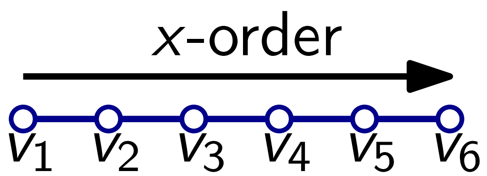
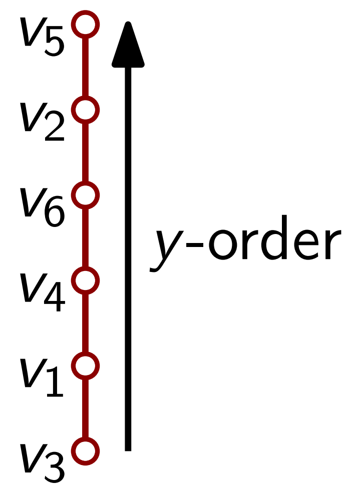
Cycle × Cycle





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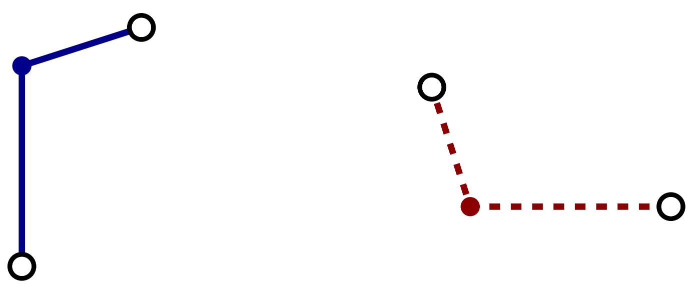
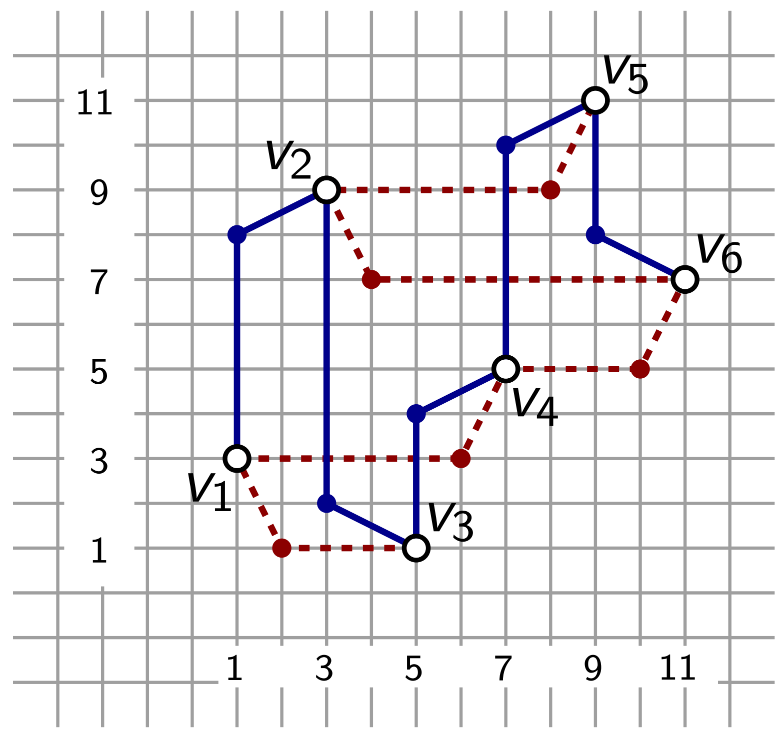
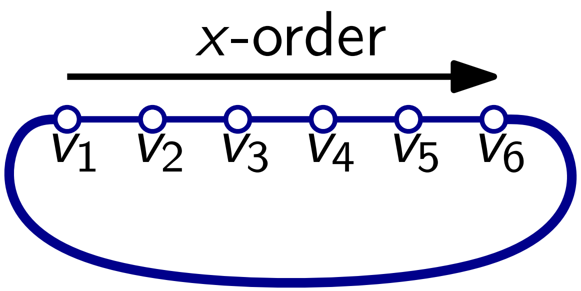
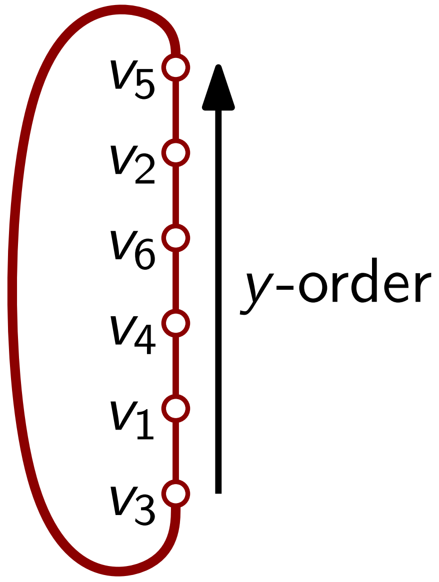




Cycle × Cycle





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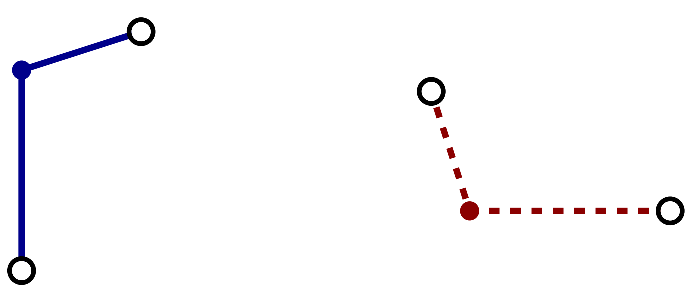

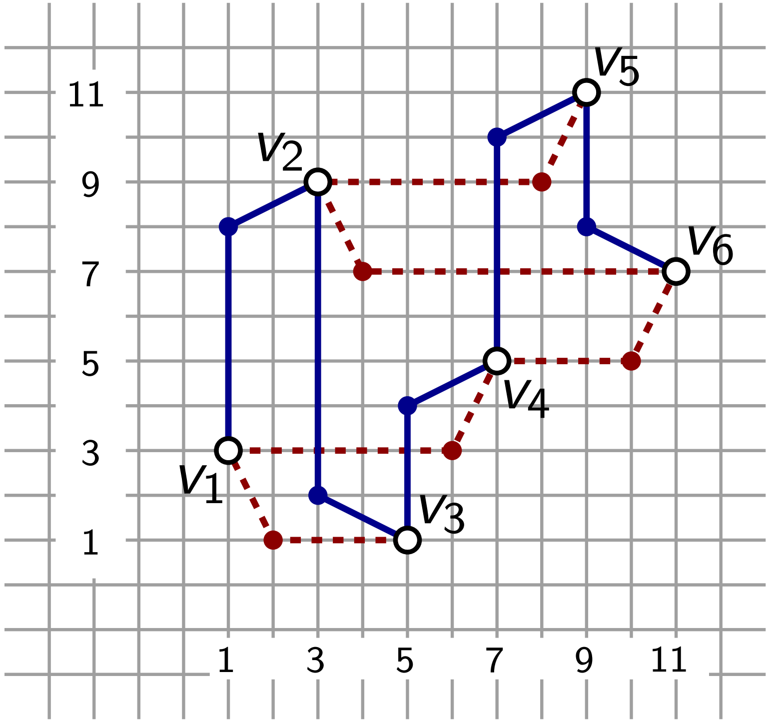

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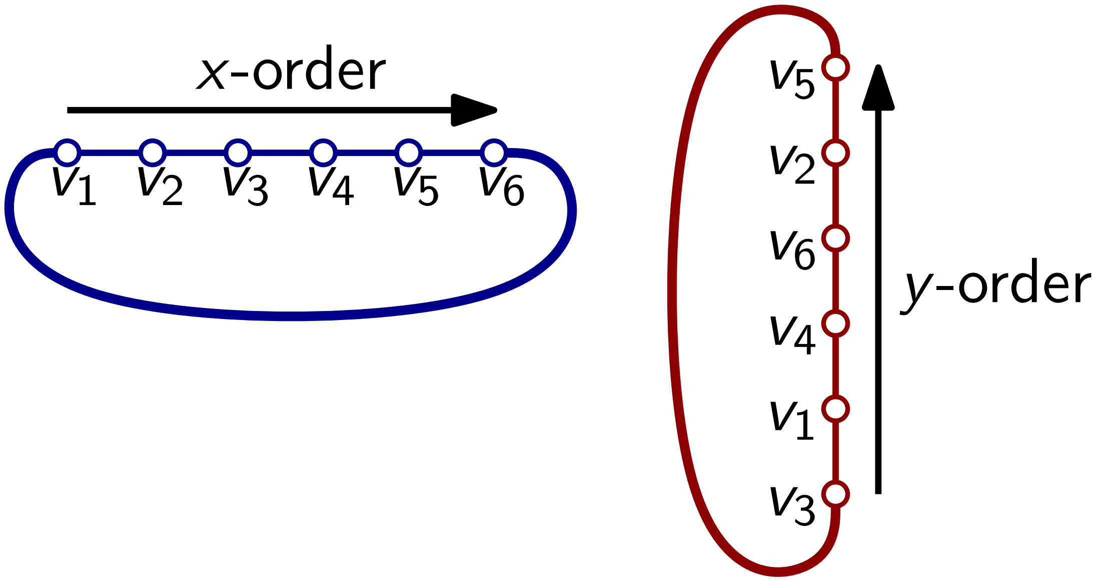

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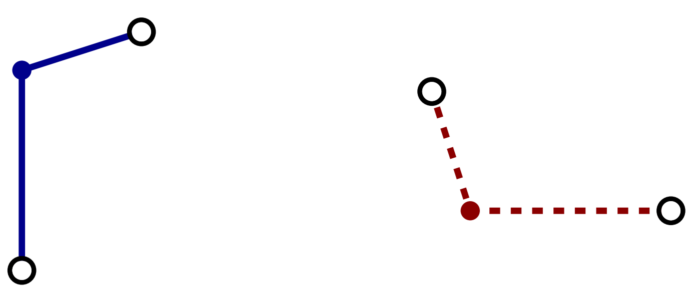

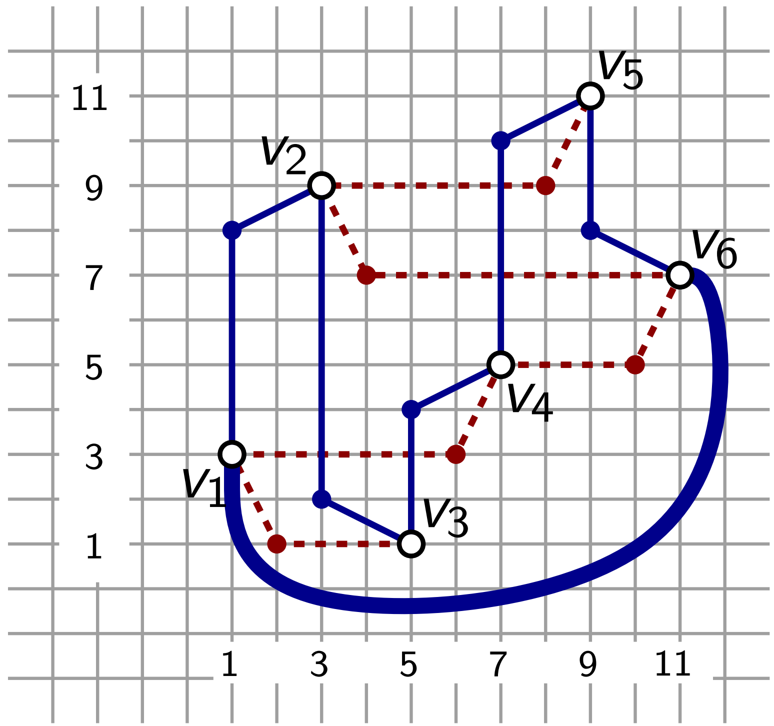

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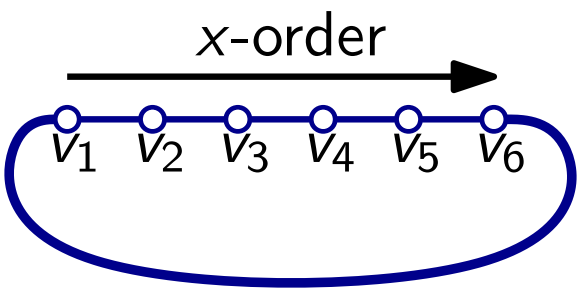

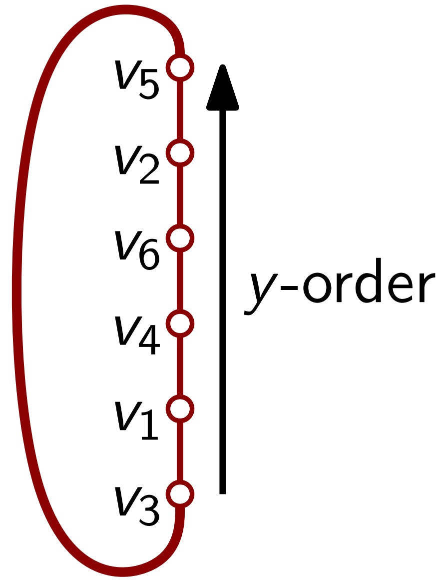

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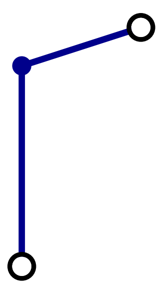

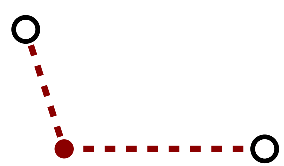

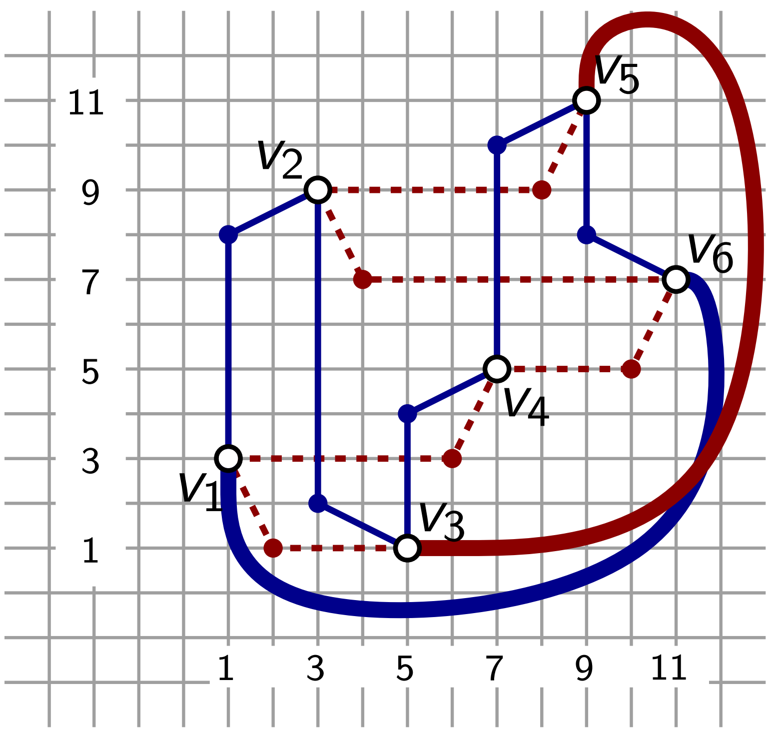

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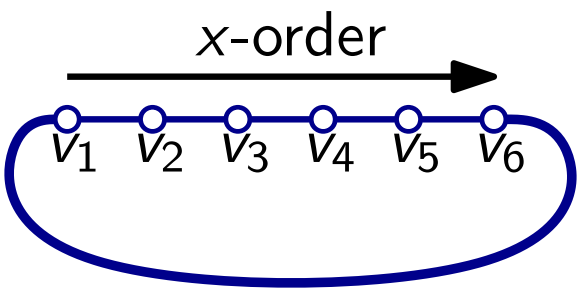

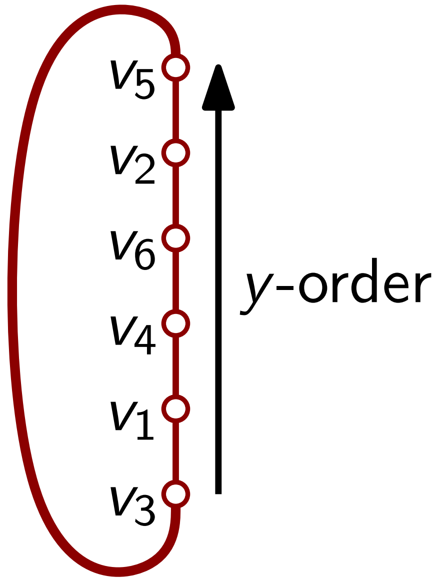

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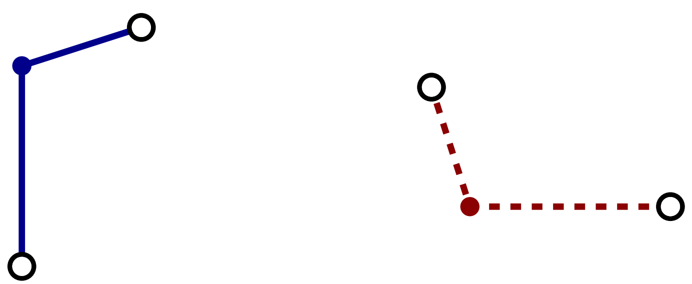

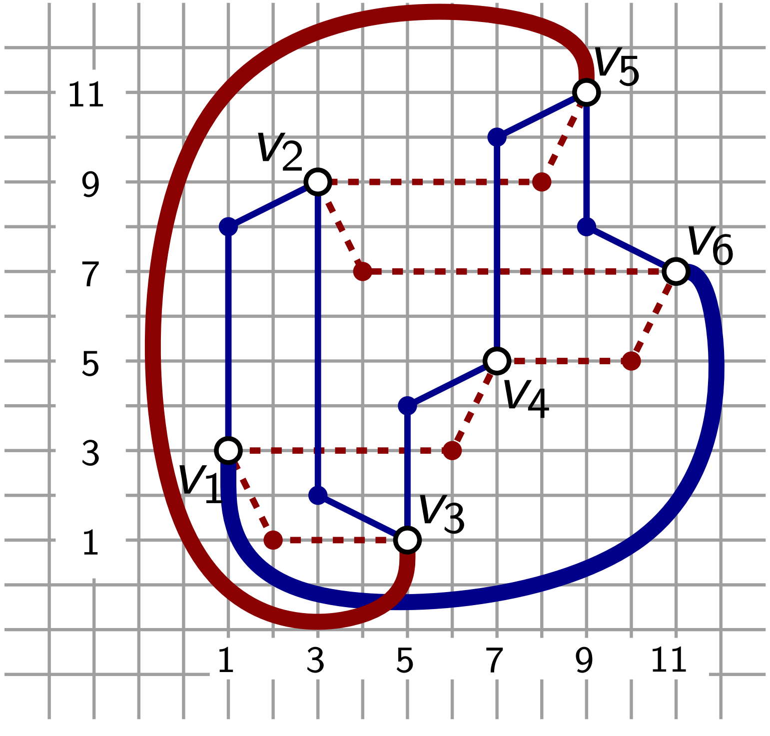

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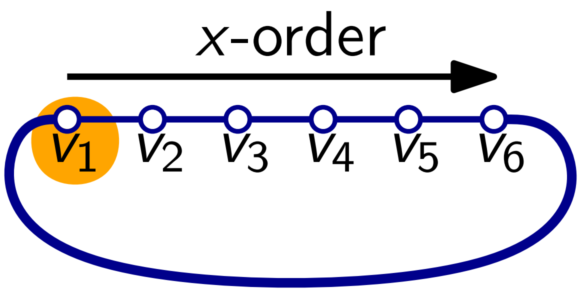

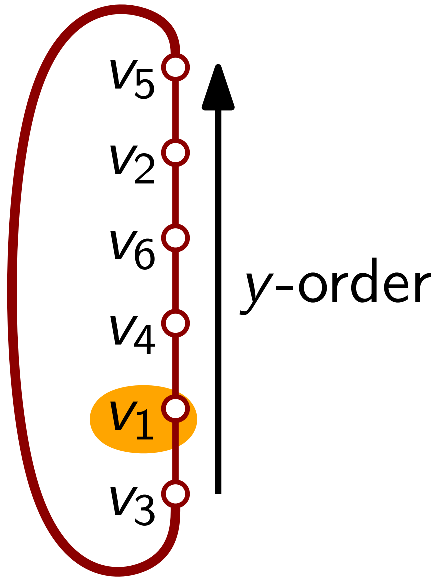

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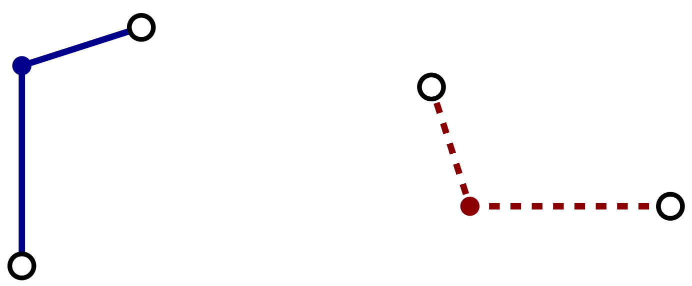

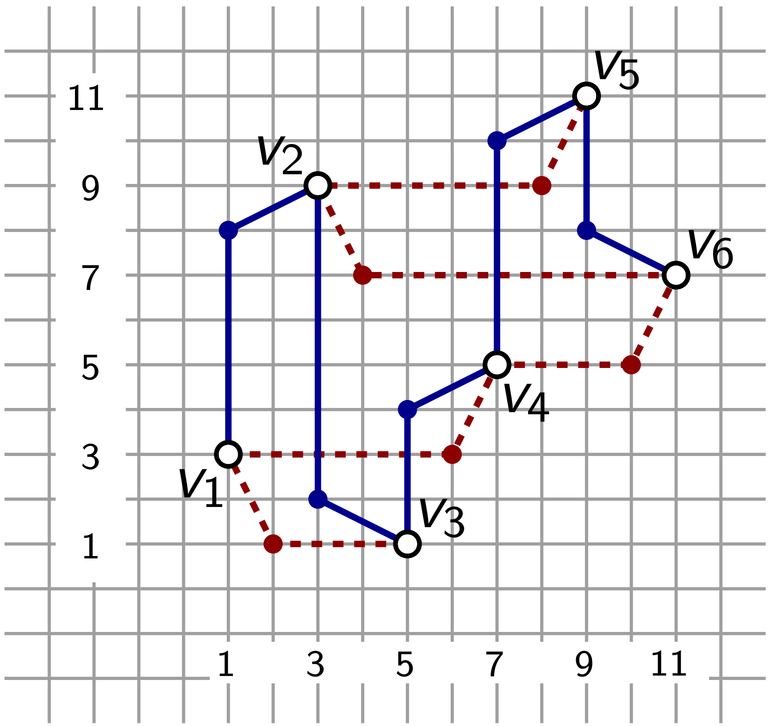

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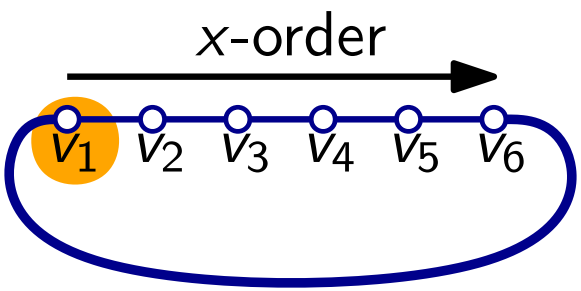

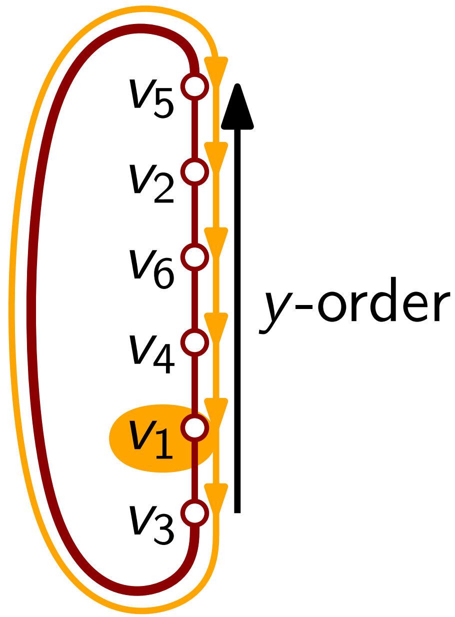

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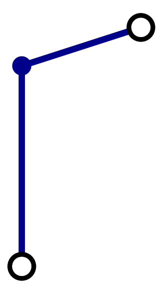

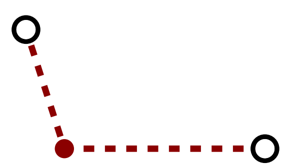

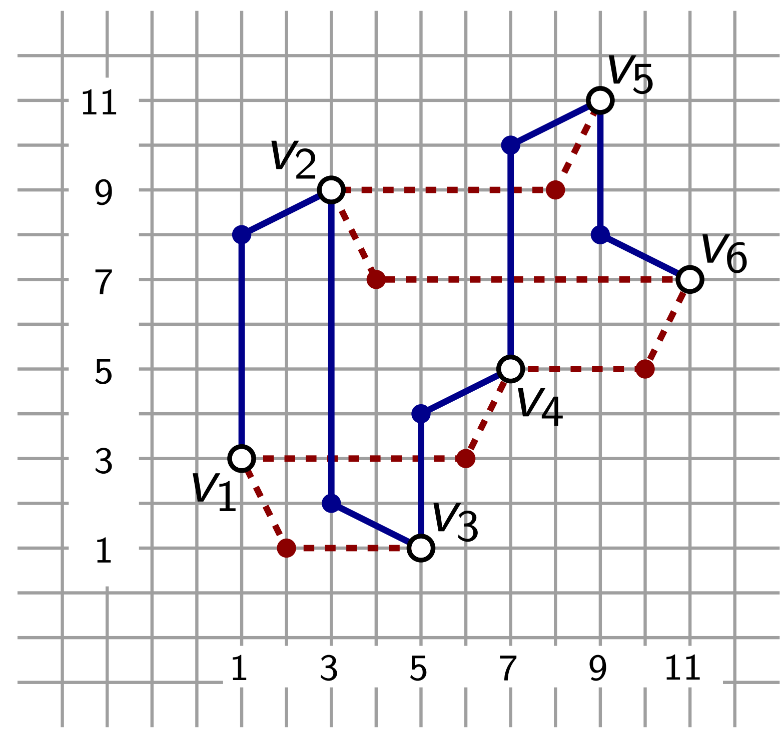

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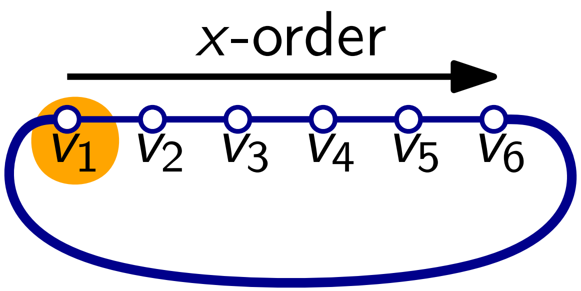

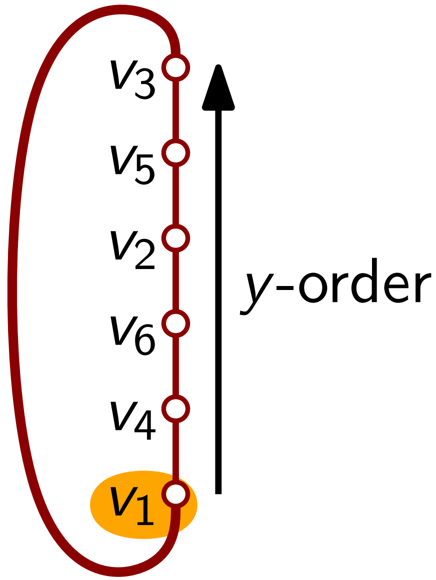

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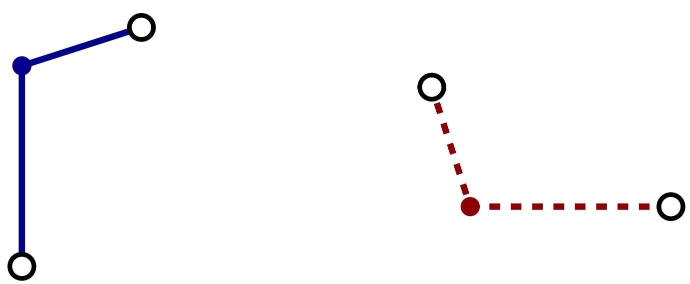

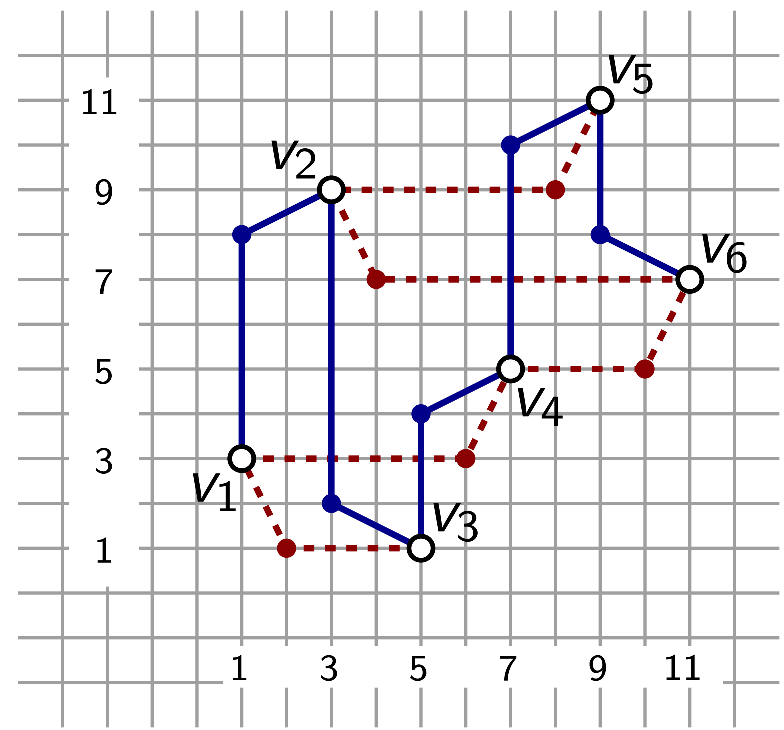

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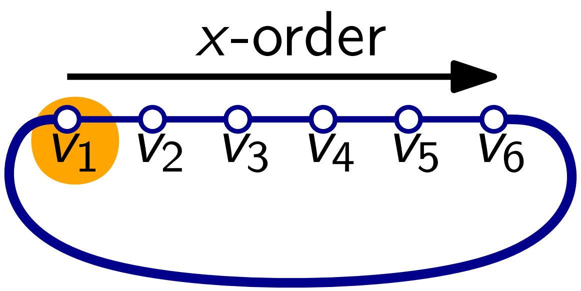

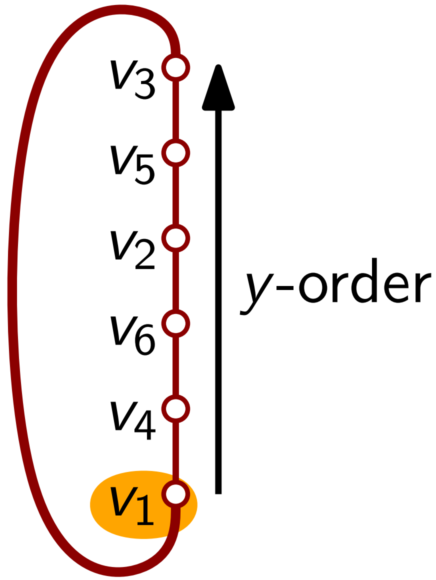

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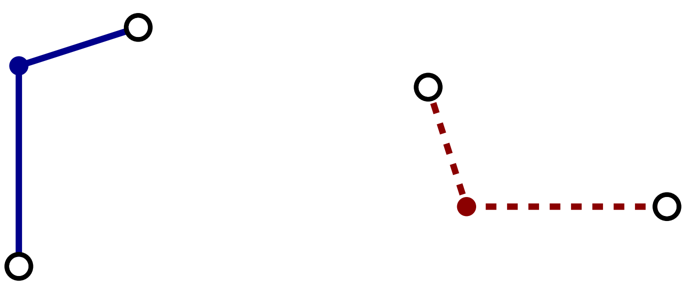

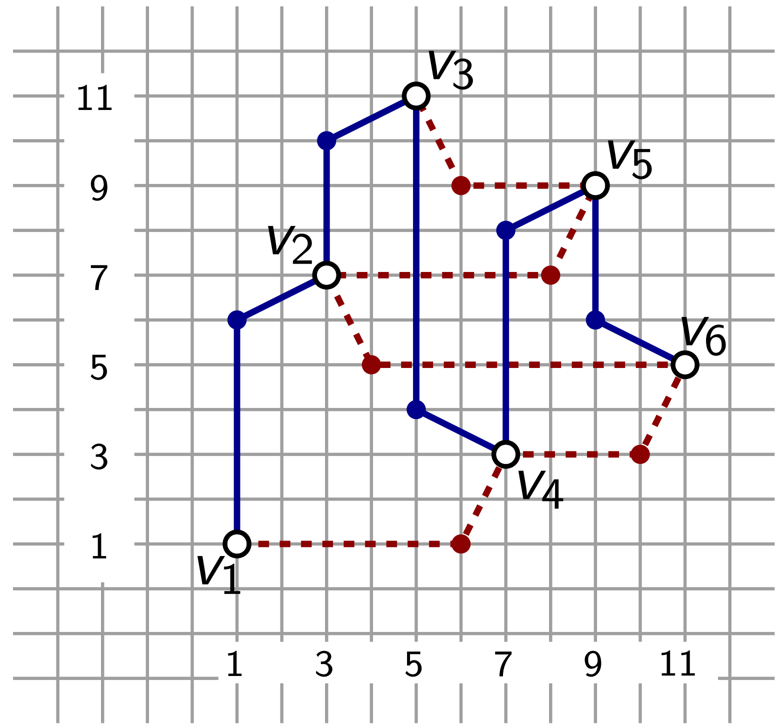

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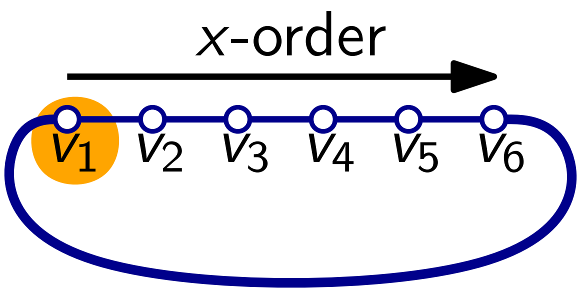

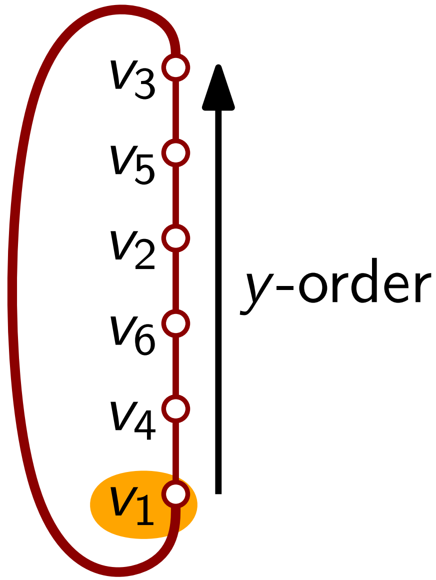

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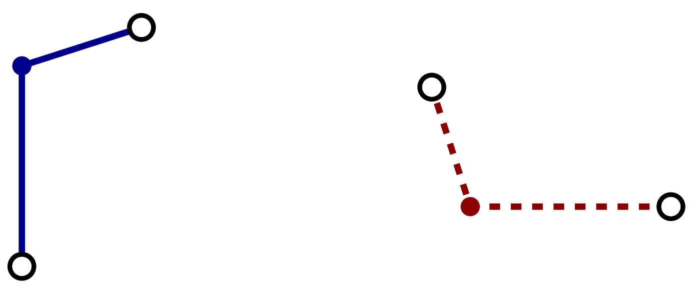

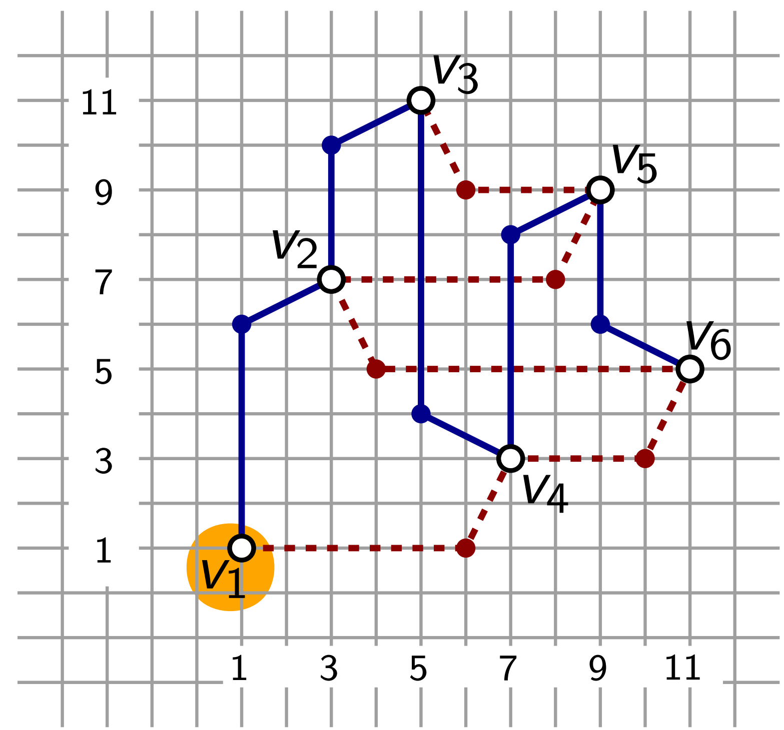

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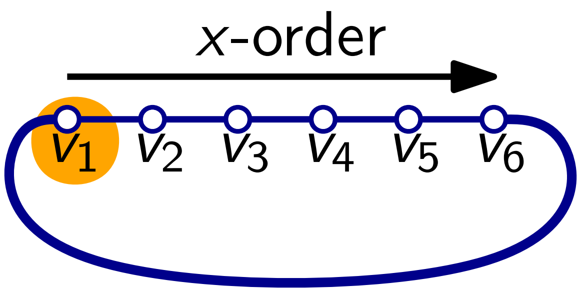

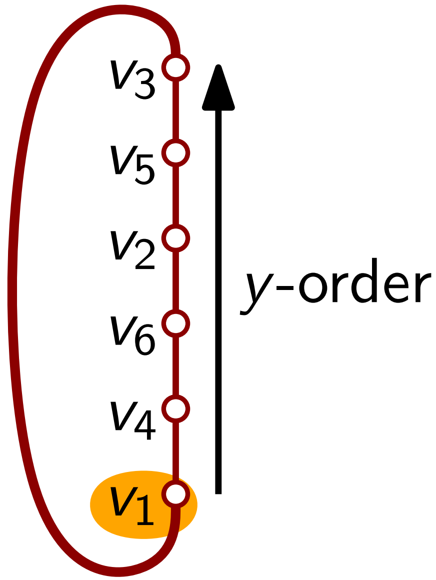

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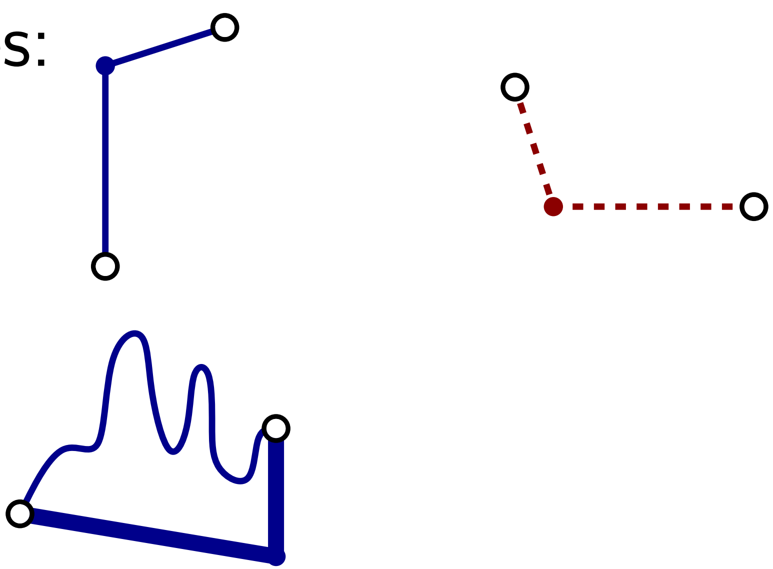

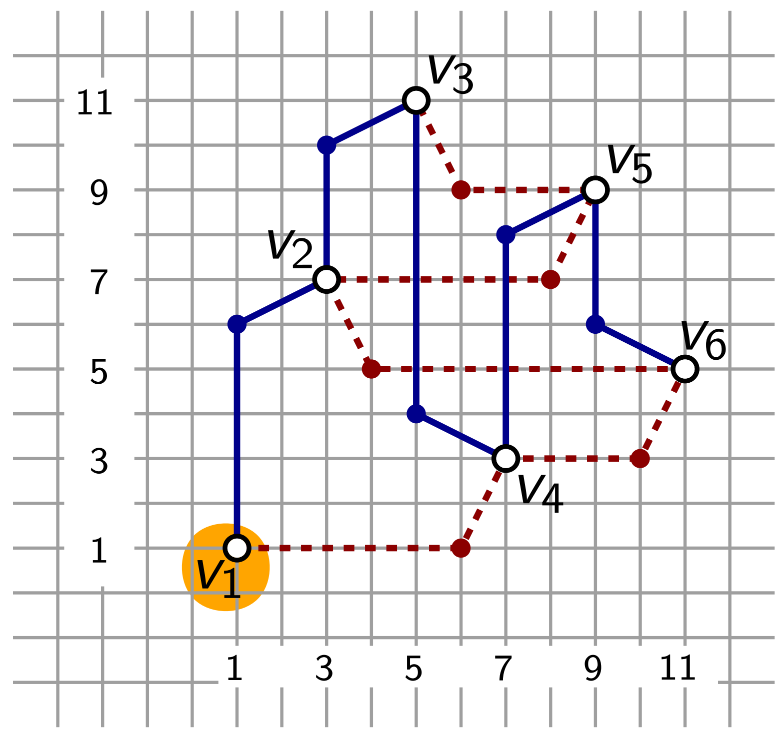

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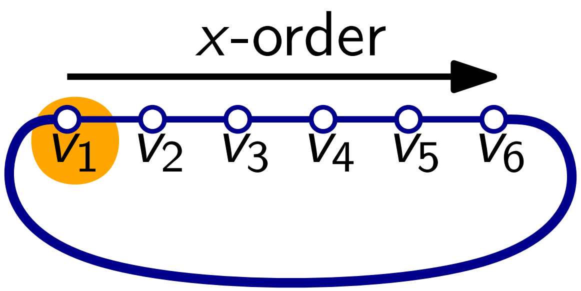

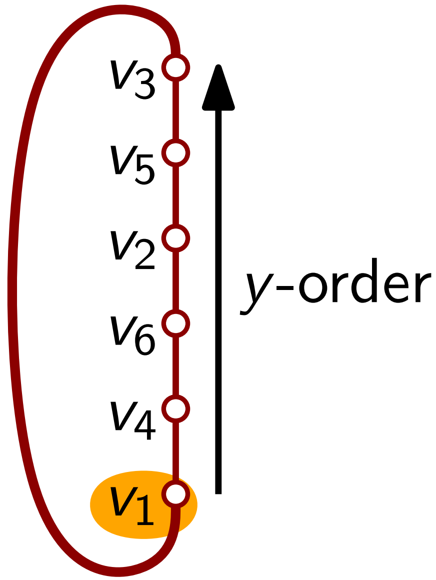

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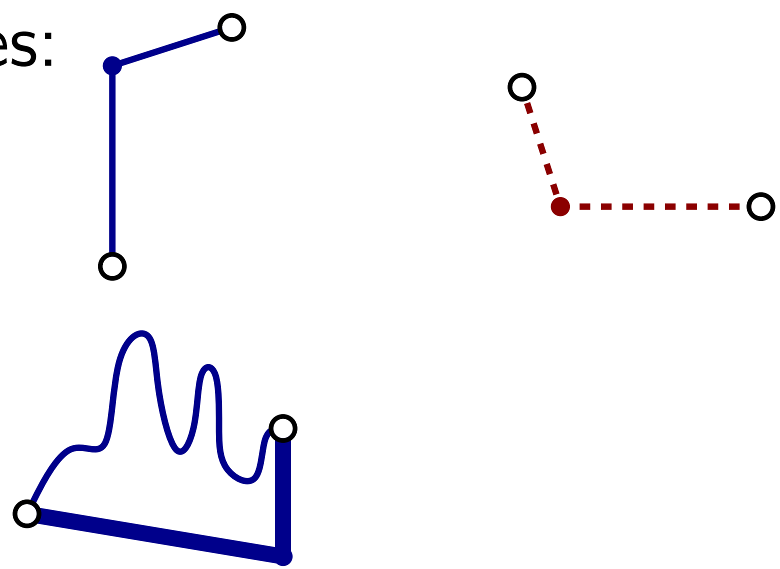

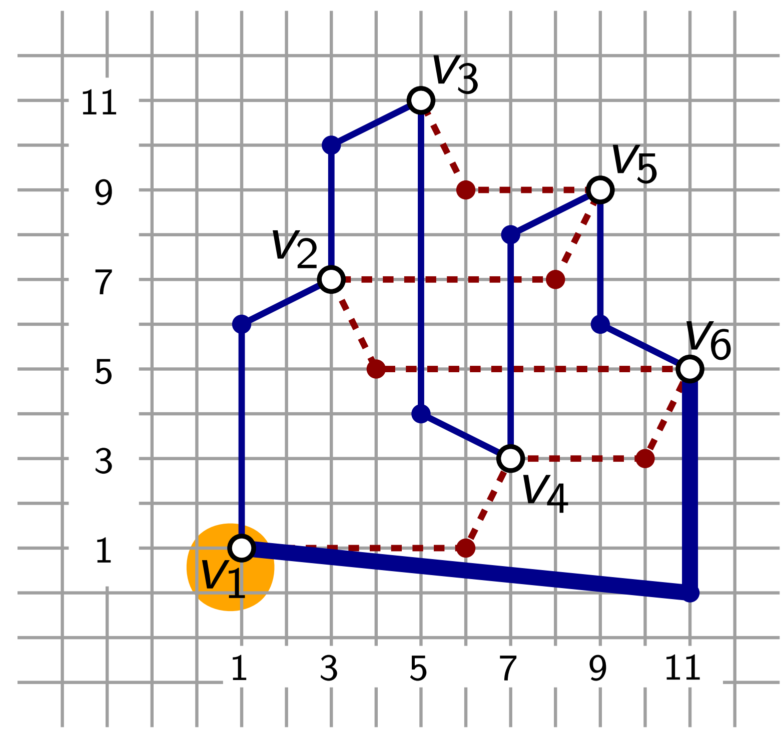

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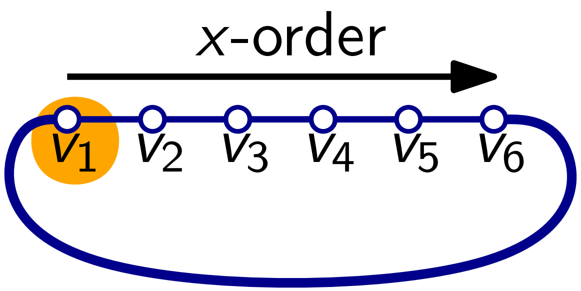

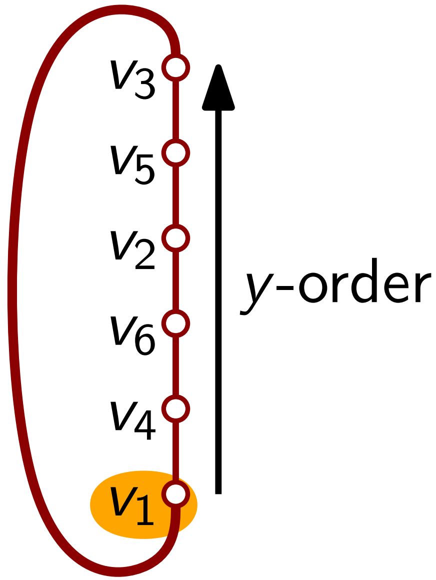

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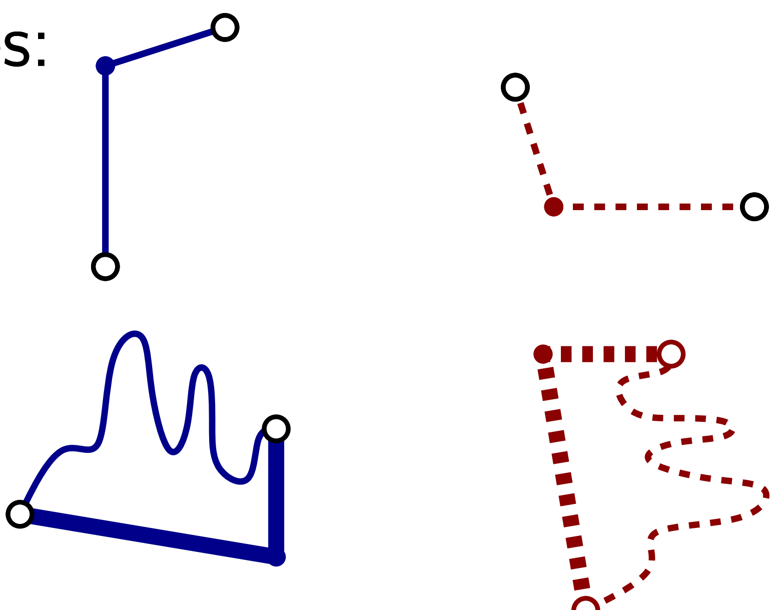

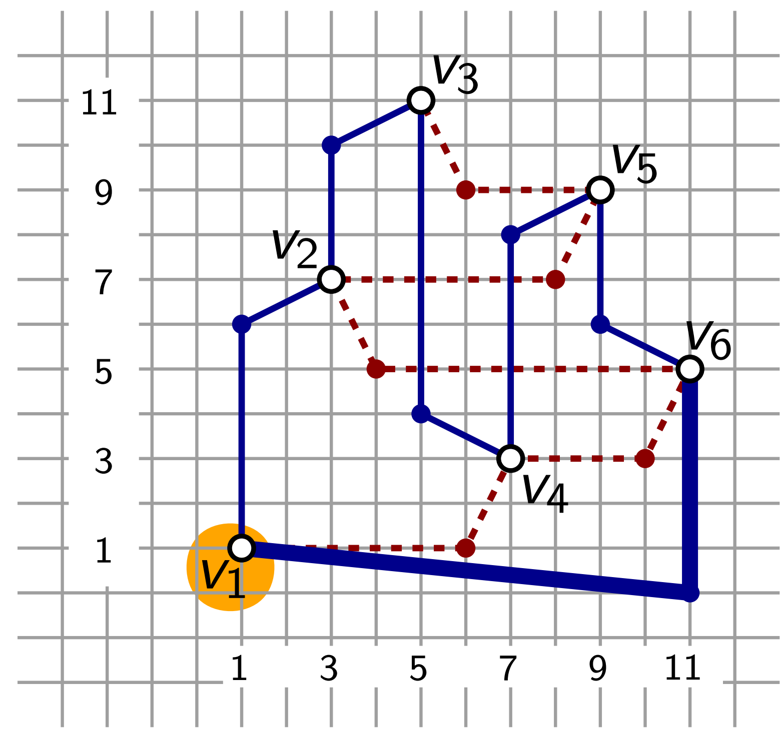

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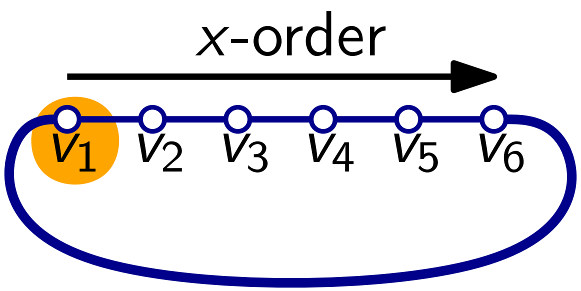

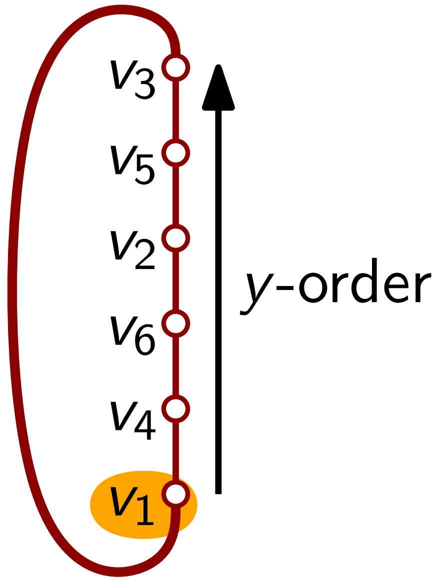

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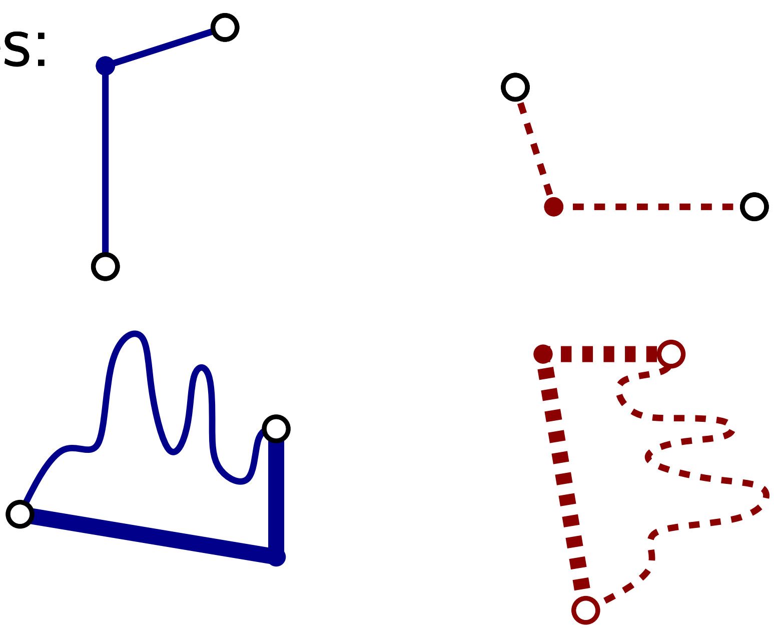

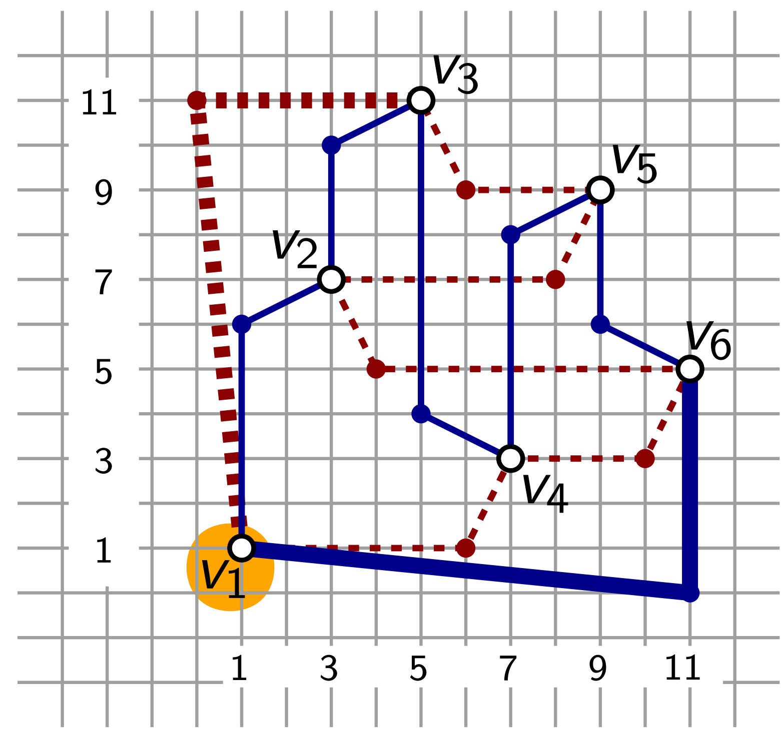

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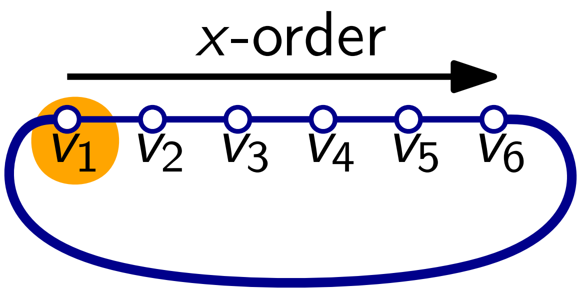

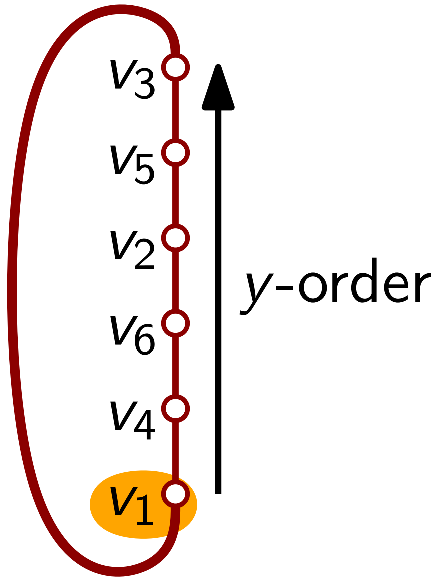

Bends: 1×1
Grid size: $(2n-1)^2$

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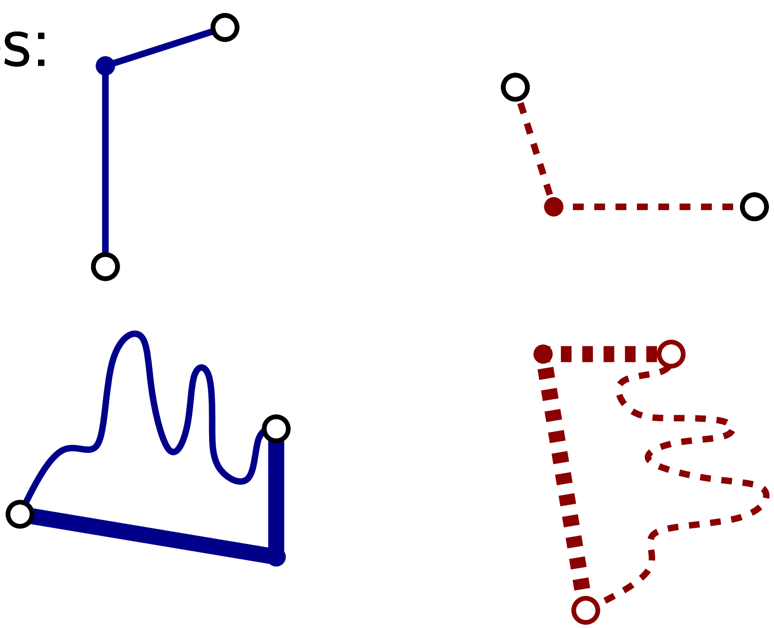
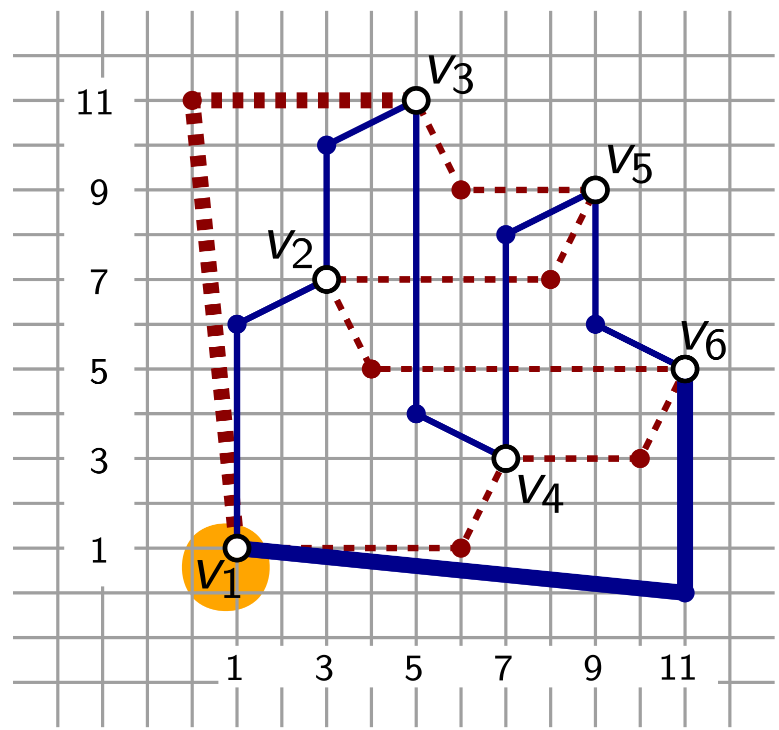
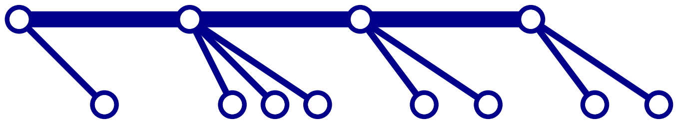
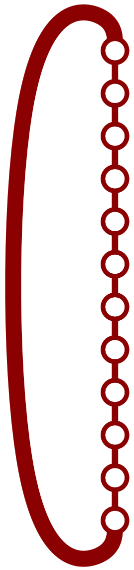




Caterpillar × Cycle





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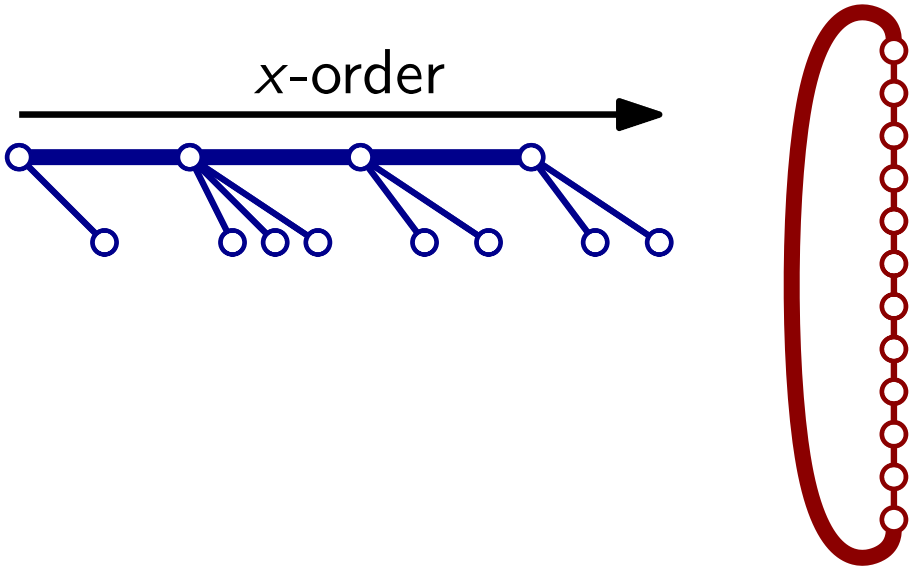

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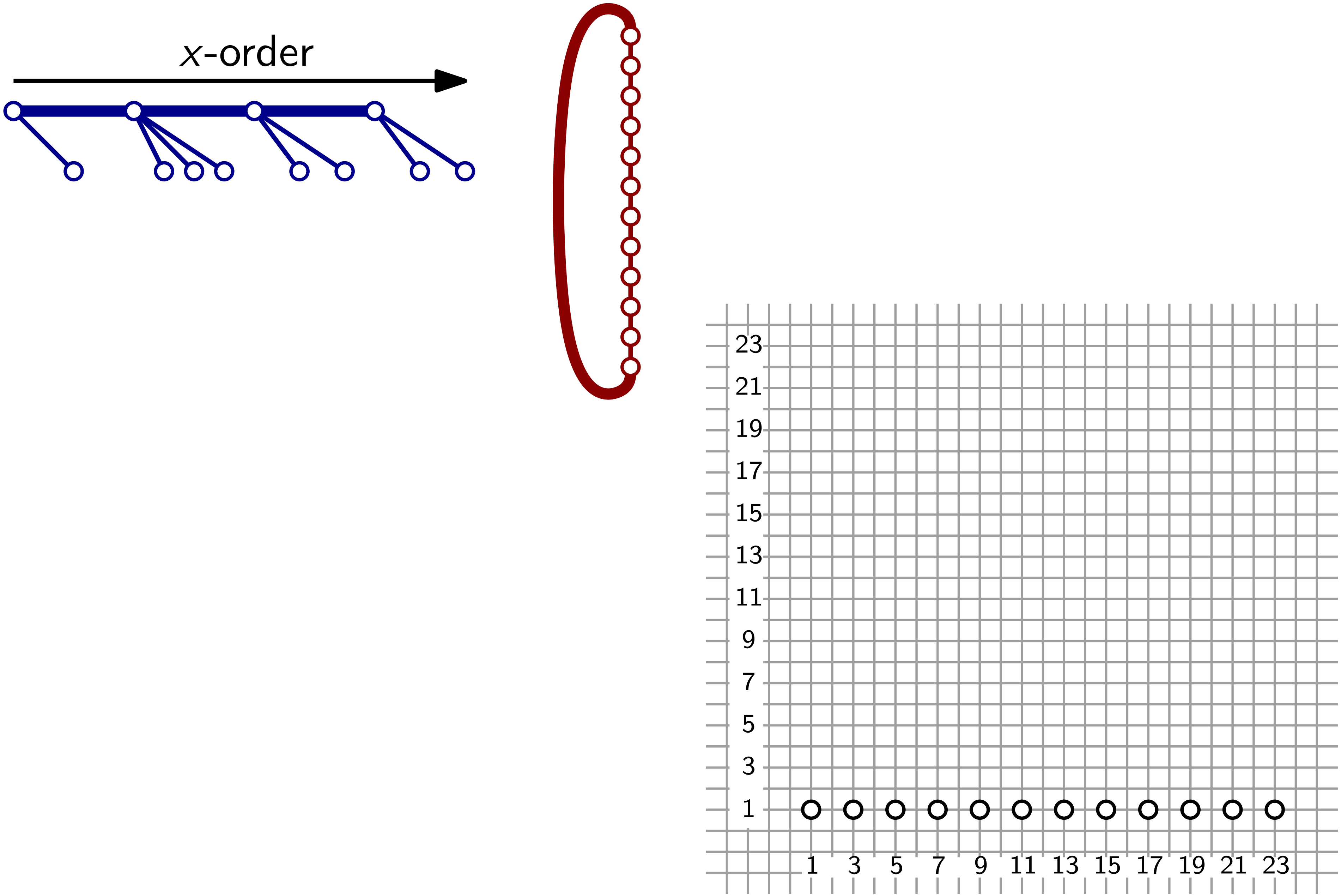

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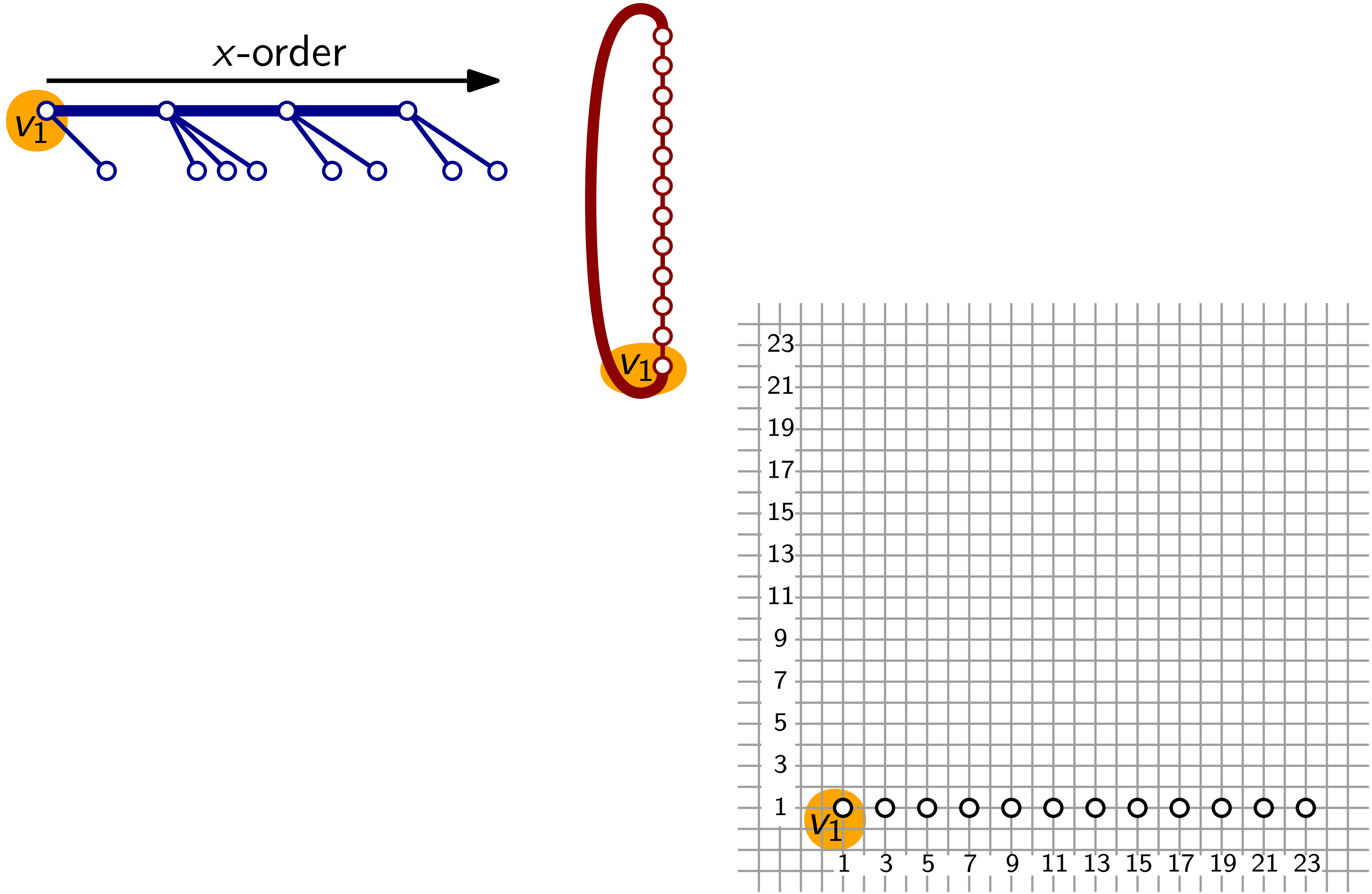

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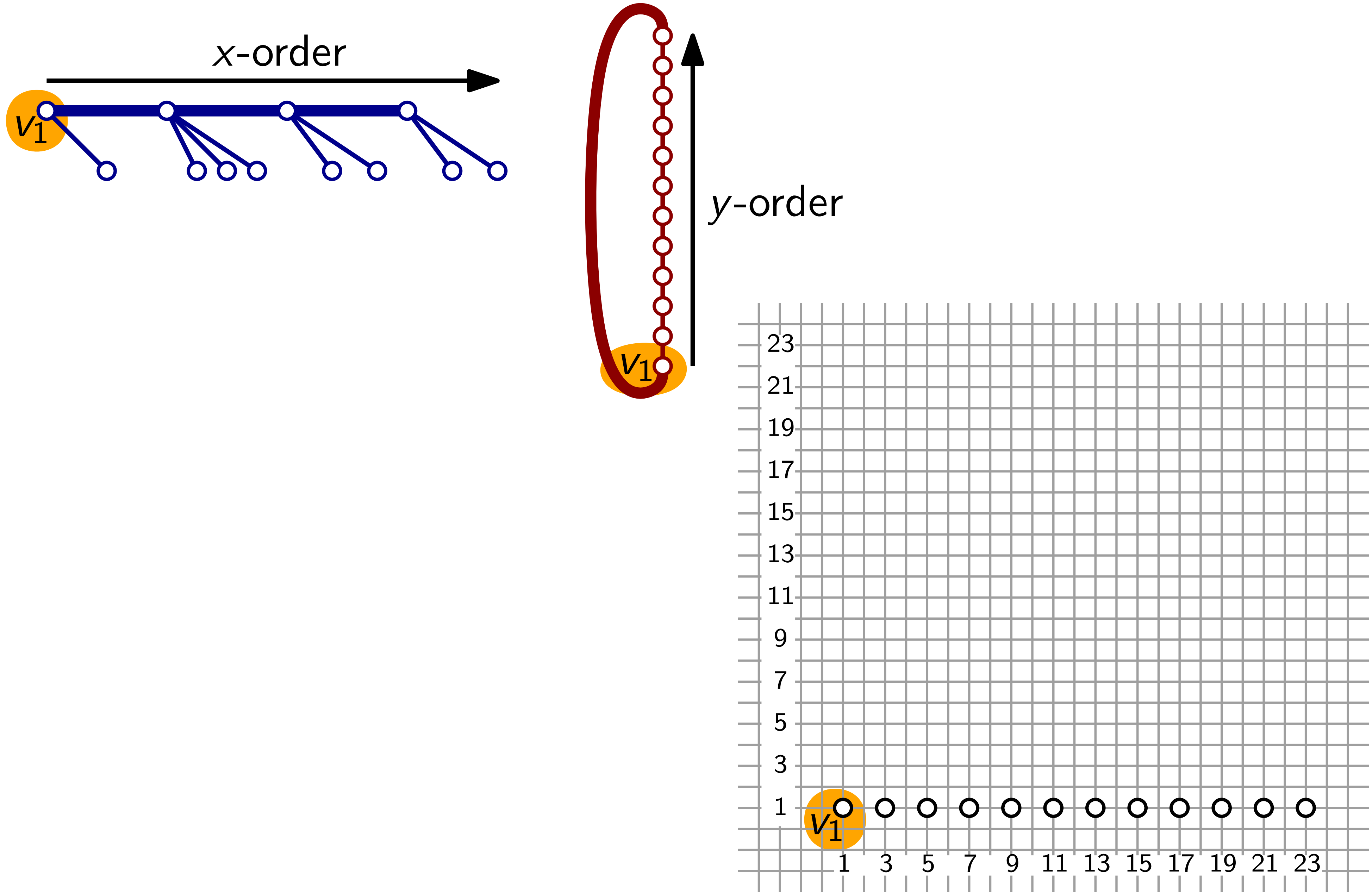

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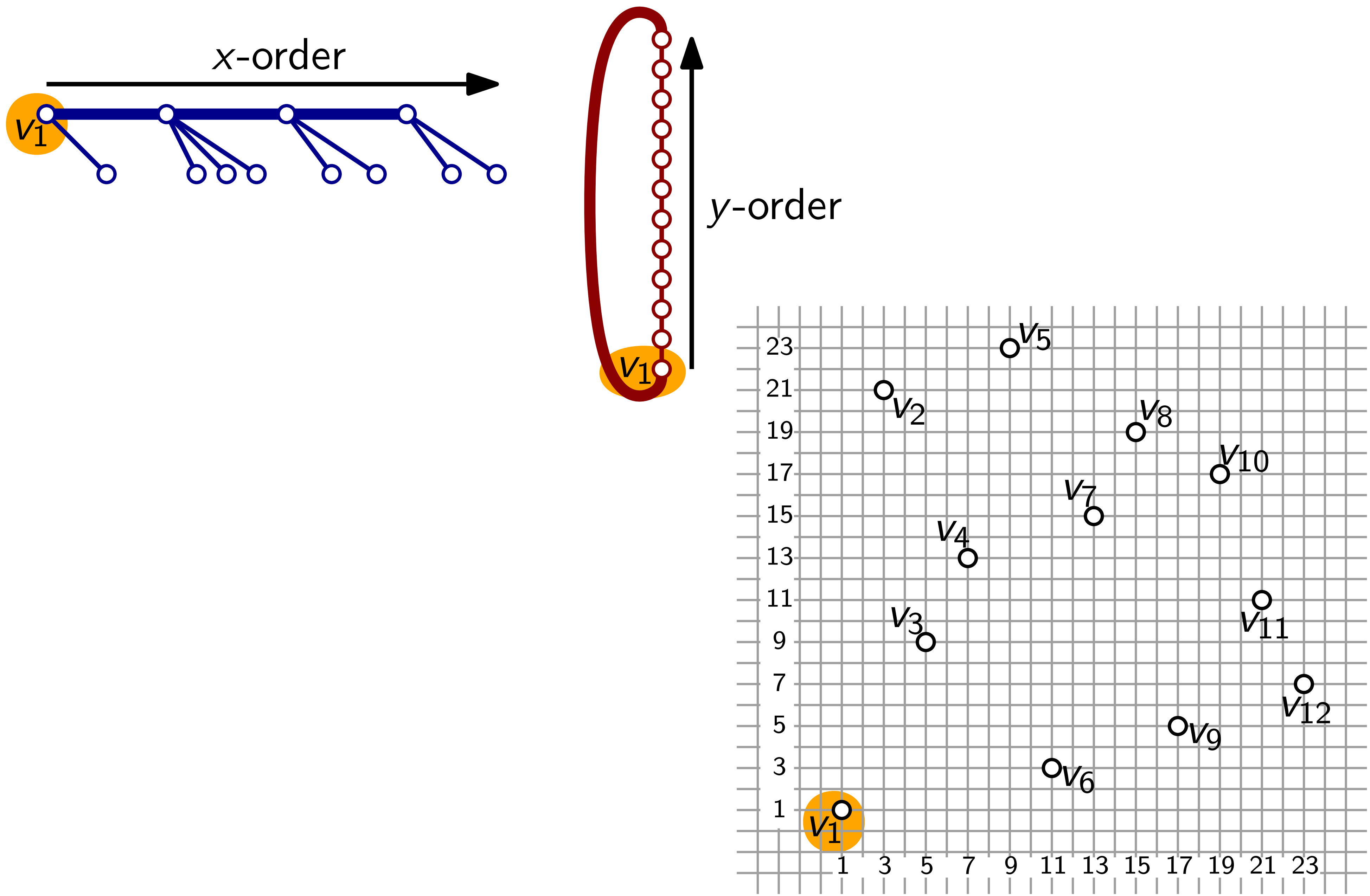

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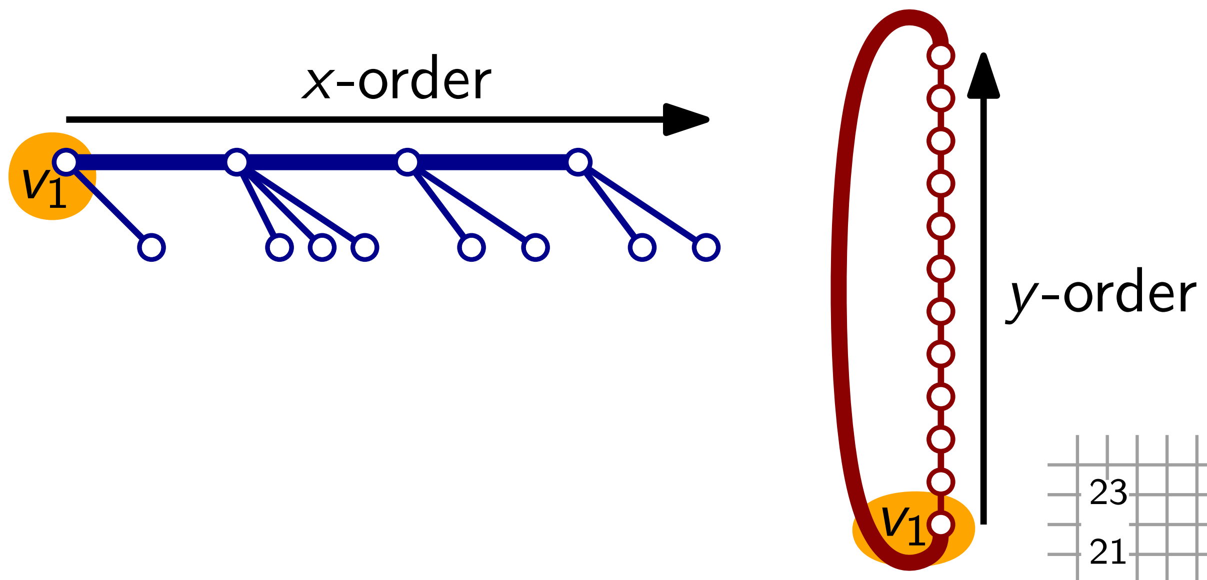

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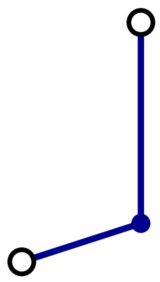

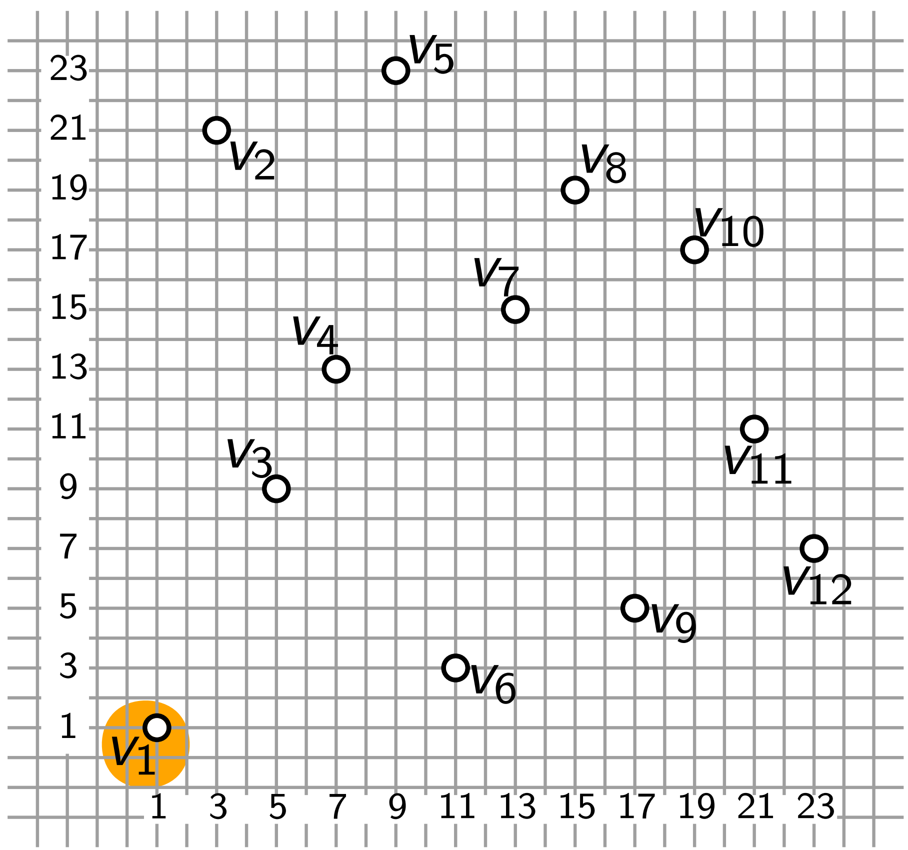

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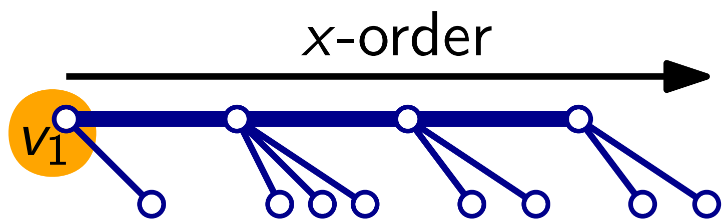

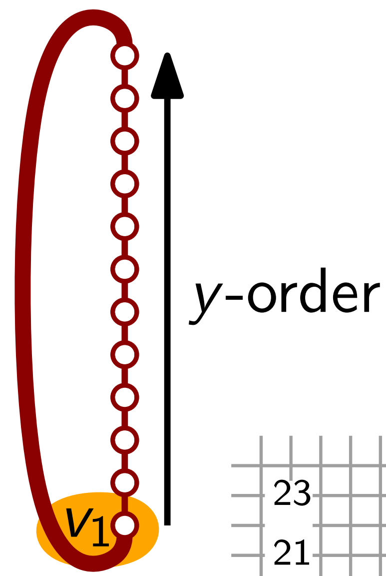

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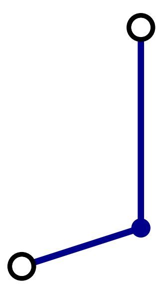

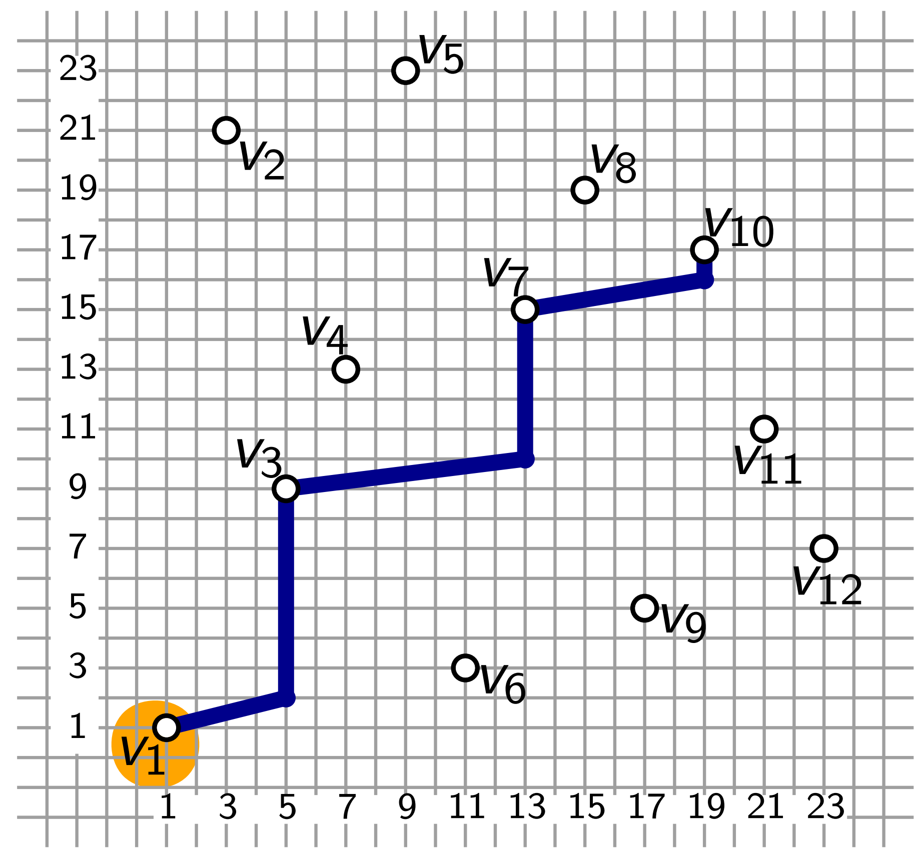

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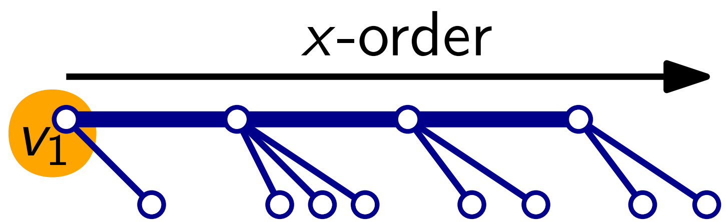

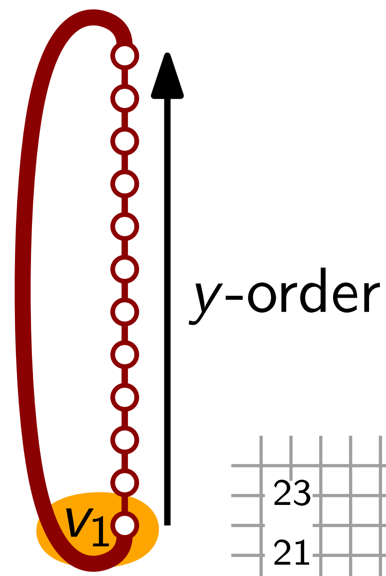

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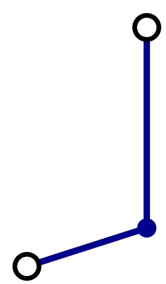
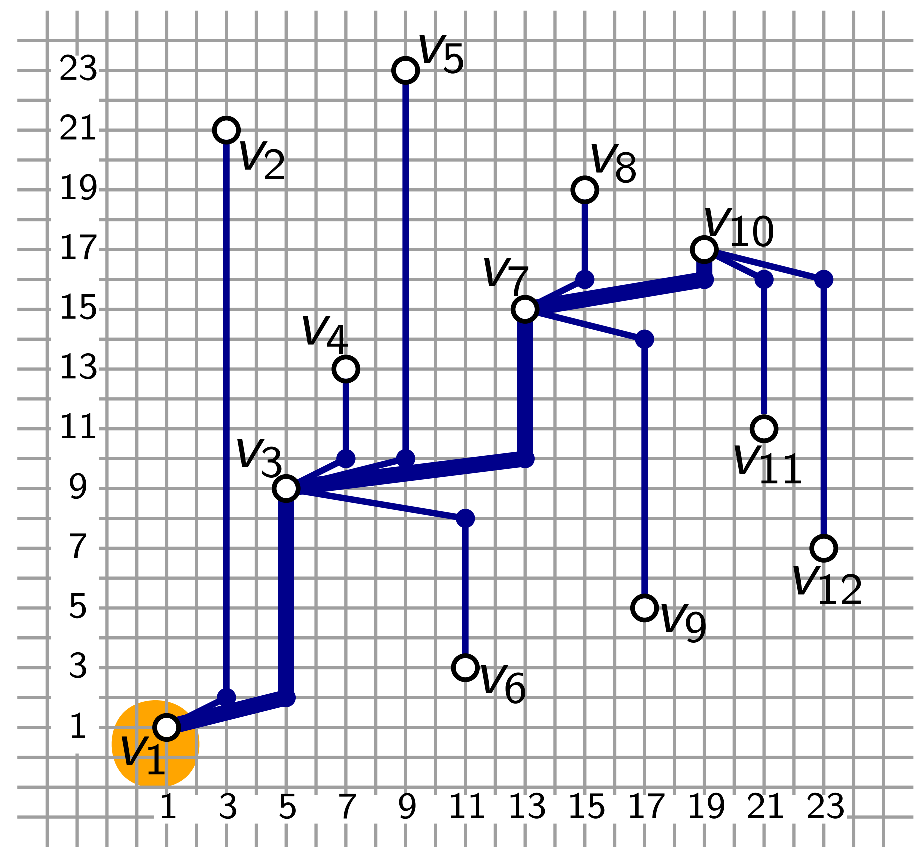
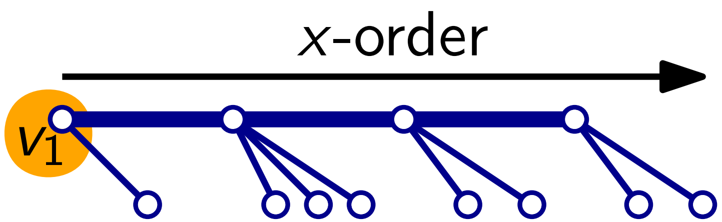
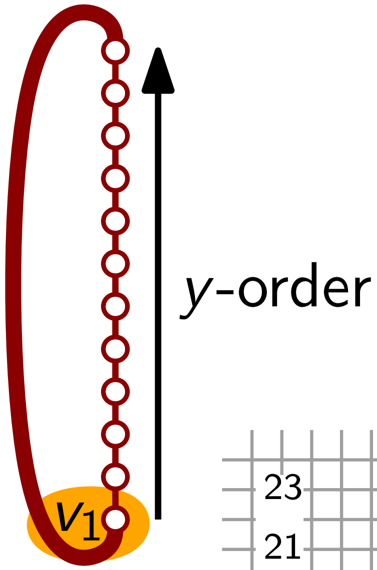




Caterpillar × Cycle





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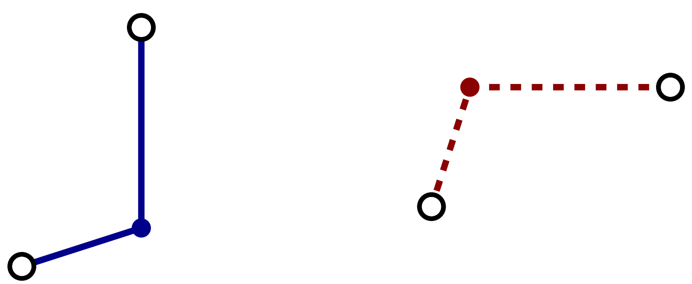
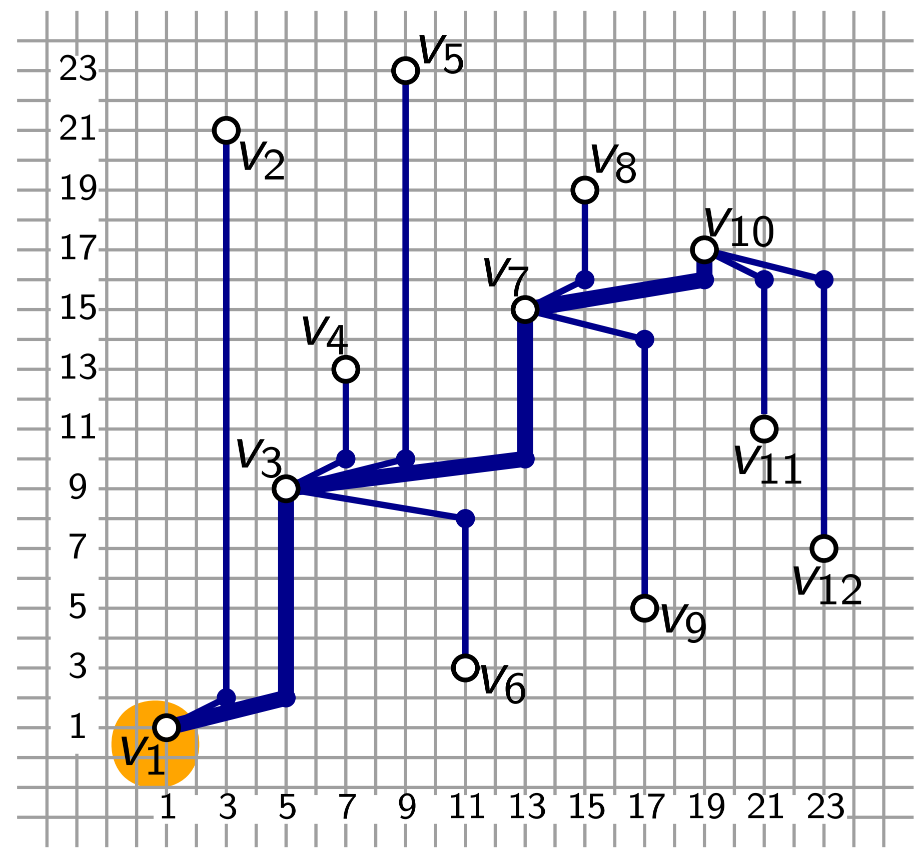
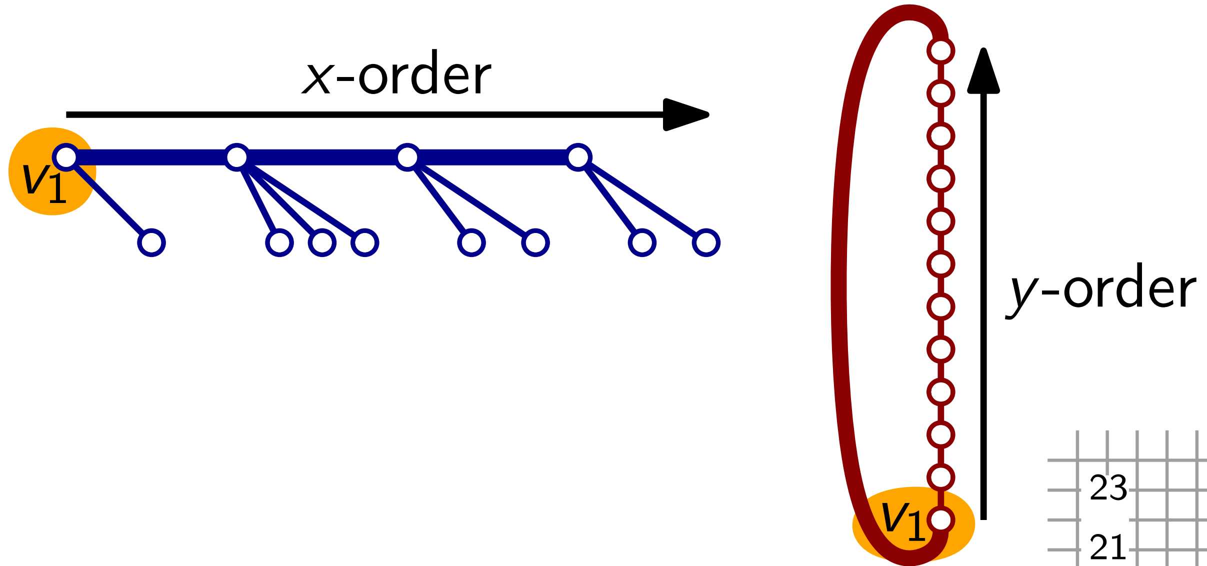




Caterpillar × Cycle



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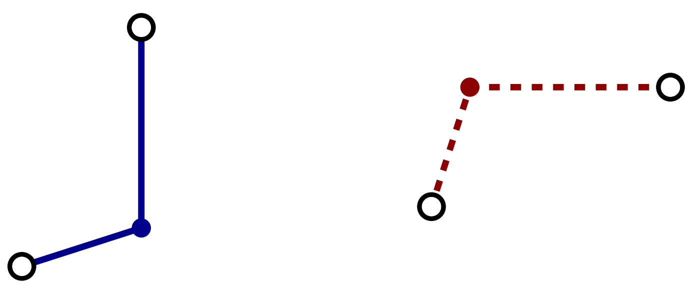
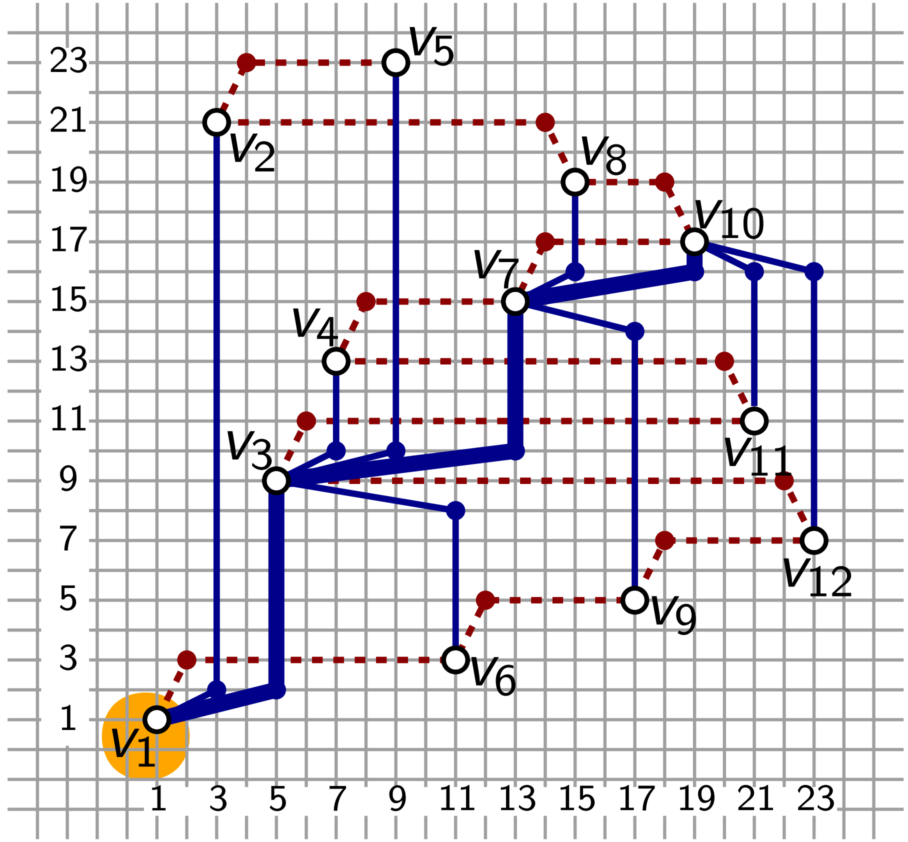
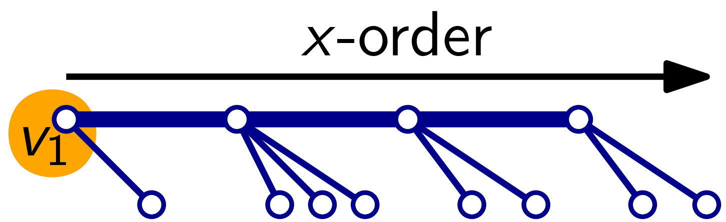
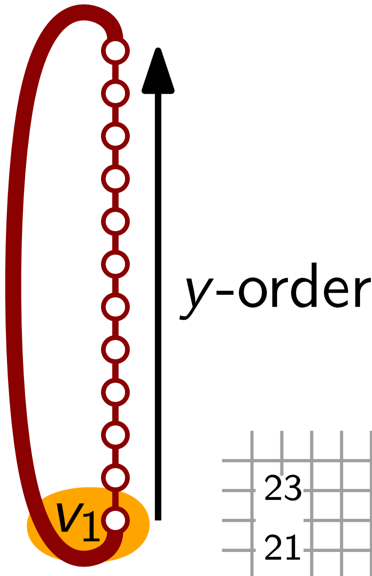




Caterpillar × Cycle





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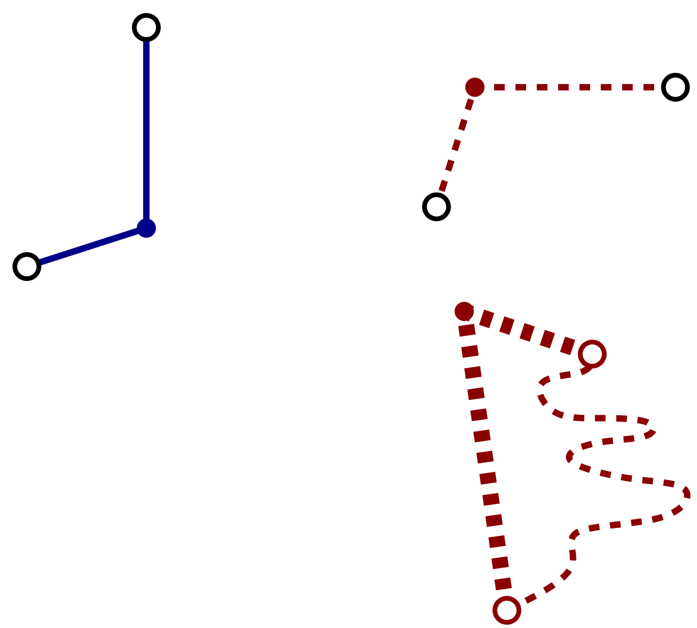
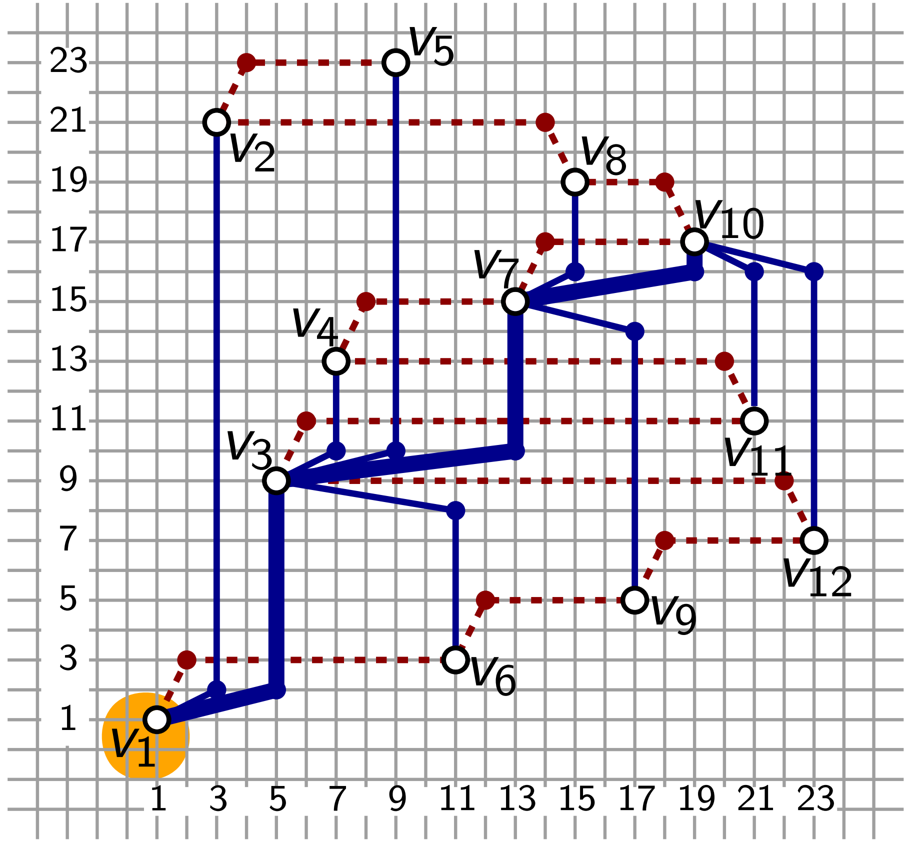
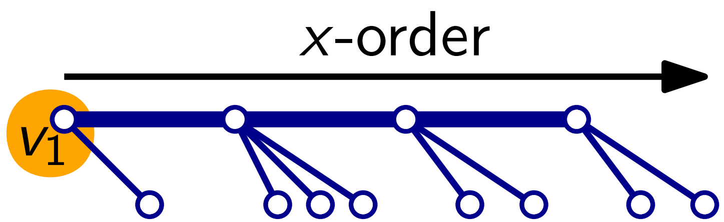
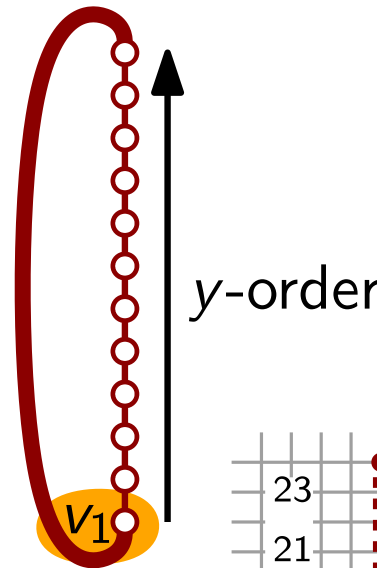




Caterpillar × Cycle





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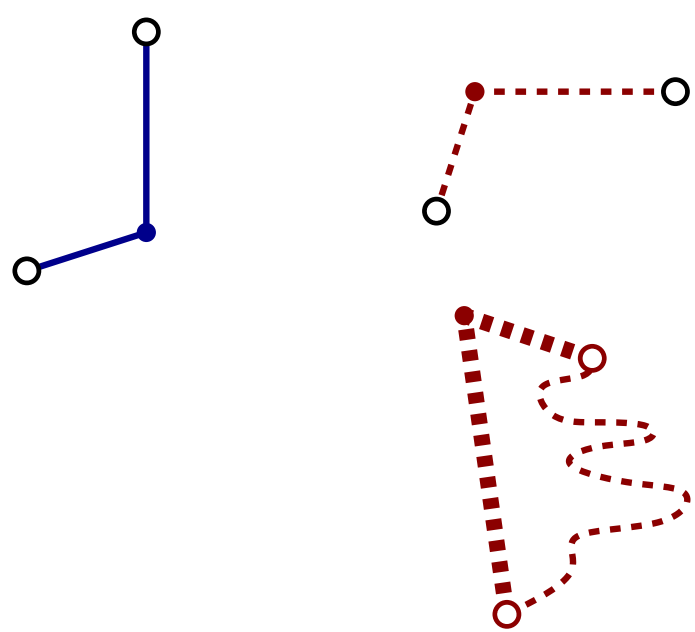
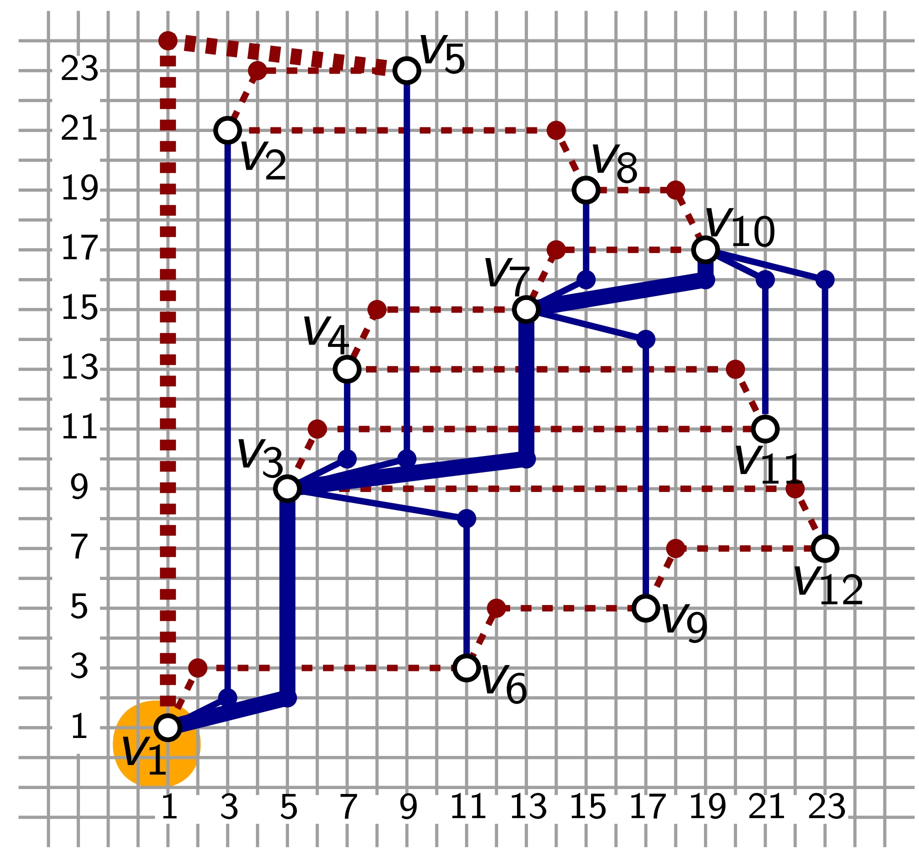
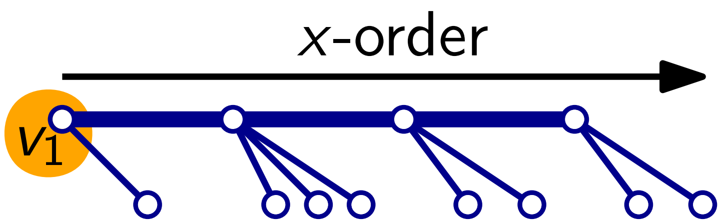
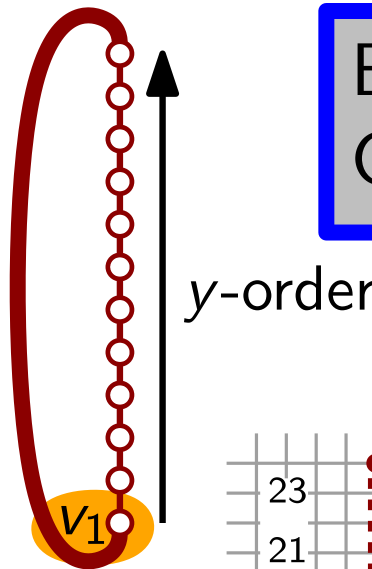


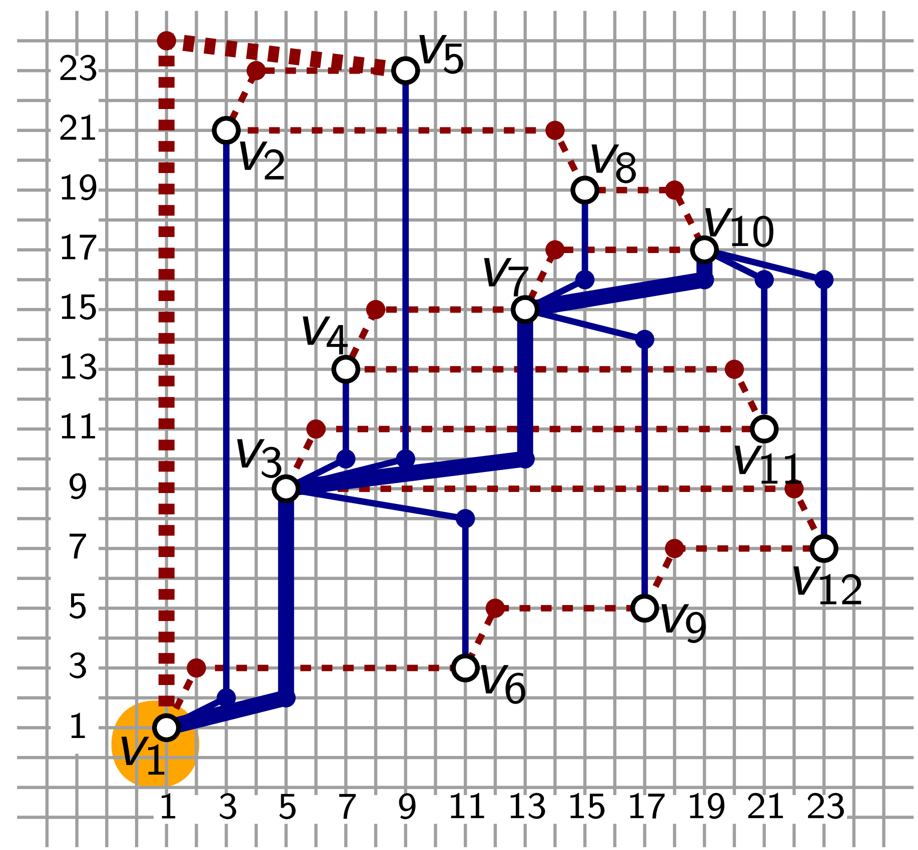


Caterpillar × Cycle

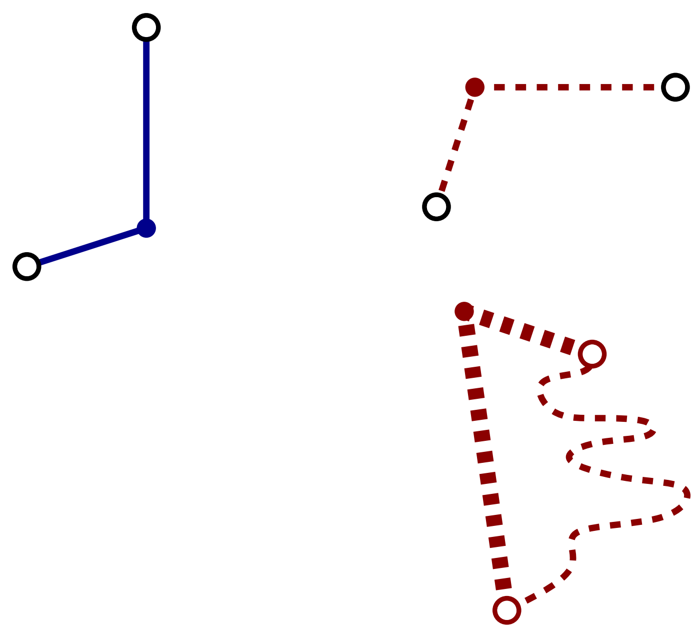




Bends: 1×1
Grid size: $(2n-1) \times 2n$



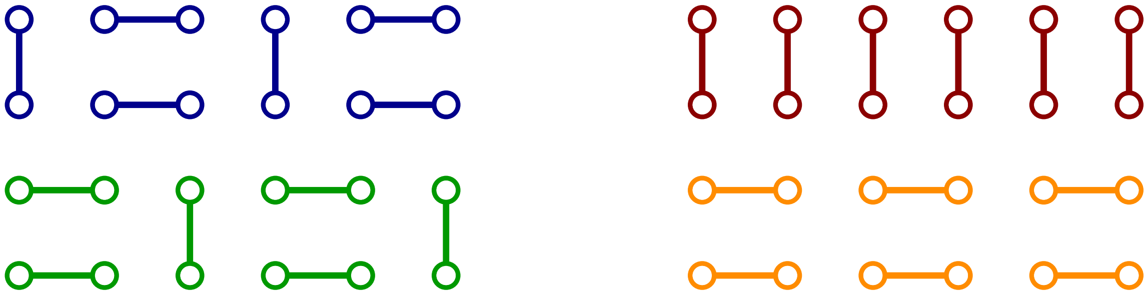
Edges:



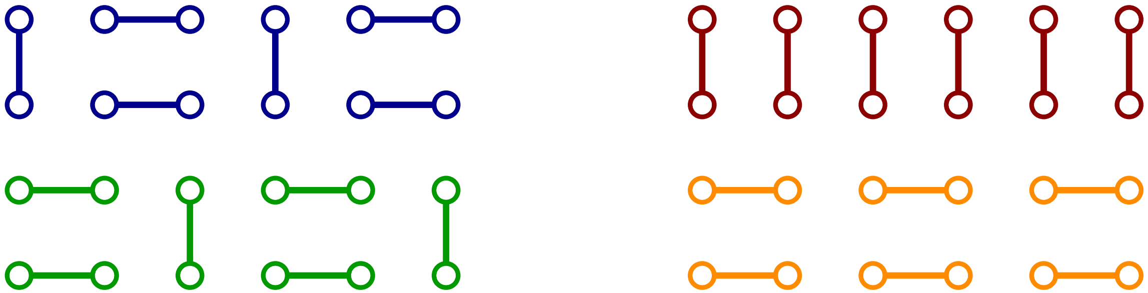
Overview

Graph classes			Number of bends
Cycle	$\times$	Cycle	1×1 ✓
Caterpillar	$\times$	Cycle	1×1 ✓
Four Matchings			$1 \times 1 \times 1 \times 1$
Tree	$\times$	Matching	1×0
Wheel	$\times$	Matching	2×0
Outerpath	$\times$	Matching	2×1
Outerplanar	$\times$	Outerplanar	3×3
2-page book emb.	$\times$	2-page book emb.	4×4
Planar	$\times$	Planar	6×6

Four Matchings



Four Matchings



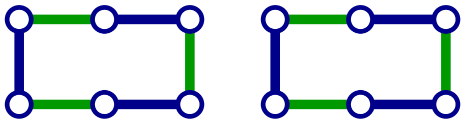
Combine $\Rightarrow$ two sets of cycles $\mathcal{C}_1$, $\mathcal{C}_2$

Four Matchings

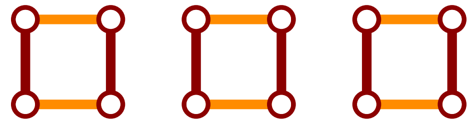
$\mathcal{C}_1$ $\mathcal{C}_2$

Combine $\Rightarrow$ two sets of cycles $\mathcal{C}_1$, $\mathcal{C}_2$

Four Matchings

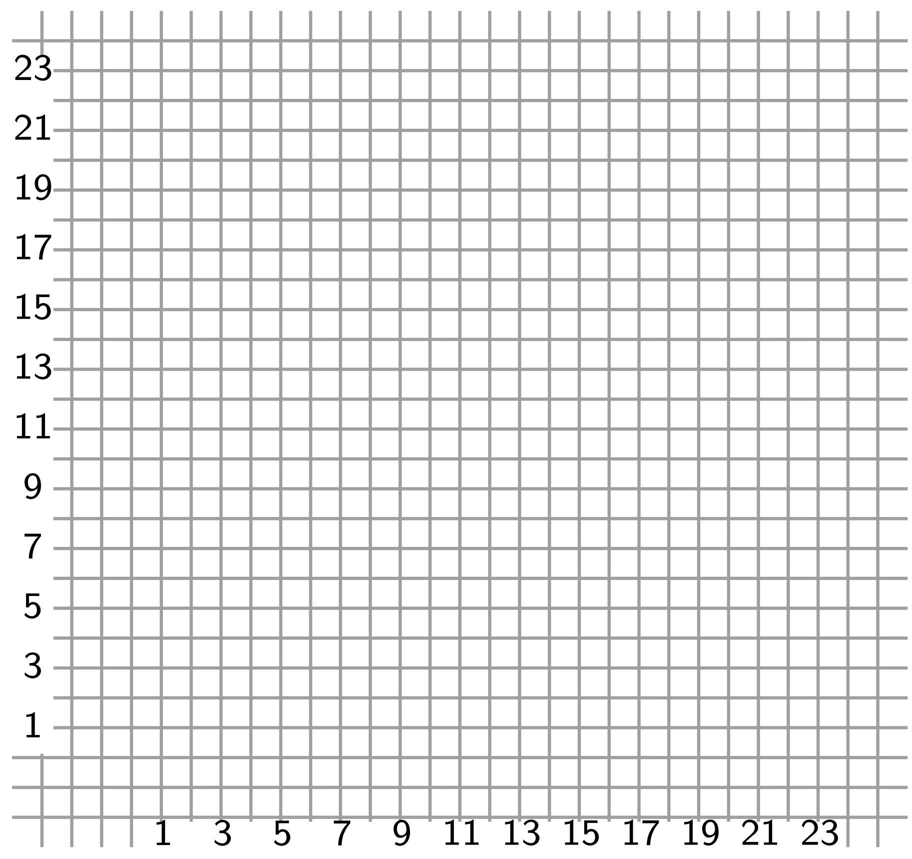


$\mathcal{C}_1$



$\mathcal{C}_2$

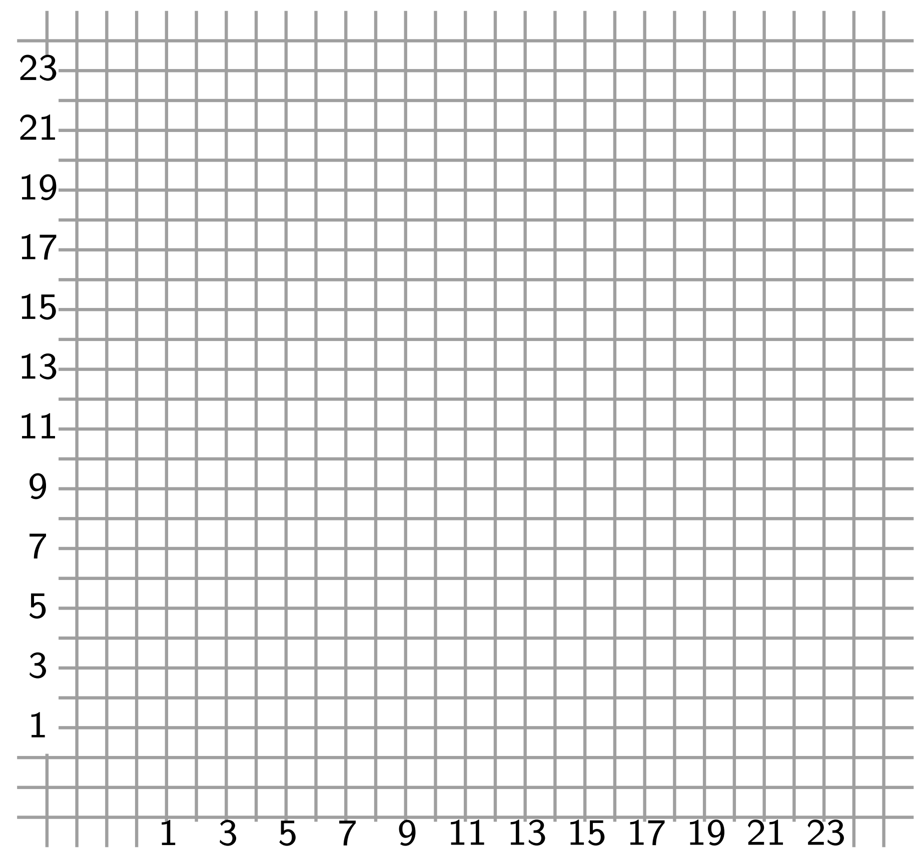
Placement algorithm:



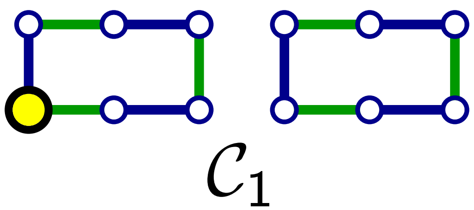
Four Matchings

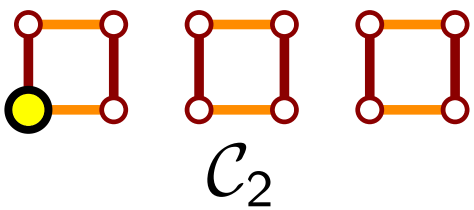
Placement algorithm:

- Pick v_1



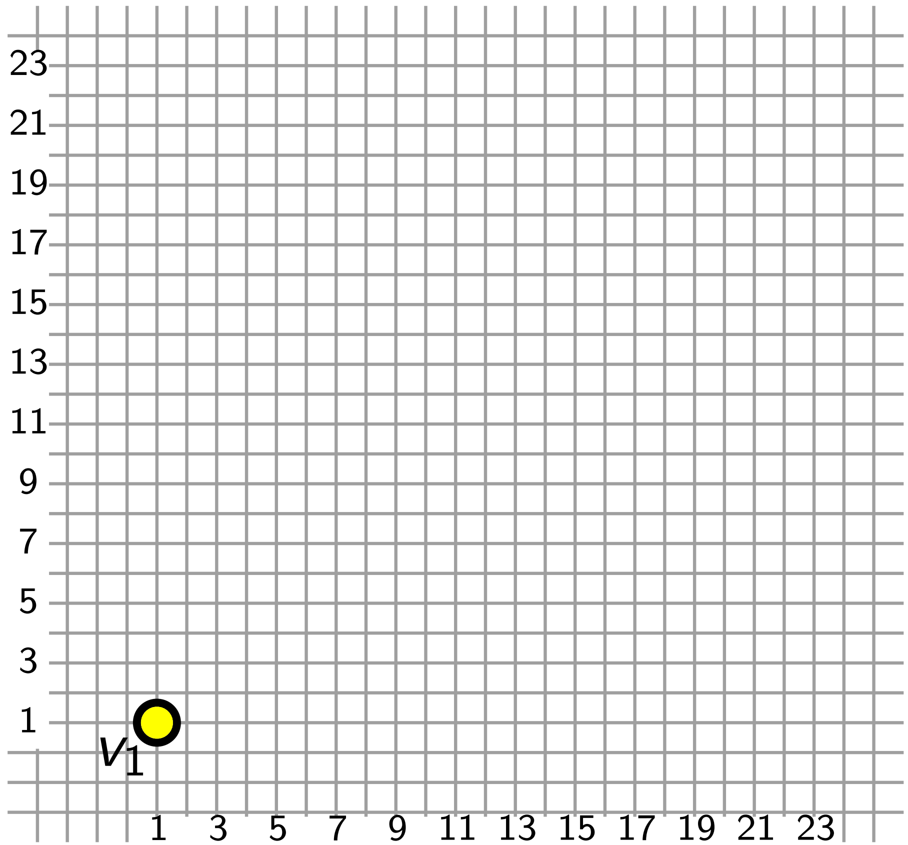
Four Matchings





Placement algorithm:

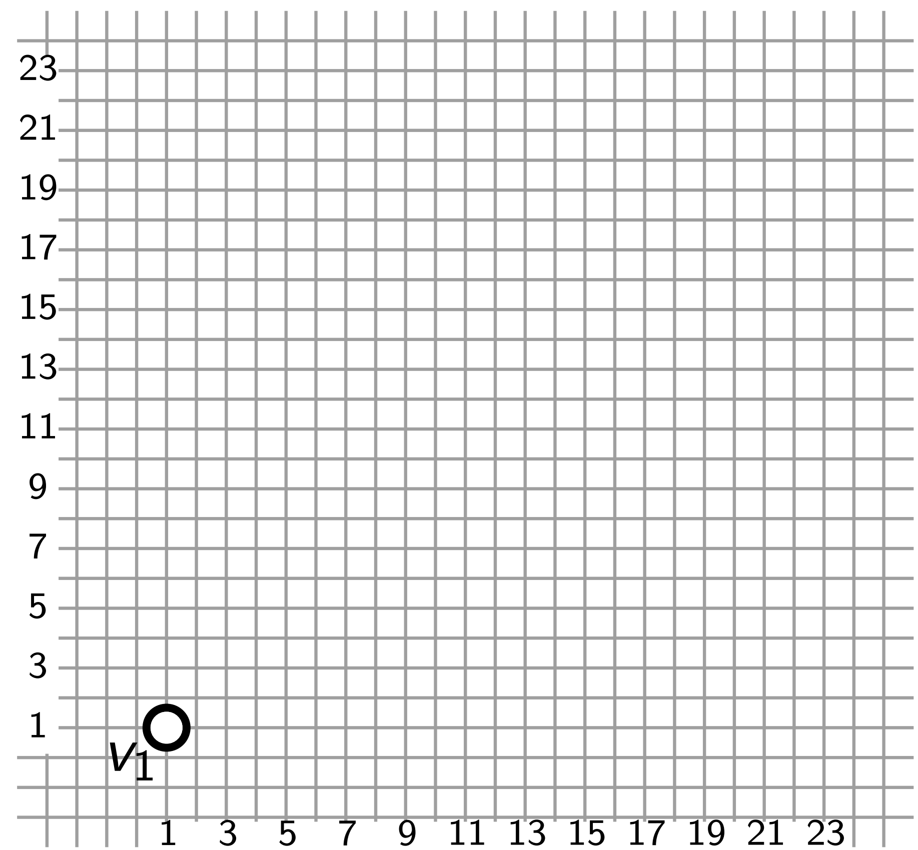
- Pick v_1, place it



Four Matchings

Placement algorithm:

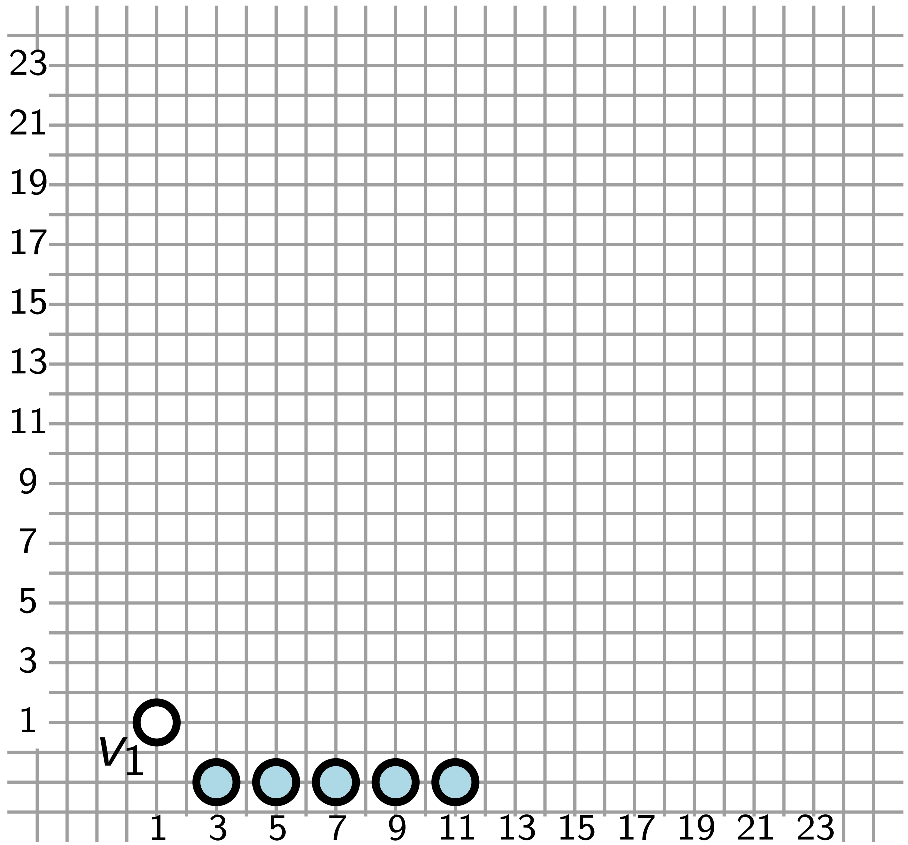
- Pick v_1, place it



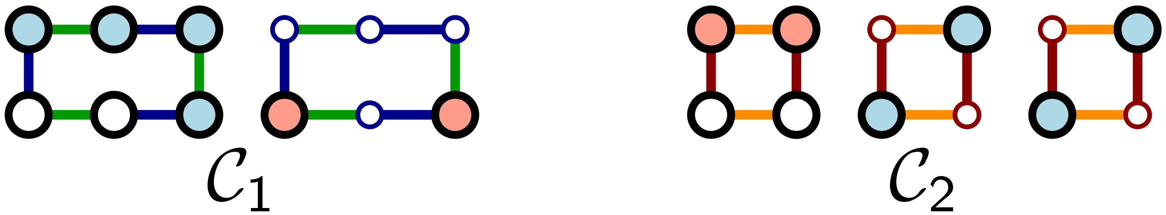
Four Matchings

Placement algorithm:

- Pick v_1, place it
- Assign x-coords. to cycle in $\mathcal{C}_1$

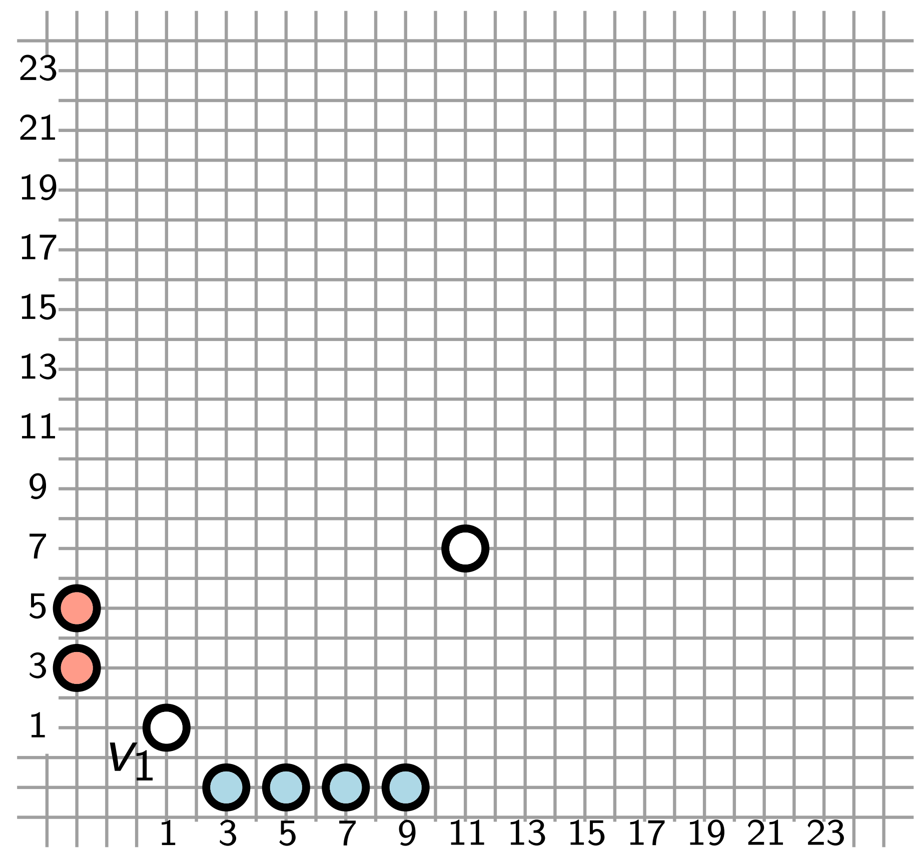


Four Matchings

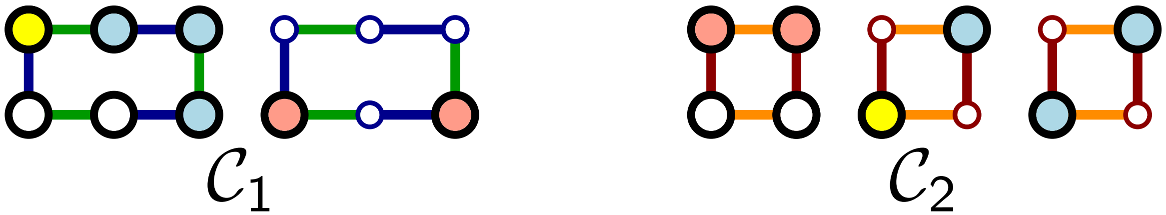


Placement algorithm:

- Pick v_1, place it
- Assign x-coords. to cycle in $\mathcal{C}_1$
- Assign y-coords. to cycle in $\mathcal{C}_2$

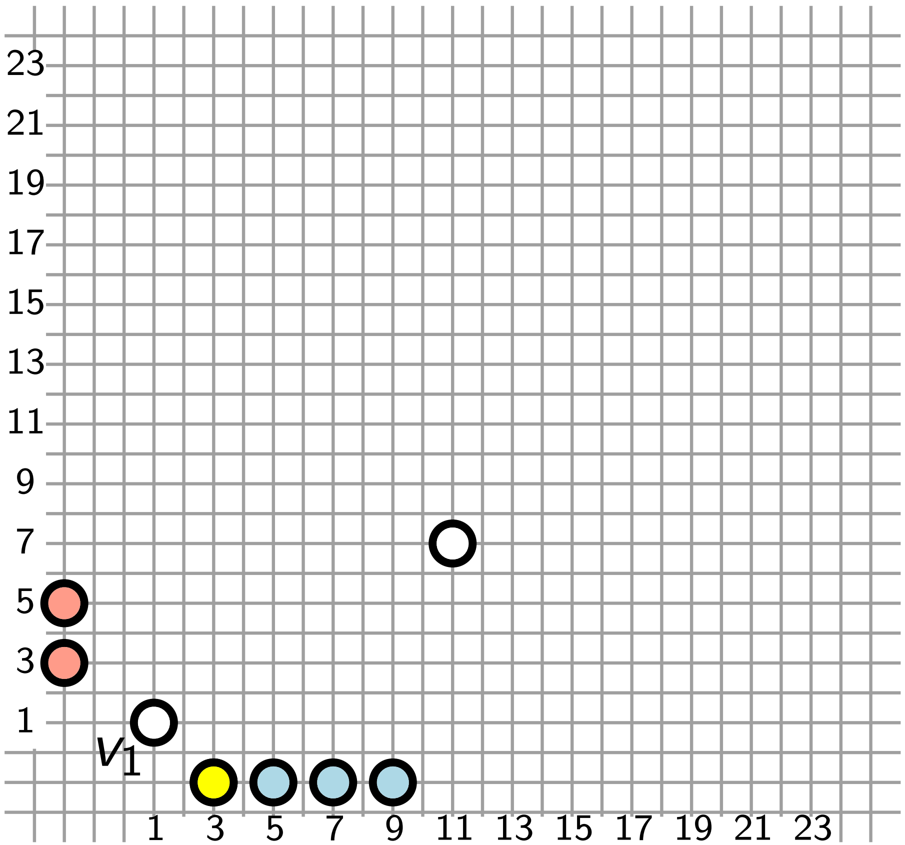


Four Matchings

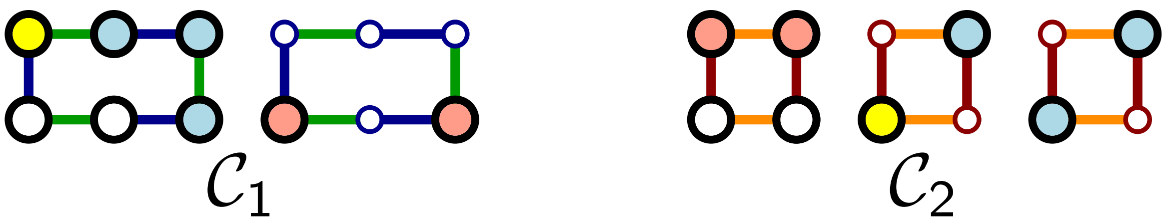


Placement algorithm:

- Pick v_1, place it
- Assign x-coords. to cycle in $\mathcal{C}_1$
- Assign y-coords. to cycle in $\mathcal{C}_2$
- Pick v with 1 assigned coord.

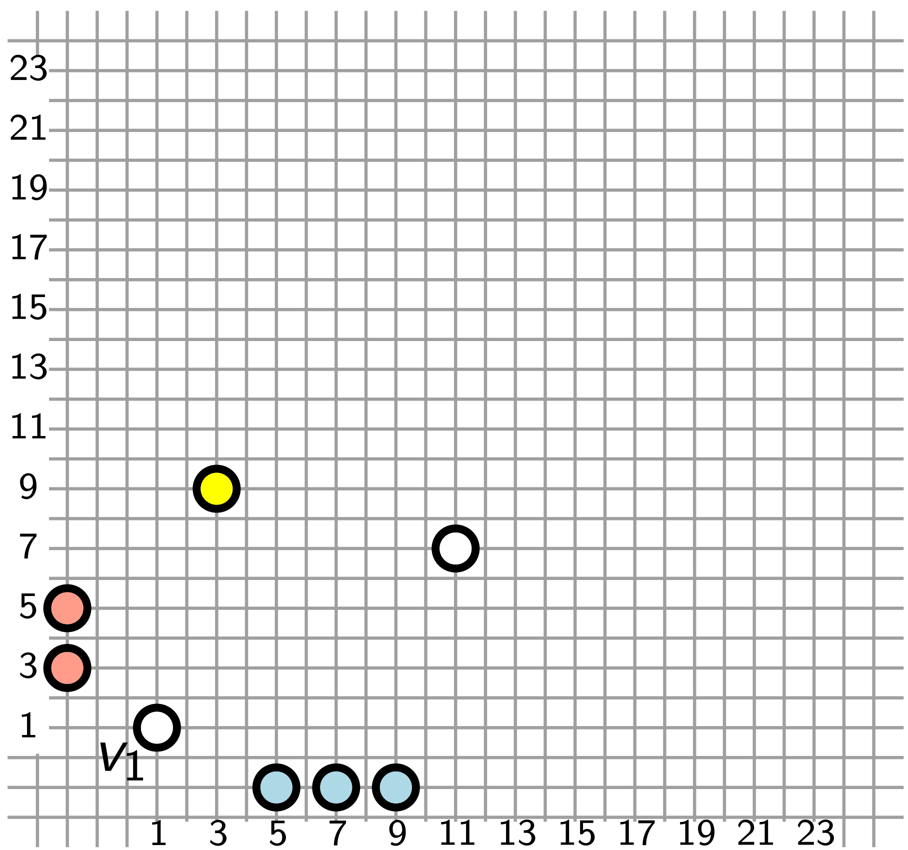


Four Matchings

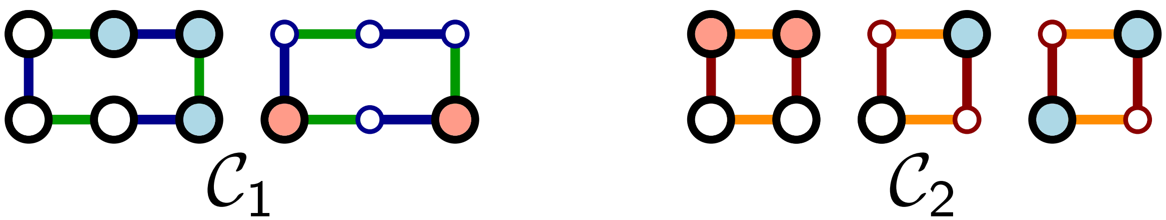


Placement algorithm:

- Pick v_1, place it
- Assign x-coords. to cycle in $\mathcal{C}_1$
- Assign y-coords. to cycle in $\mathcal{C}_2$
- Pick v with 1 assigned coord.
- Place v



Four Matchings



Placement algorithm:

- Pick v_1, place it
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- Assign y-coords. to cycle in $\mathcal{C}_2$
- Pick v with 1 assigned coord.
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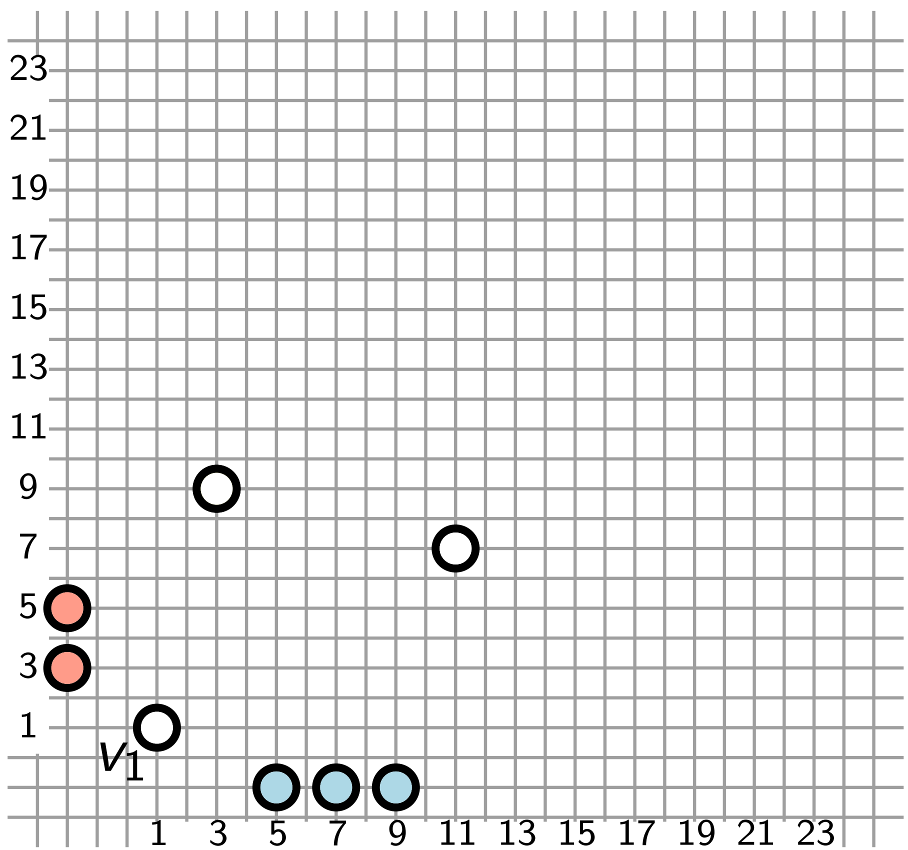
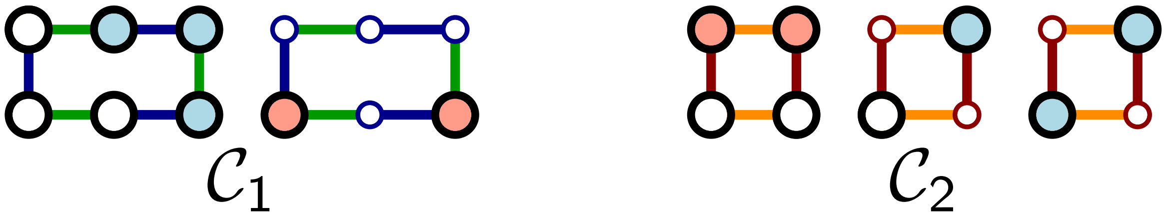


Four Matchings



Placement algorithm:

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- Assign x-coords. to cycle in $\mathcal{C}_1$
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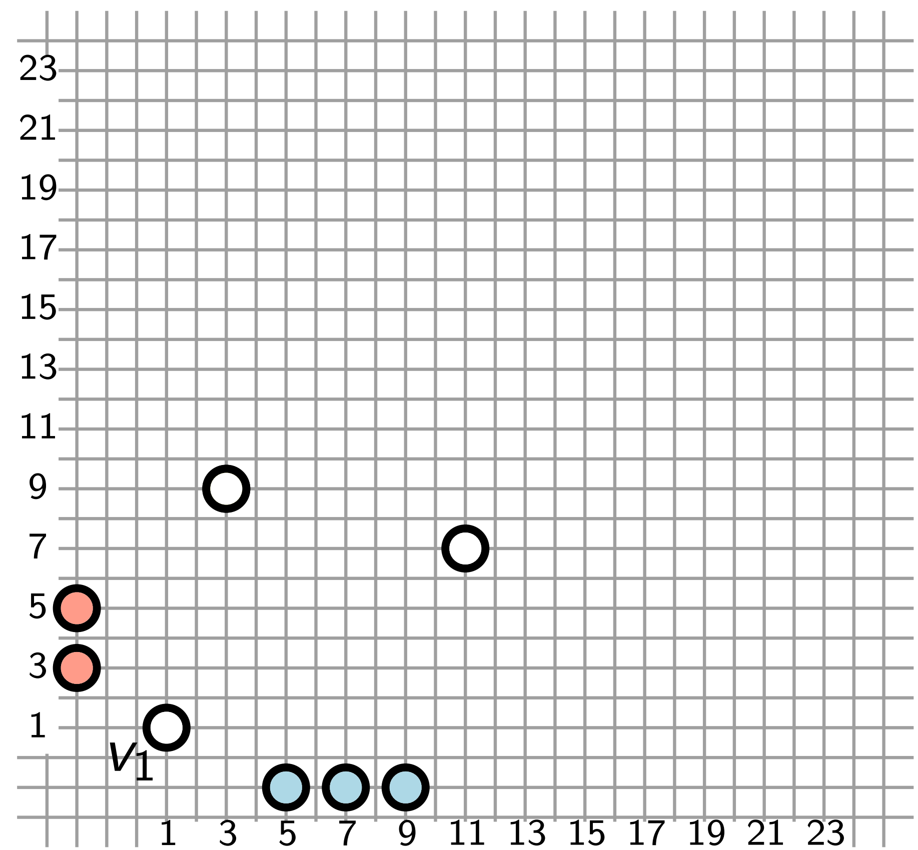
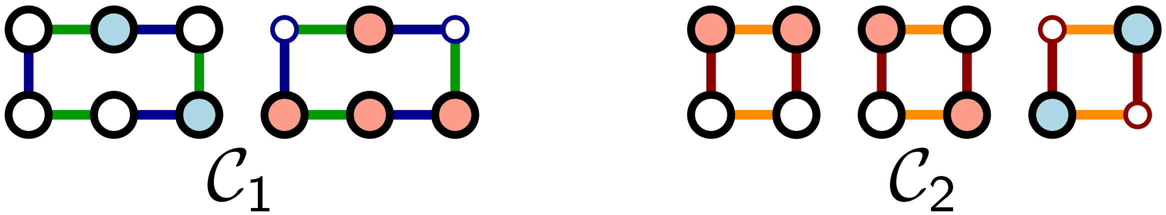
- Place v



Four Matchings



Placement algorithm:

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- Assign x-coords. to cycle in $\mathcal{C}_1$
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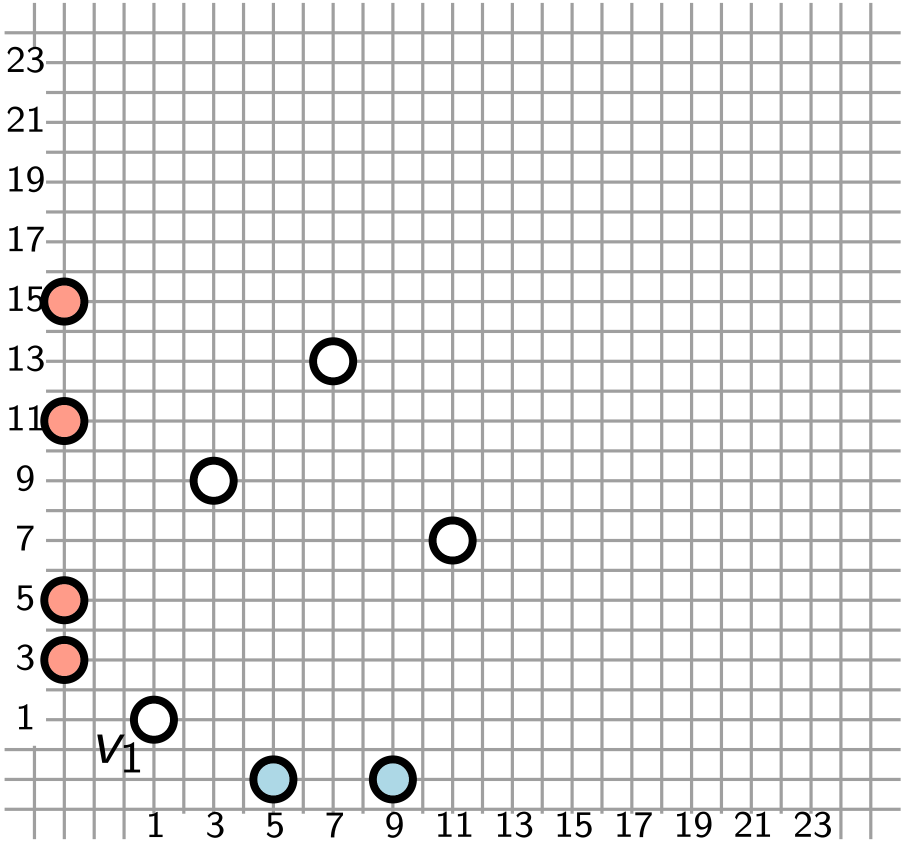
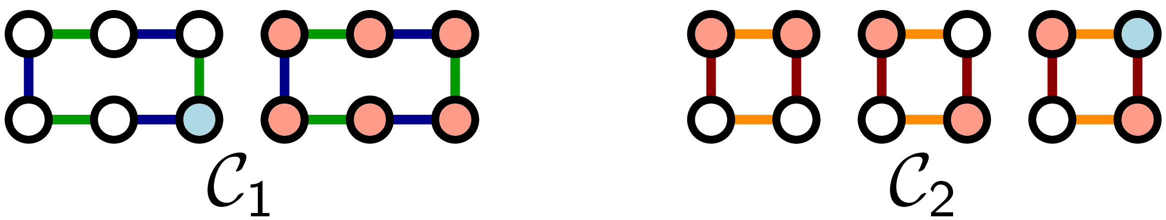
- Pick v with 1 assigned coord.
- Place v



Four Matchings



Placement algorithm:

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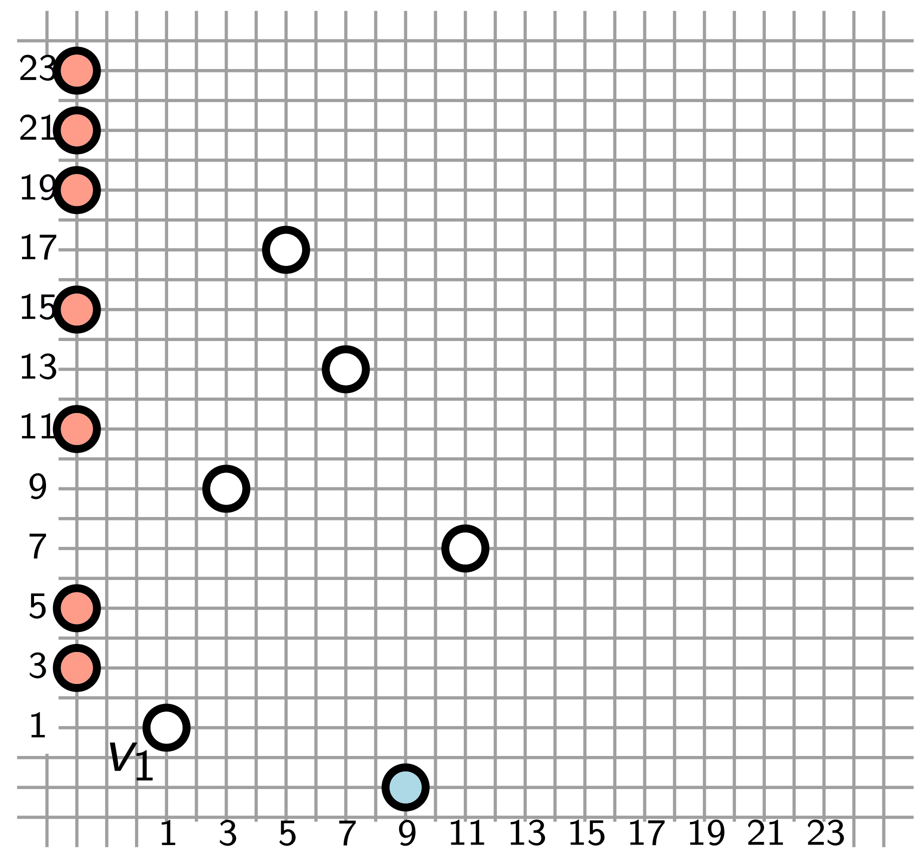
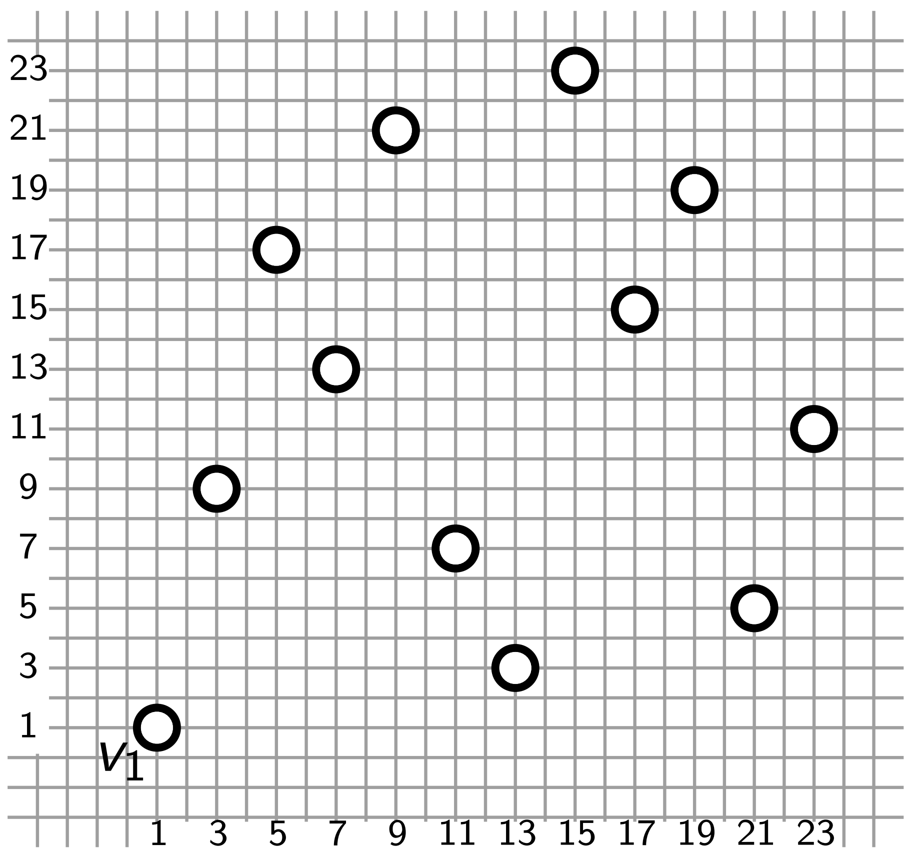


Four Matchings

Placement algorithm:

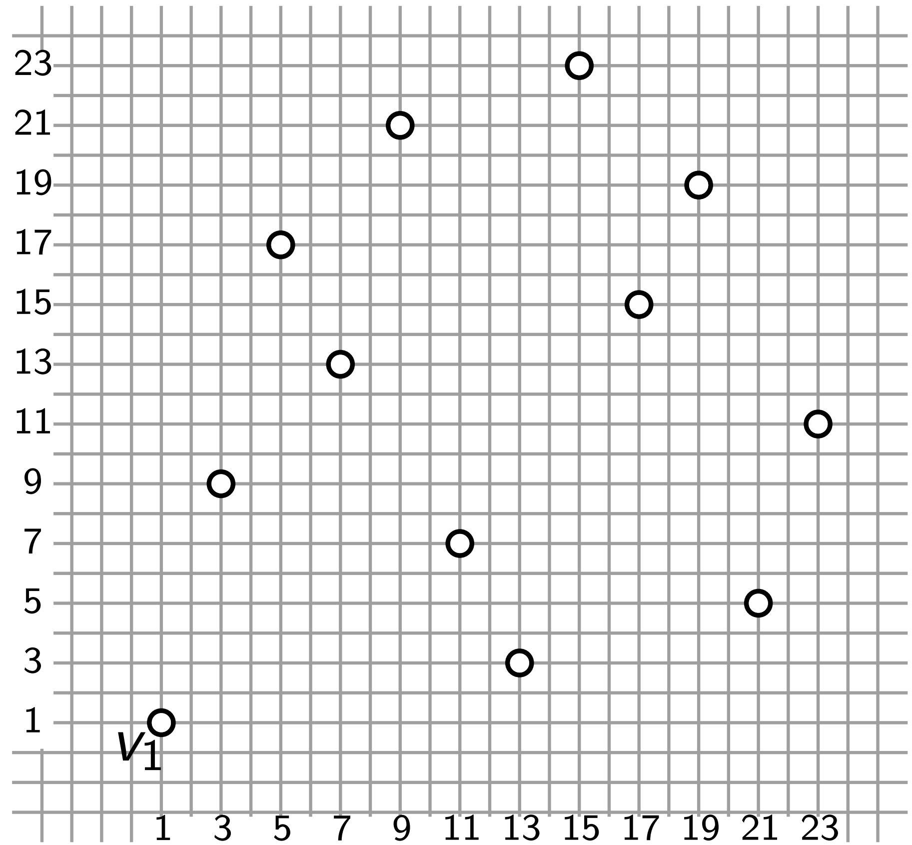
- Pick v_1, place it
- Assign x-coords. to cycle in $\mathcal{C}_1$
- Assign y-coords. to cycle in $\mathcal{C}_2$
- Pick v with 1 assigned coord.
- Place v



Four Matchings

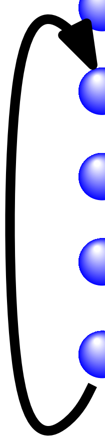
Placement algorithm:

- Pick v_1, place it
- Assign x-coords. to cycle in $\mathcal{C}_1$
- Assign y-coords. to cycle in $\mathcal{C}_2$
- Pick v with 1 assigned coord.
- Place v

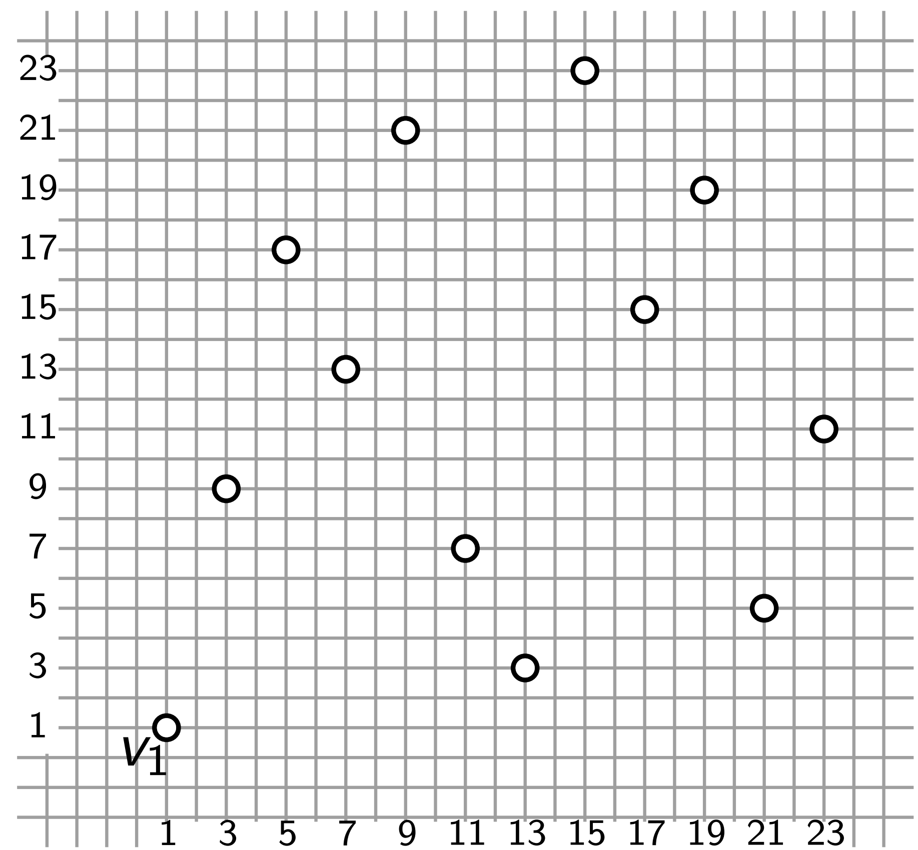


Four Matchings

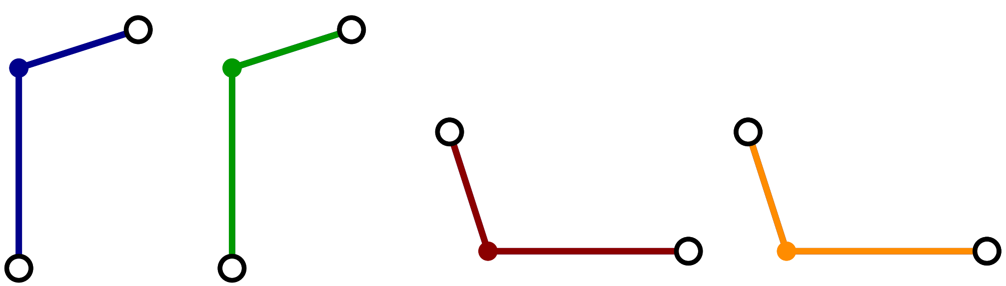
Placement algorithm:



- Pick v_1, place it
- Assign x-coords. to cycle in $\mathcal{C}_1$
- Assign y-coords. to cycle in $\mathcal{C}_2$
- Pick v with 1 assigned coord.
- Place v



Edges:

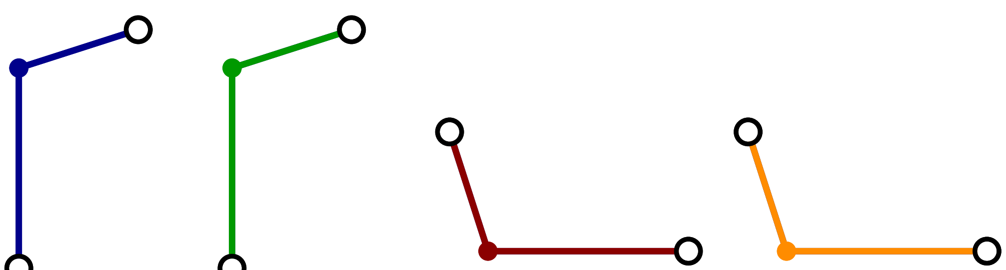


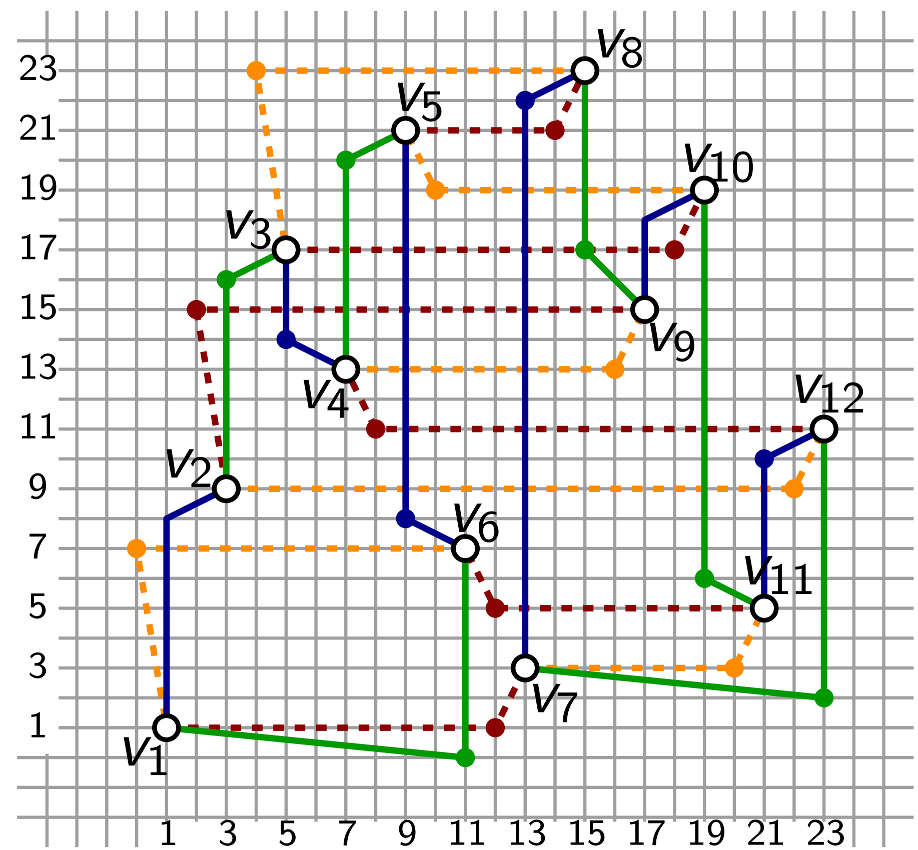
Four Matchings

Placement algorithm:

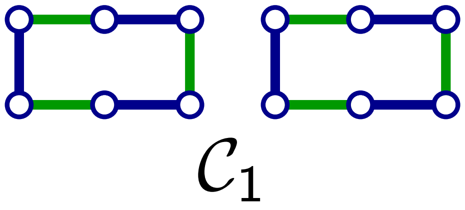
- Pick v_1, place it
- Assign x-coords. to cycle in $\mathcal{C}_1$
- Assign y-coords. to cycle in $\mathcal{C}_2$
- Pick v with 1 assigned coord.
- Place v

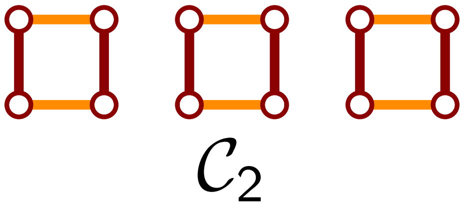
Edges:

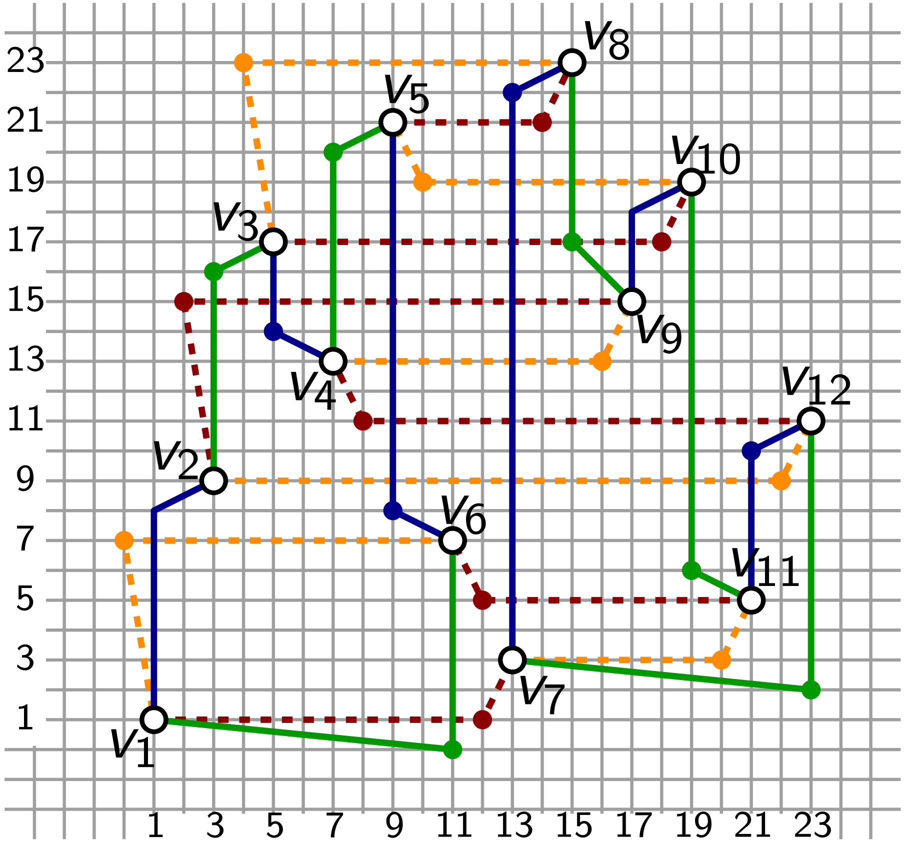




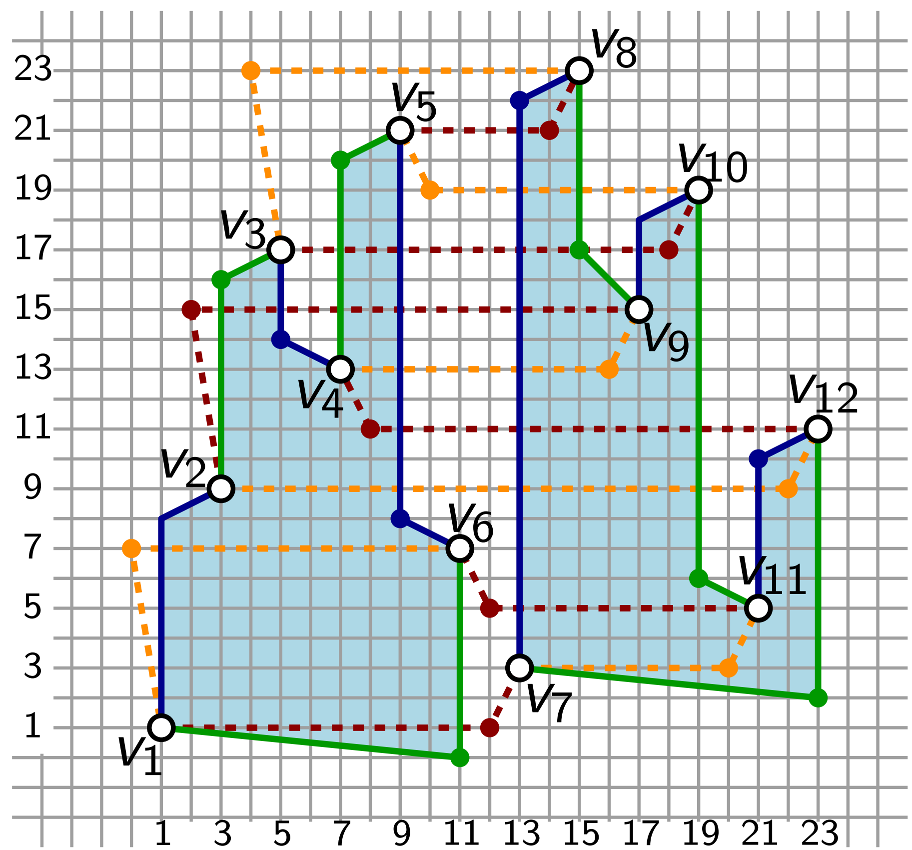
Four Matchings







Four Matchings



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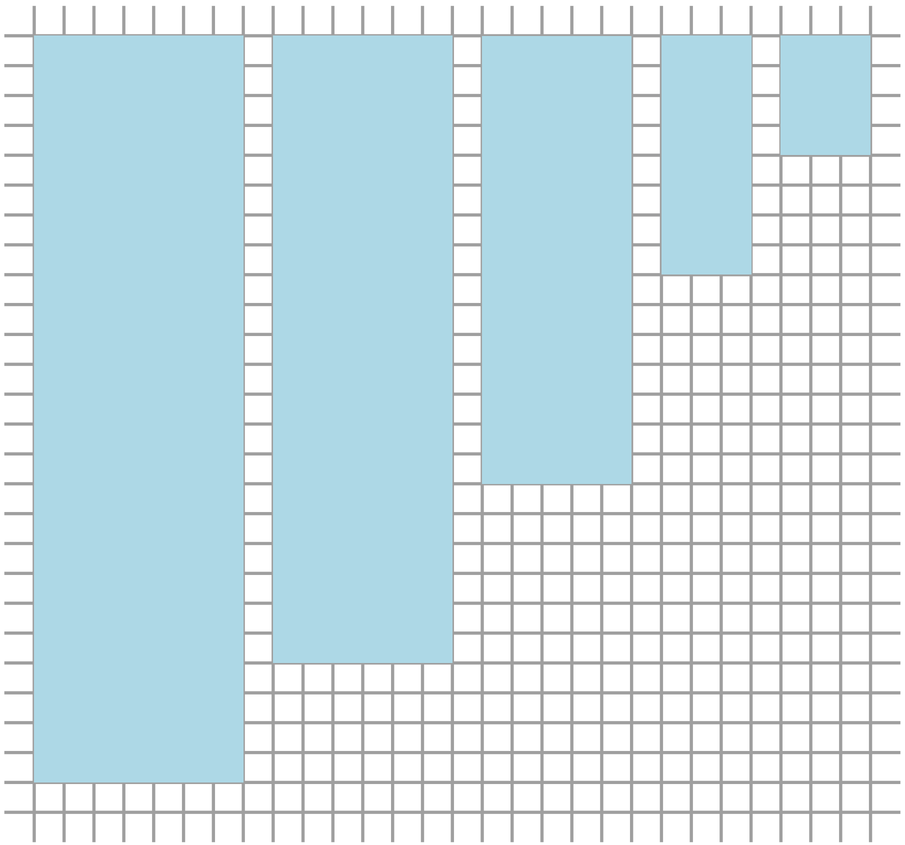
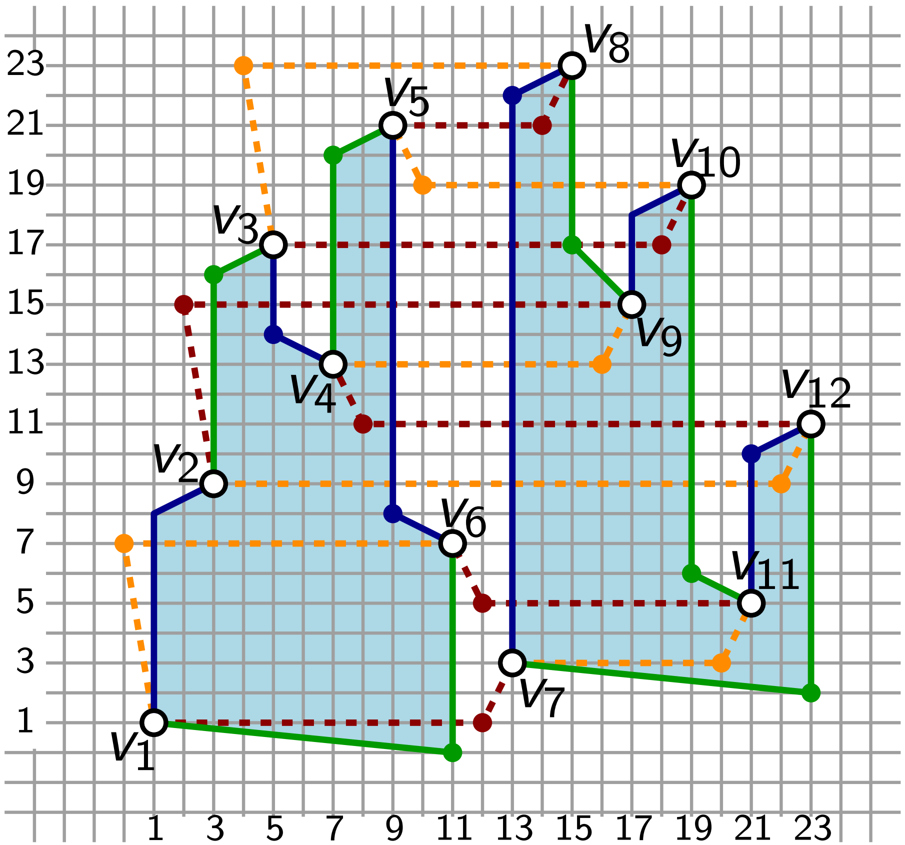
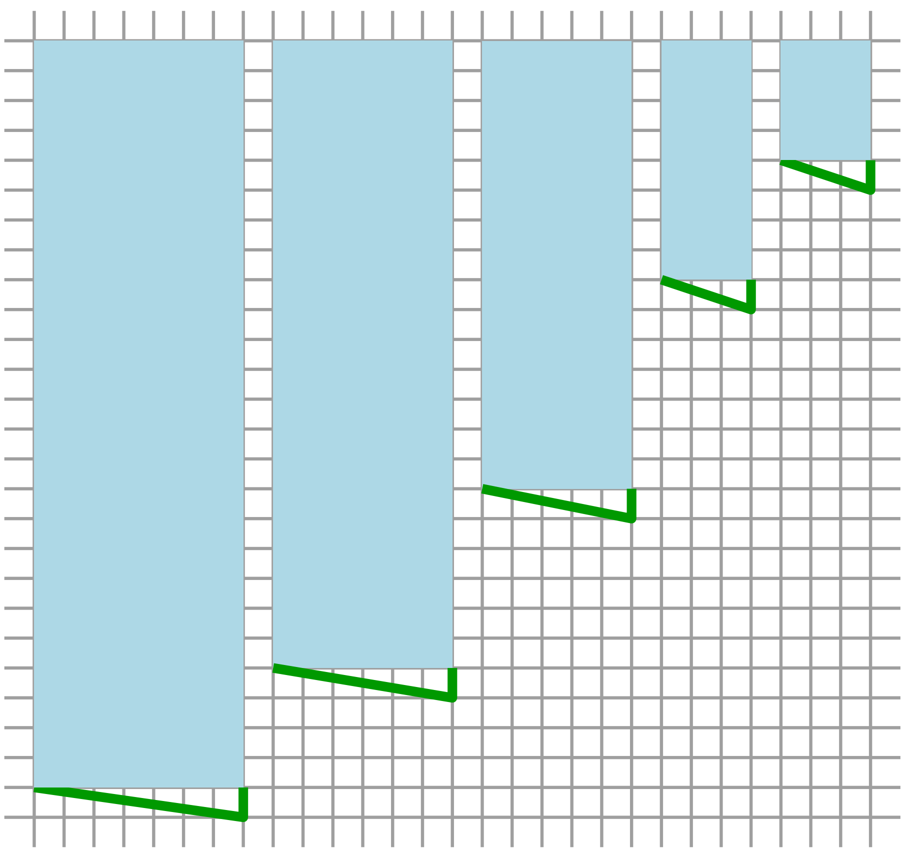
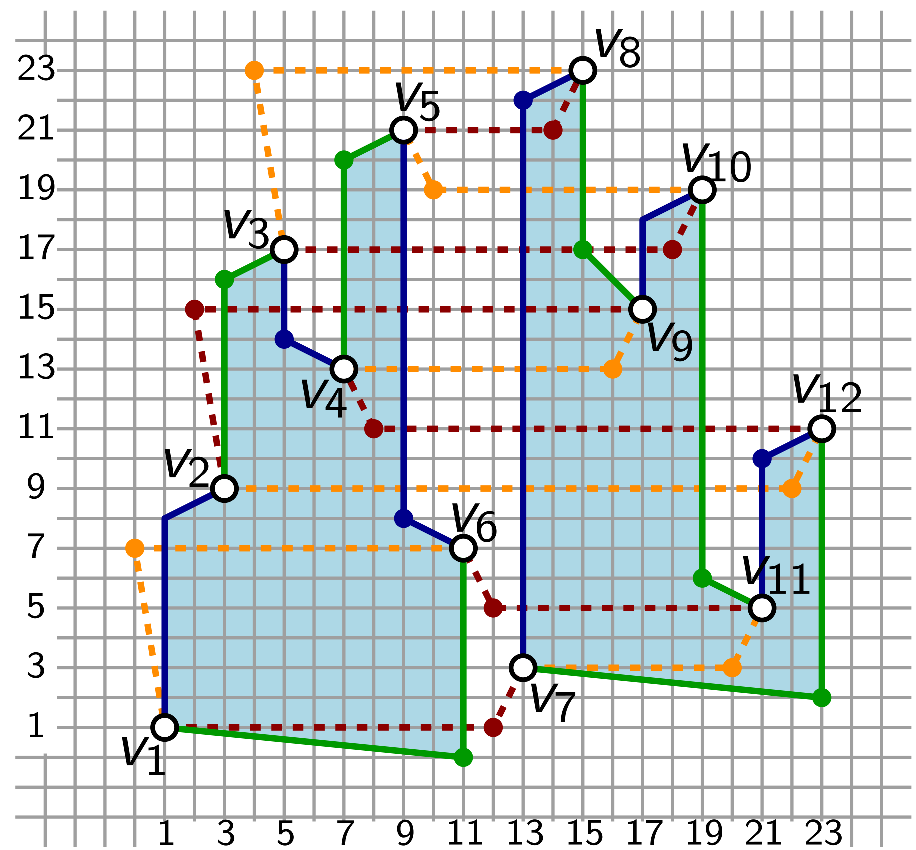


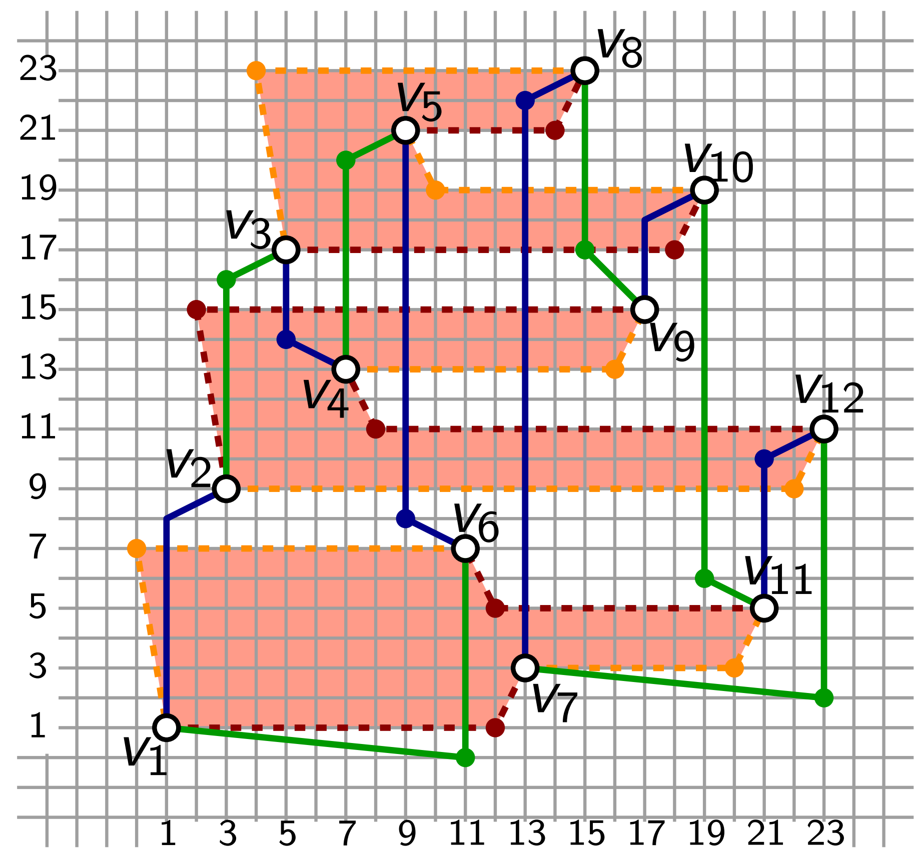


Four Matchings

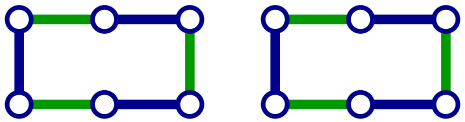




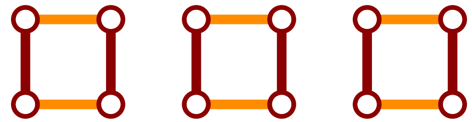
Four Matchings



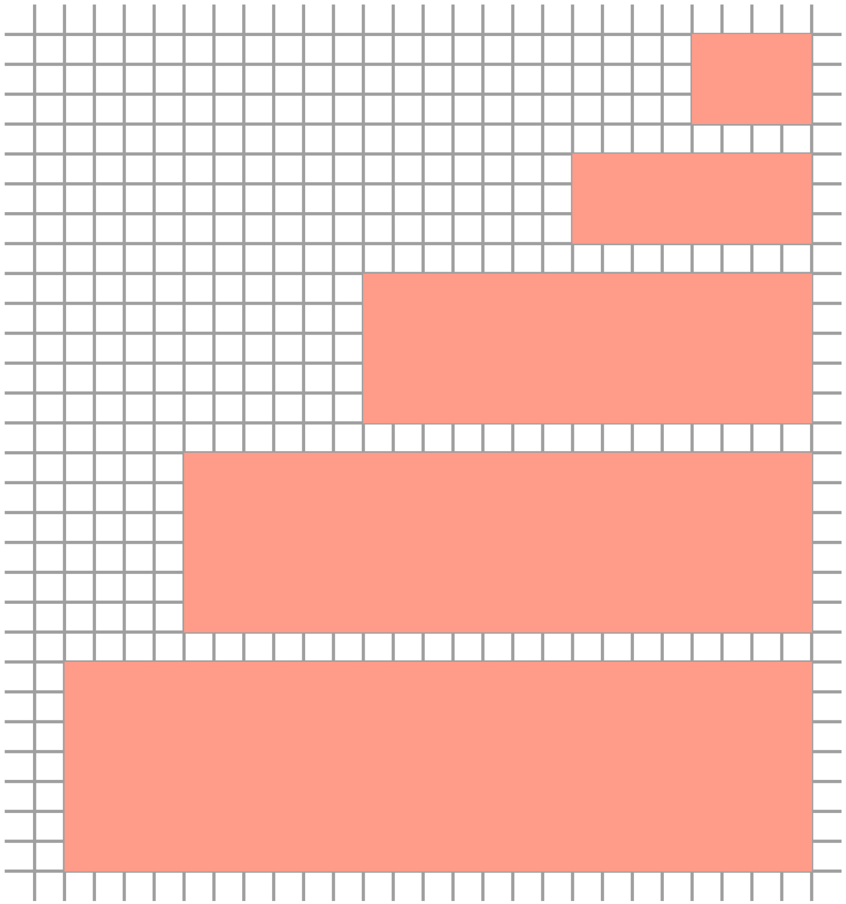
Four Matchings

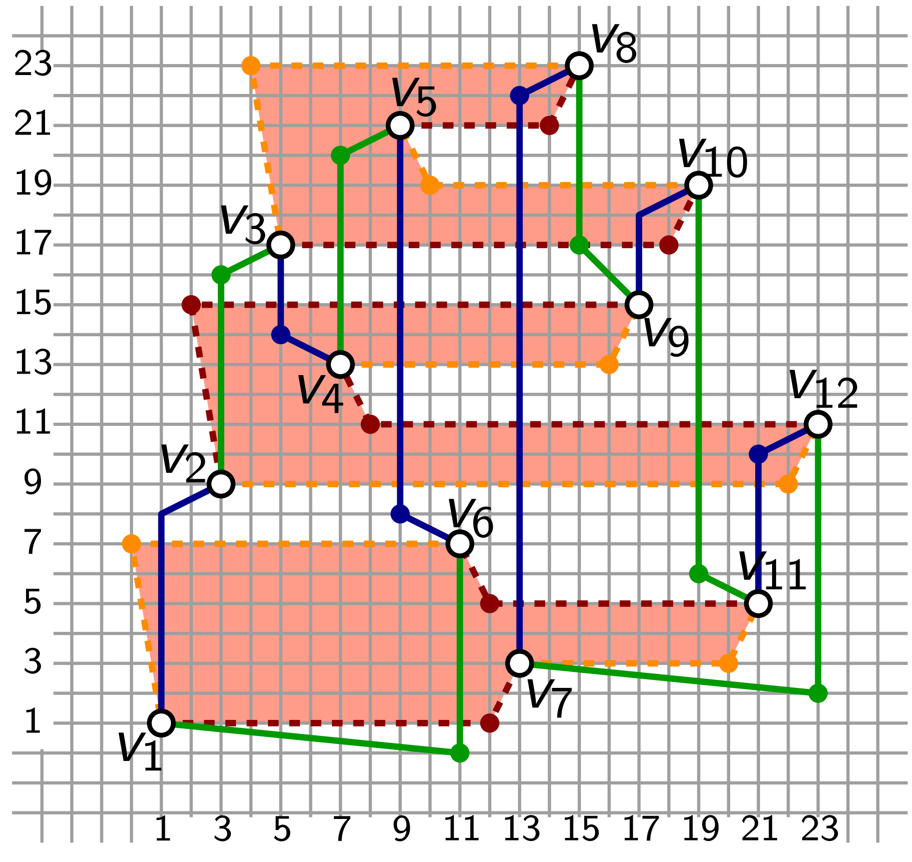


$\mathcal{C}_1$

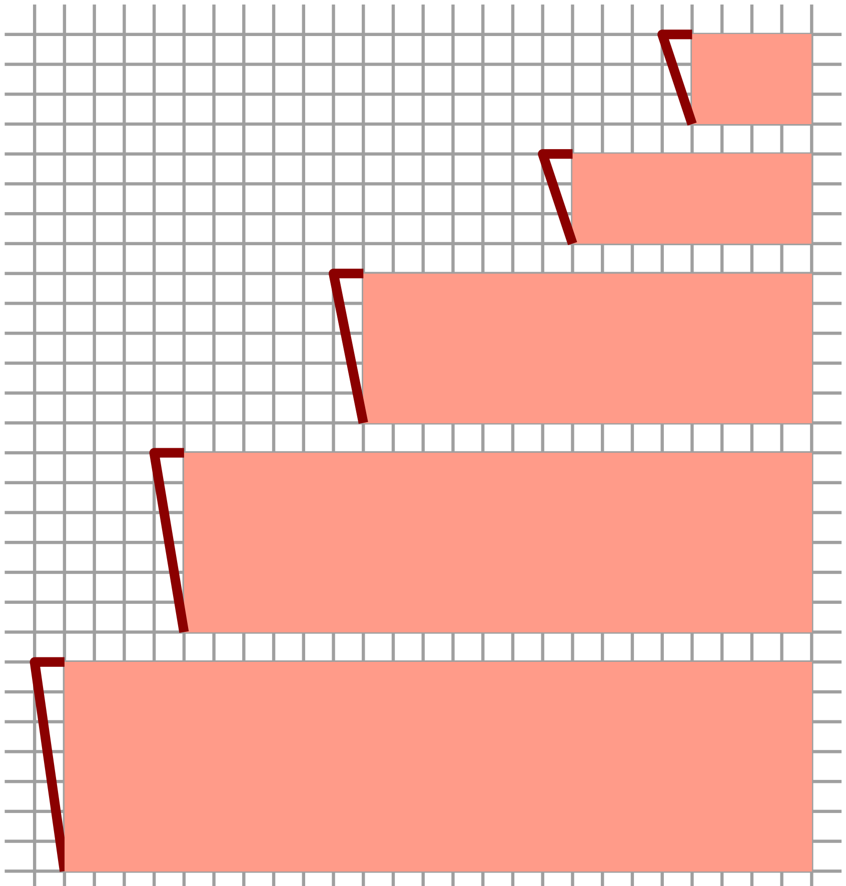


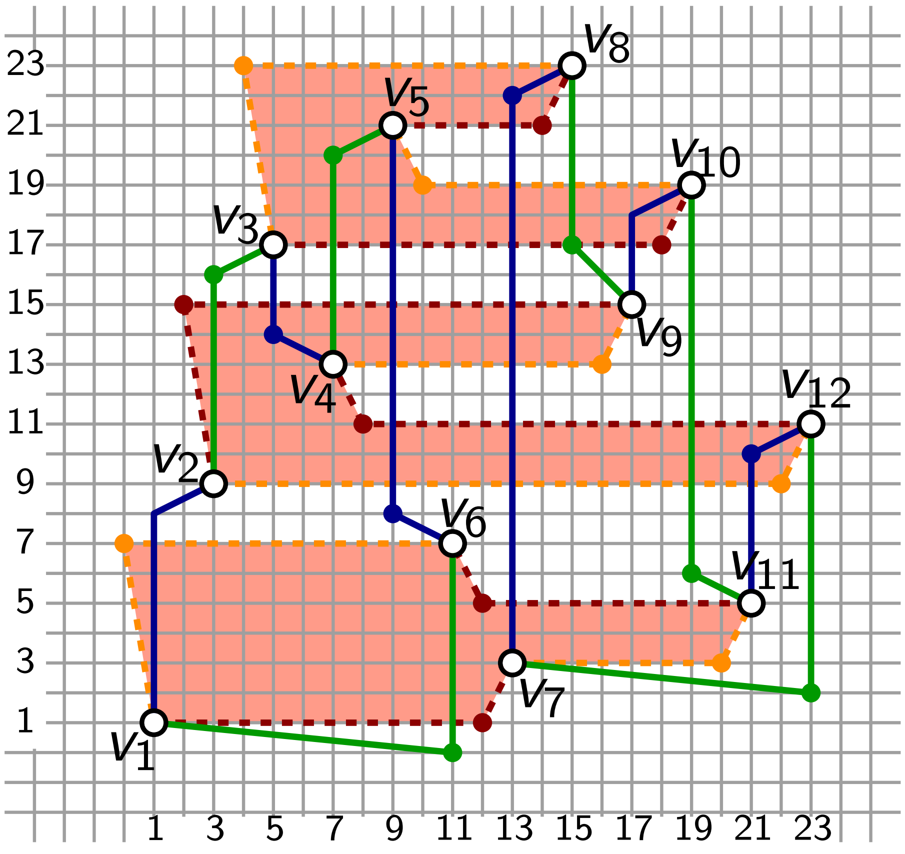
$\mathcal{C}_2$



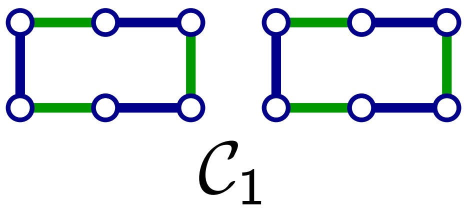


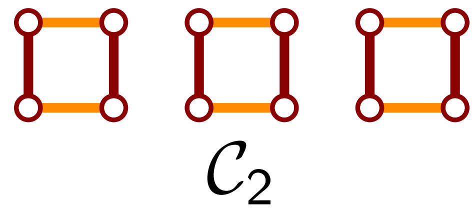
Four Matchings





Four Matchings

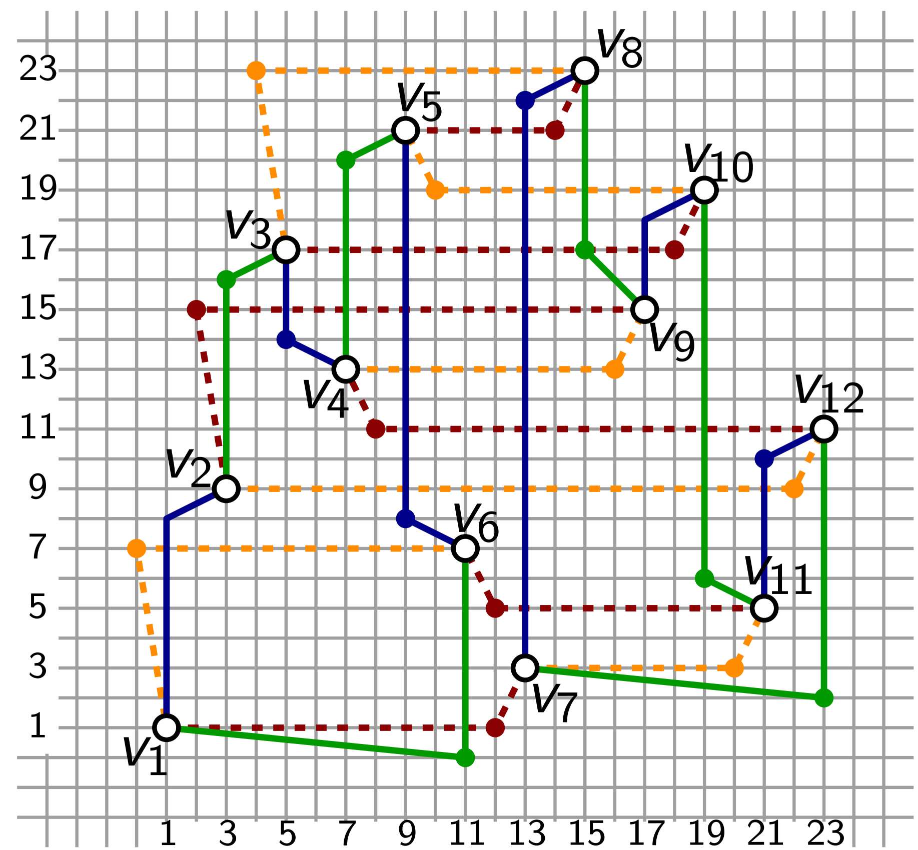




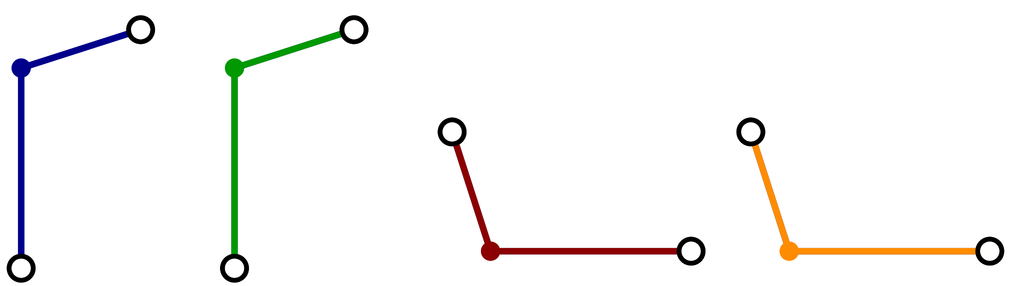
Bends:	1×1
Grid size:	$2n \times 2n$

Placement algorithm:

- Pick v_1 , place it
- Assign x-coords. to cycle in $\mathcal{C}_1$
- Assign y-coords. to cycle in $\mathcal{C}_2$
- Pick v with 1 assigned coord.
- Place v



Edges:



Overview

Graph classes			Number of bends
Cycle	$\times$	Cycle	1×1
Caterpillar	$\times$	Cycle	1×1
Four Matchings			$1 \times 1 \times 1 \times 1$ ✓
Tree	$\times$	Matching	1×0
Wheel	$\times$	Matching	2×0
Outerpath	$\times$	Matching	2×1
Outerplanar	$\times$	Outerplanar	3×3
2-page book emb.	$\times$	2-page book emb.	4×4
Planar	$\times$	Planar	6×6

Tree × Matching

Idea: • Matching edges horizontal, tree edges with 1 bend

Tree $\times$ Matching

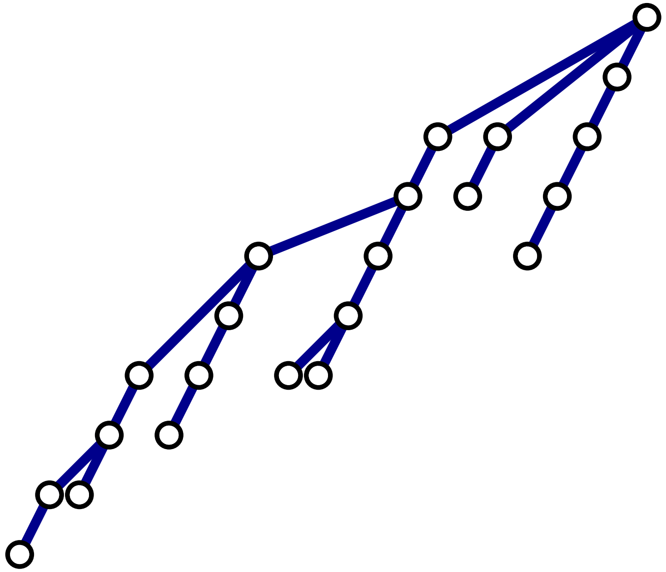
Idea:
- Matching edges horizontal, tree edges with 1 bend
- Place matching edges inductively $\Rightarrow$ y-coord.

Tree × Matching

Idea:
- Matching edges horizontal, tree edges with 1 bend
- Place matching edges inductively $\Rightarrow$ y-coord.
- Use post-order on tree $\Rightarrow$ x-coord.

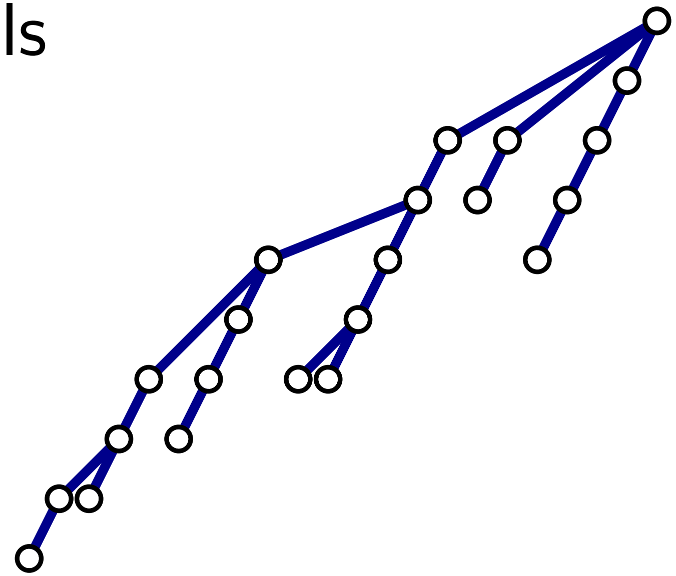
Tree × Matching

Idea:
- Matching edges horizontal, tree edges with 1 bend
- Place matching edges inductively ⇒ y-coord.
- Use post-order on tree ⇒ x-coord.



Tree × Matching

Idea:
- Matching edges horizontal, tree edges with 1 bend
- Place matching edges inductively ⇒ y-coord.
- Use post-order on tree ⇒ x-coord.
- ⇒ Subtrees in disjoint x-intervals



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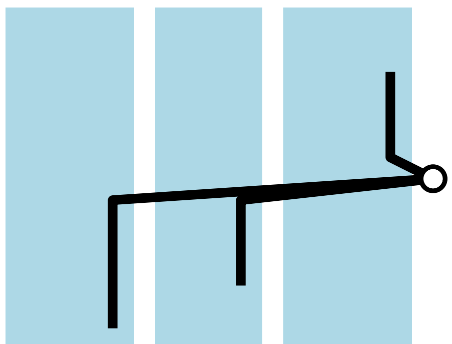

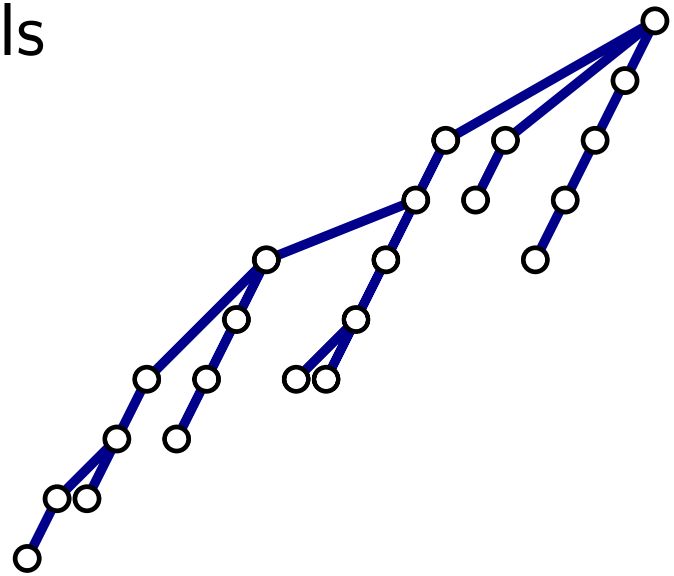

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Idea:
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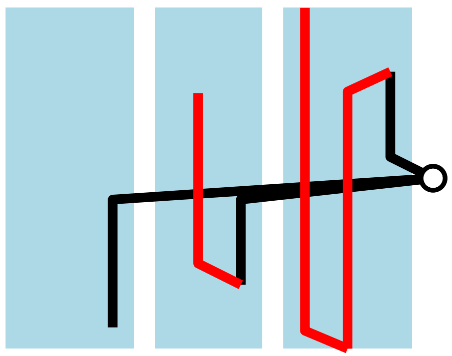
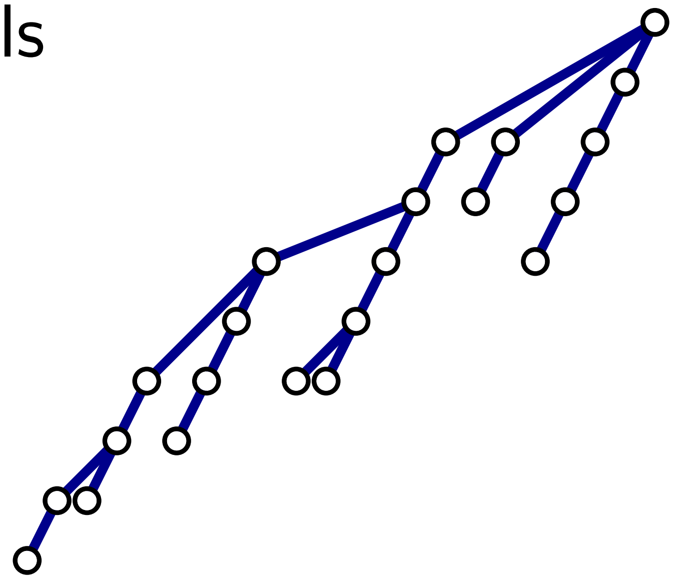
Problem:





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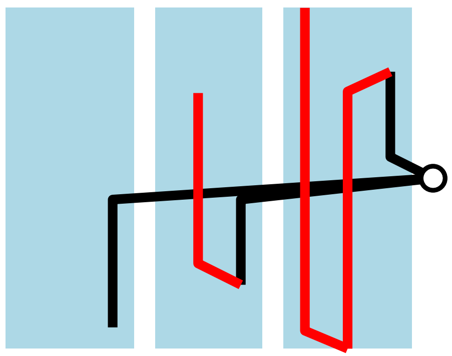
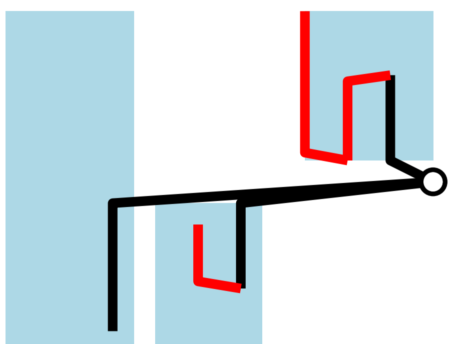
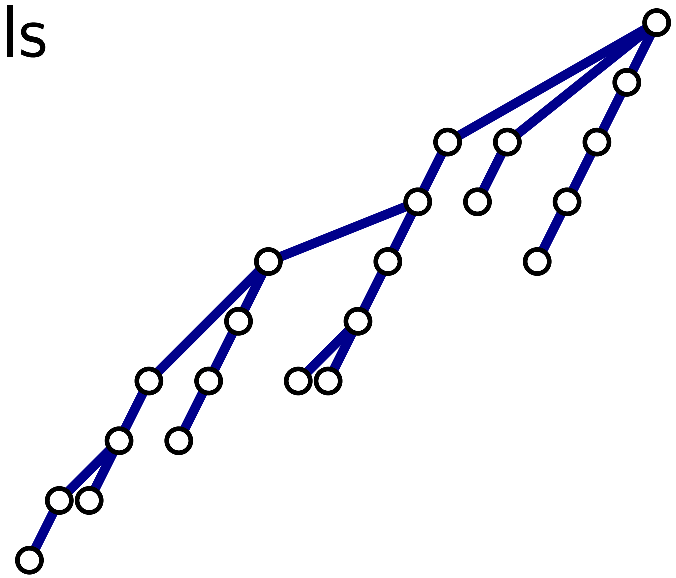


Solution:





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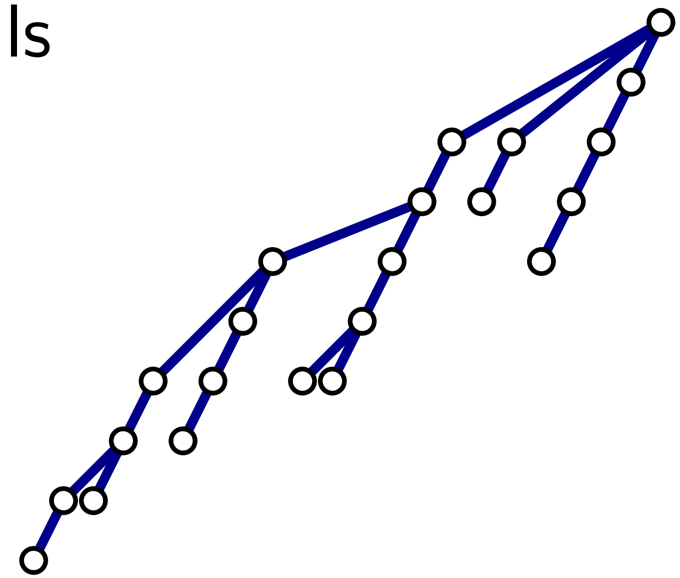

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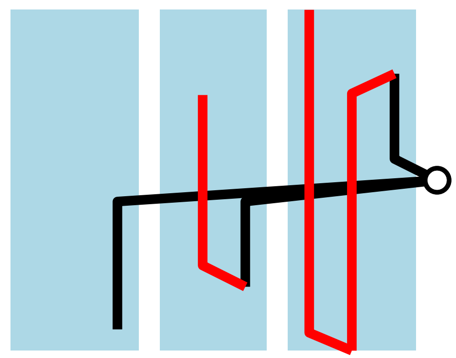
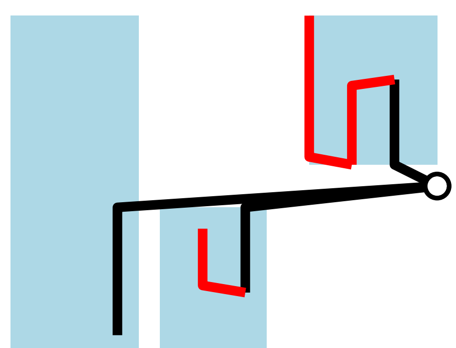


Solution:



All but one subtree completely above or completely below.

Tree × Matching

Idea:
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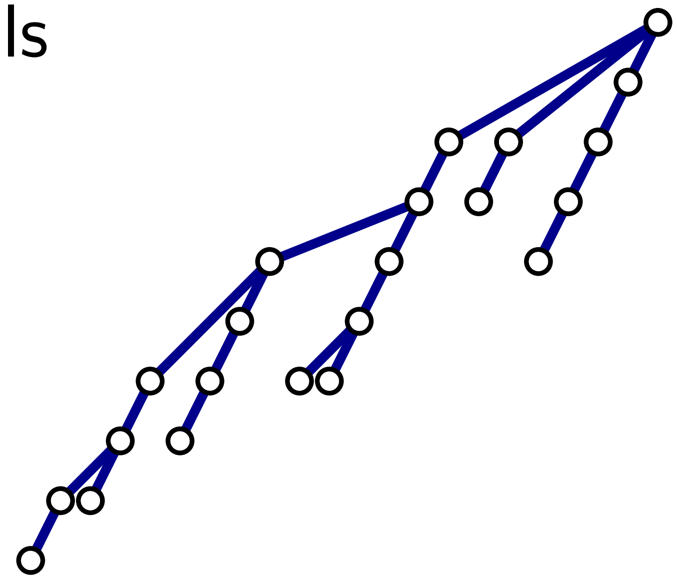

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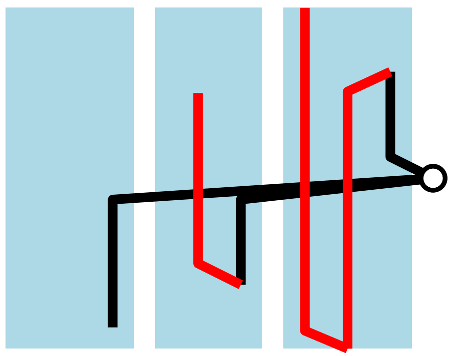
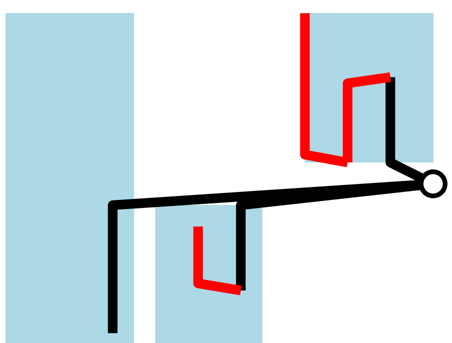


Solution:

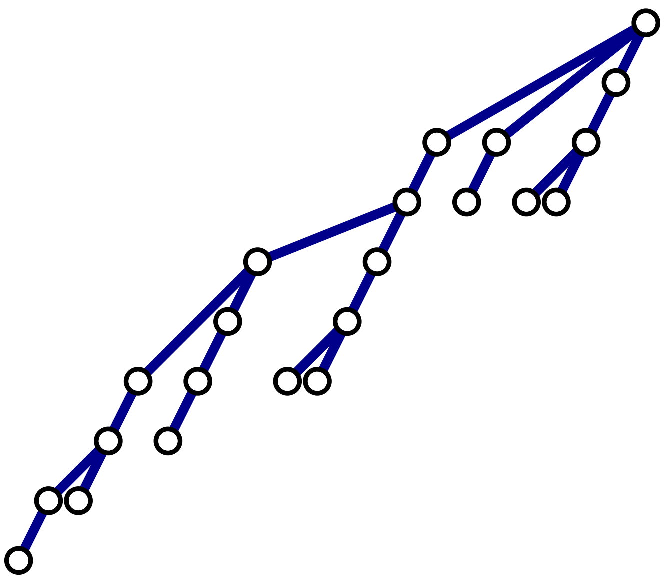


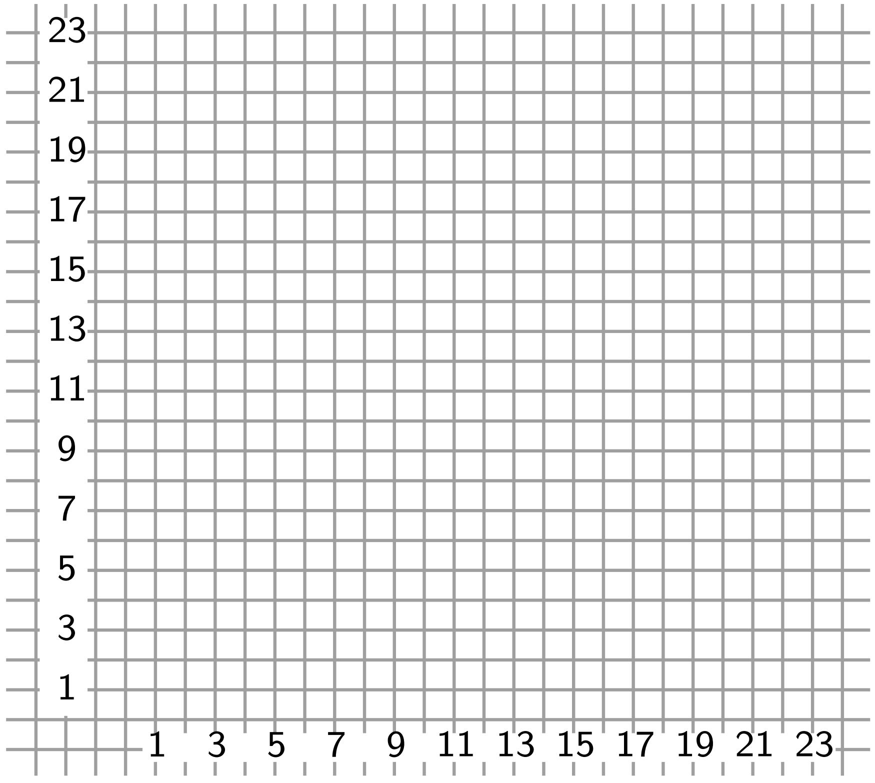
All but one subtree completely above or completely below.

Main ideas adopted from [Cabello et al. JGAA'11, Di Giacomo et al. JGAA'09].

Tree × Matching

- Place root + matching at the top





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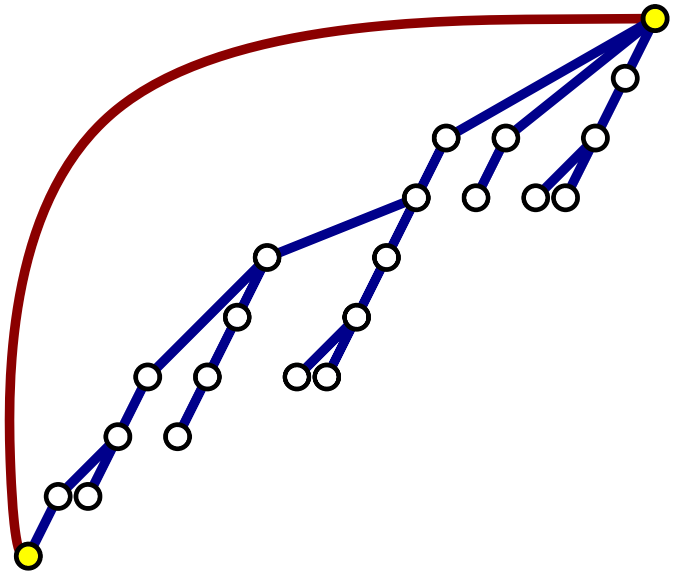

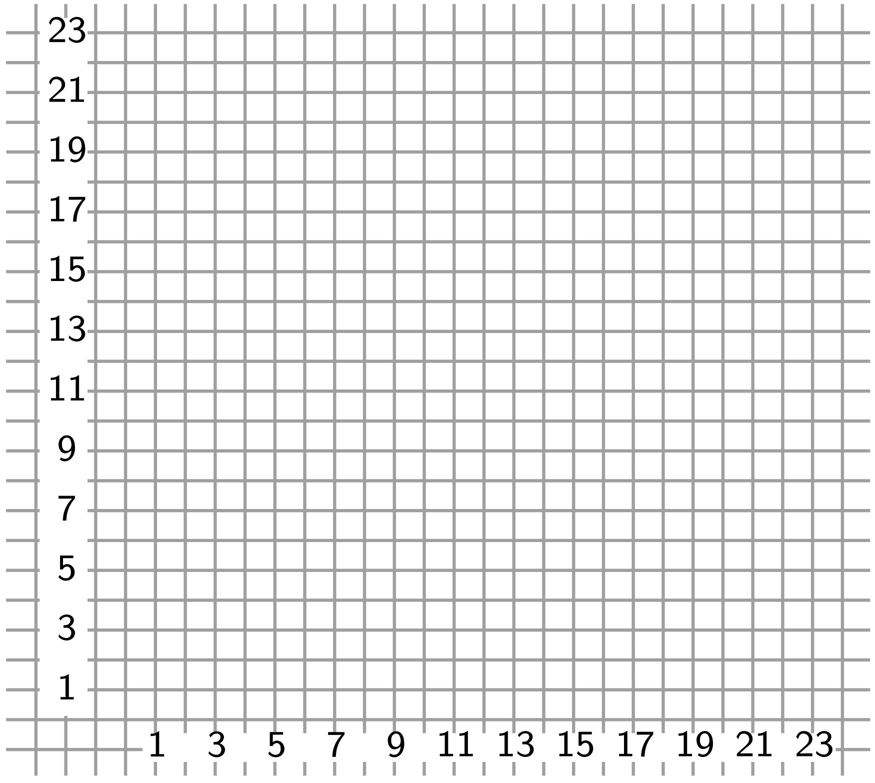

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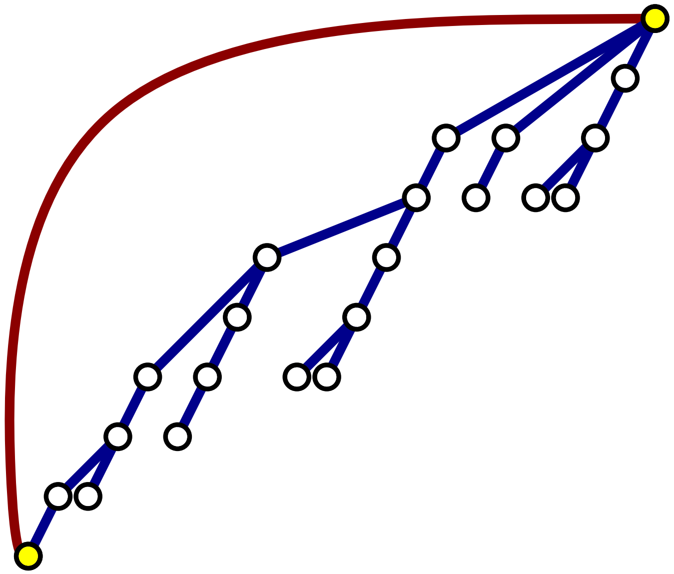
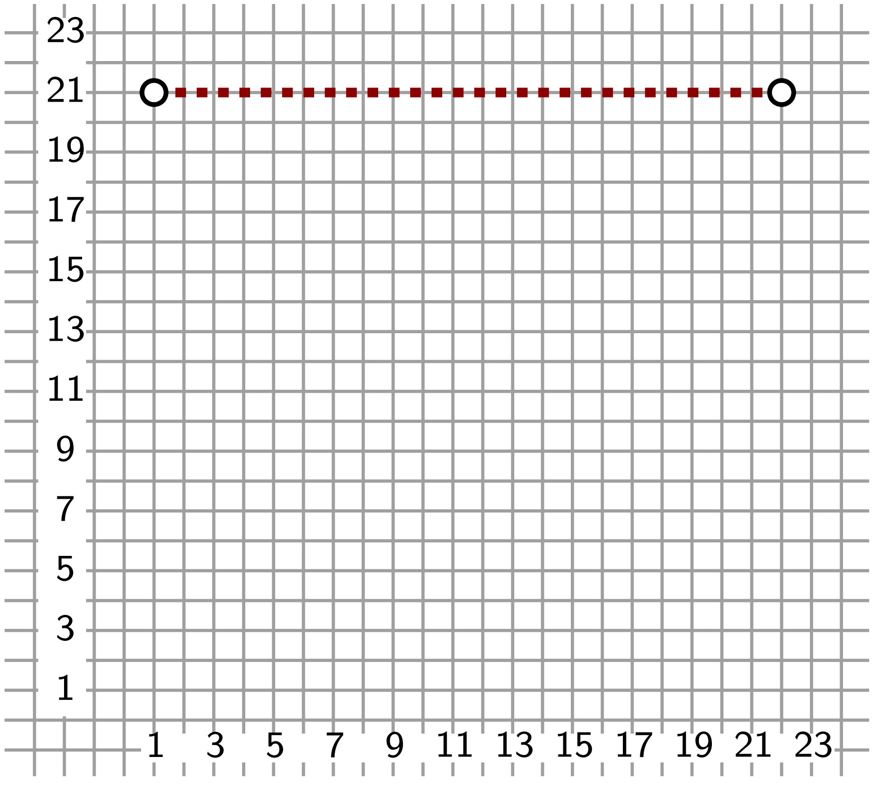
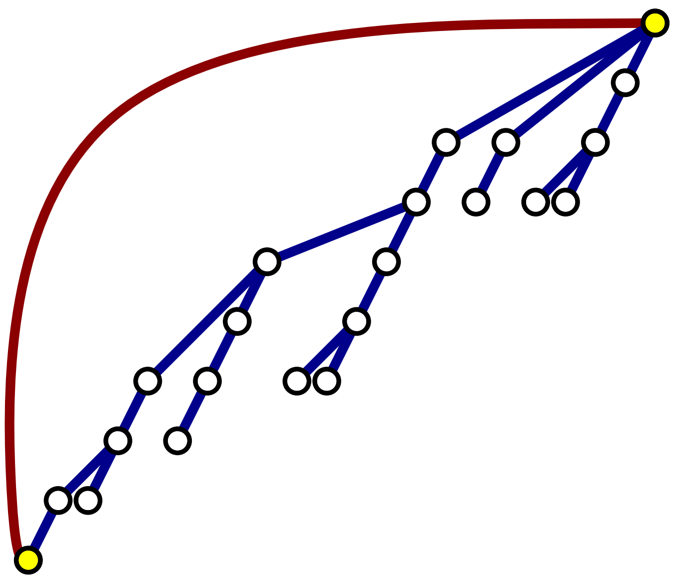
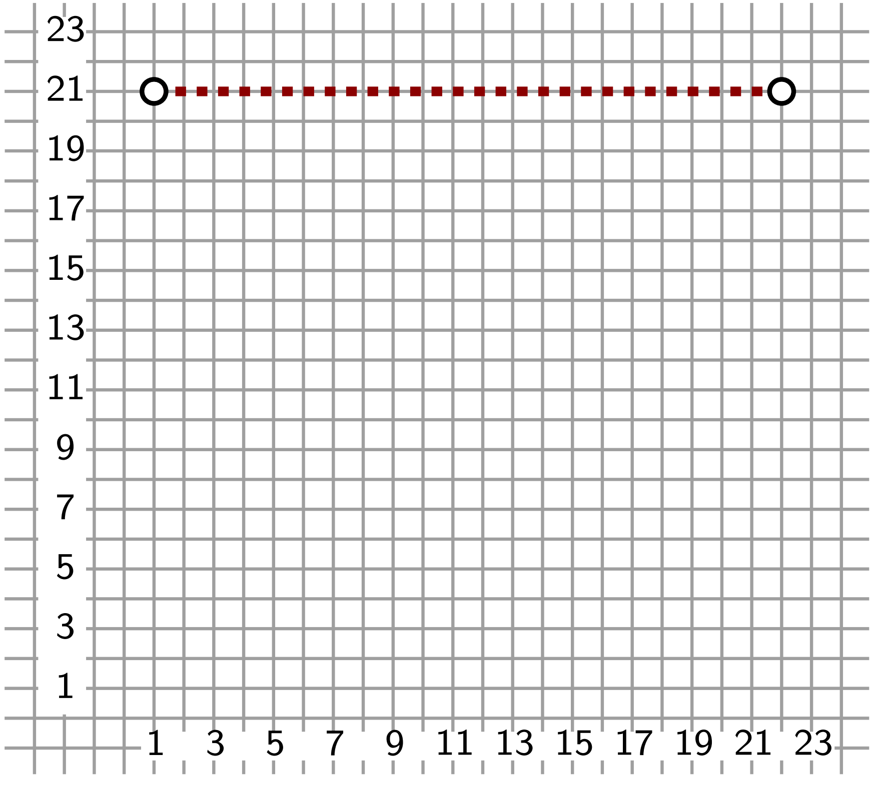




Tree × Matching

- Place root + matching at the top
- Split the tree





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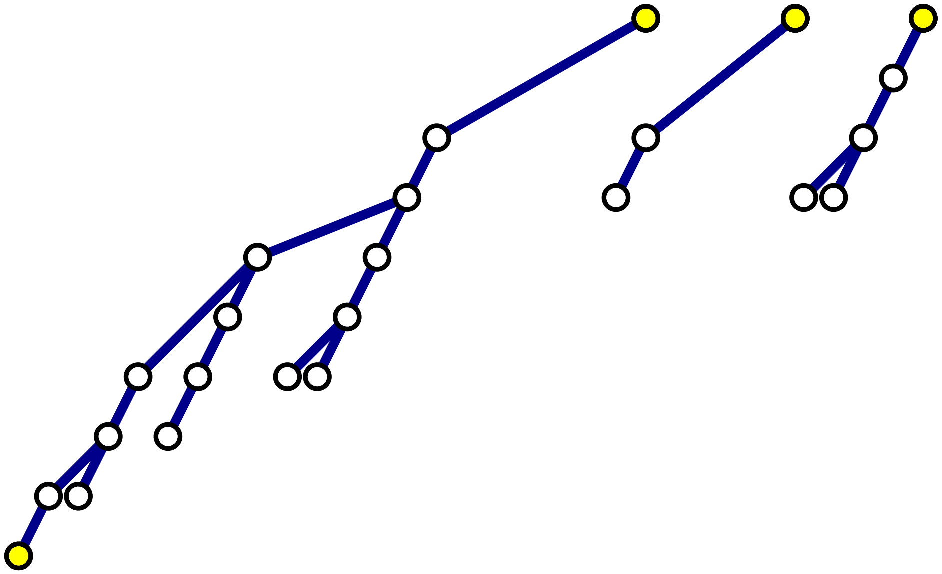
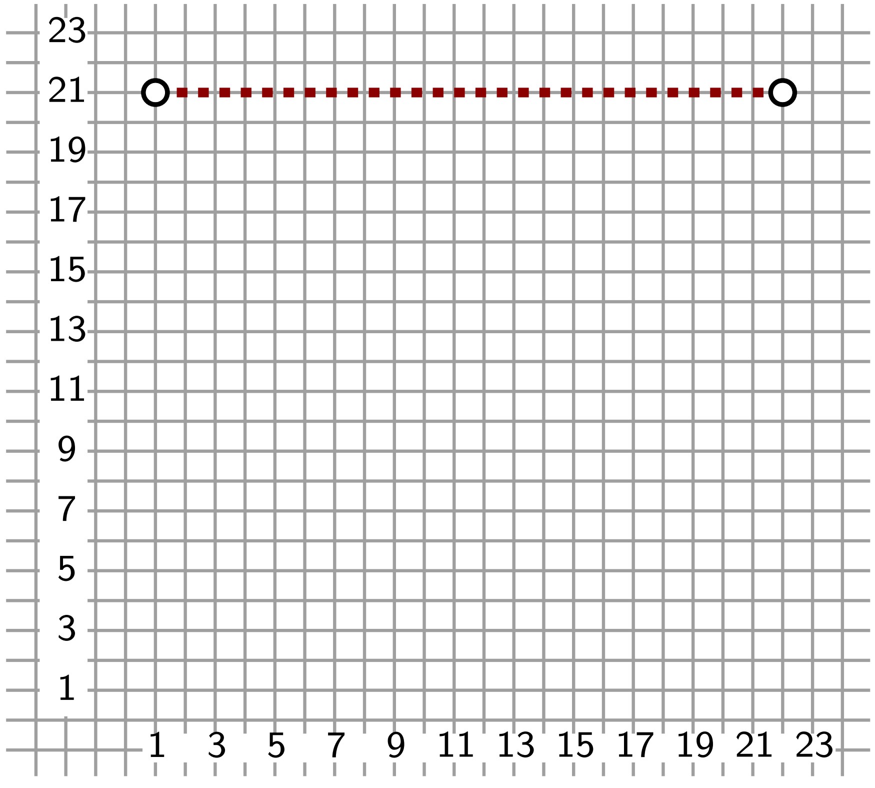
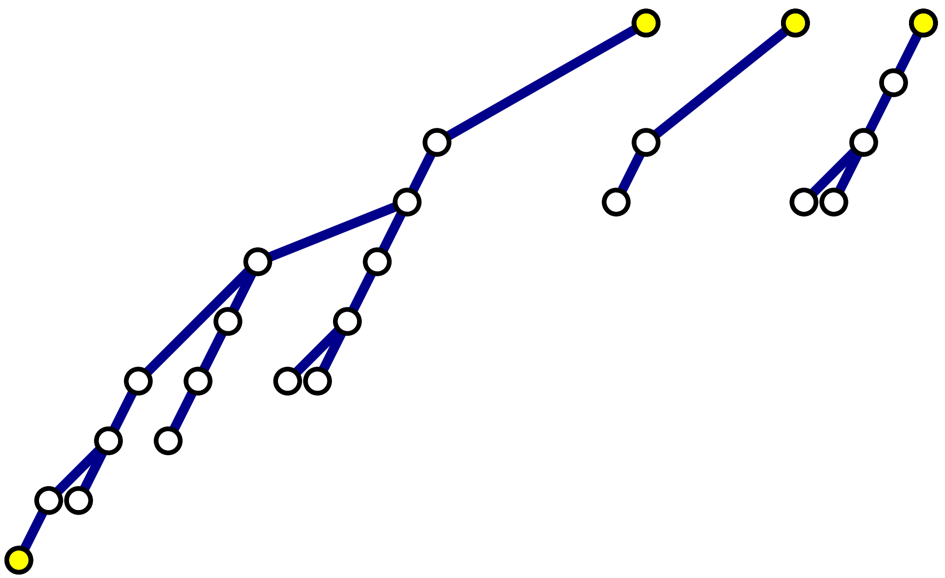
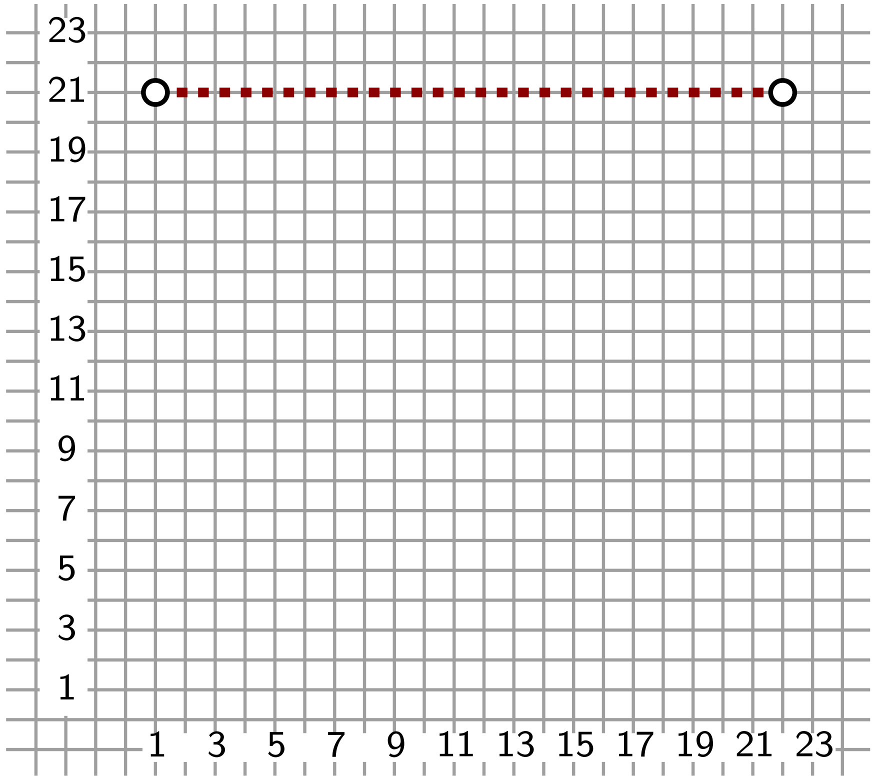




Tree × Matching

- Place root + matching at the top
- Split the tree
- Place vertex adj. to placed vertex (+ matching) at the top





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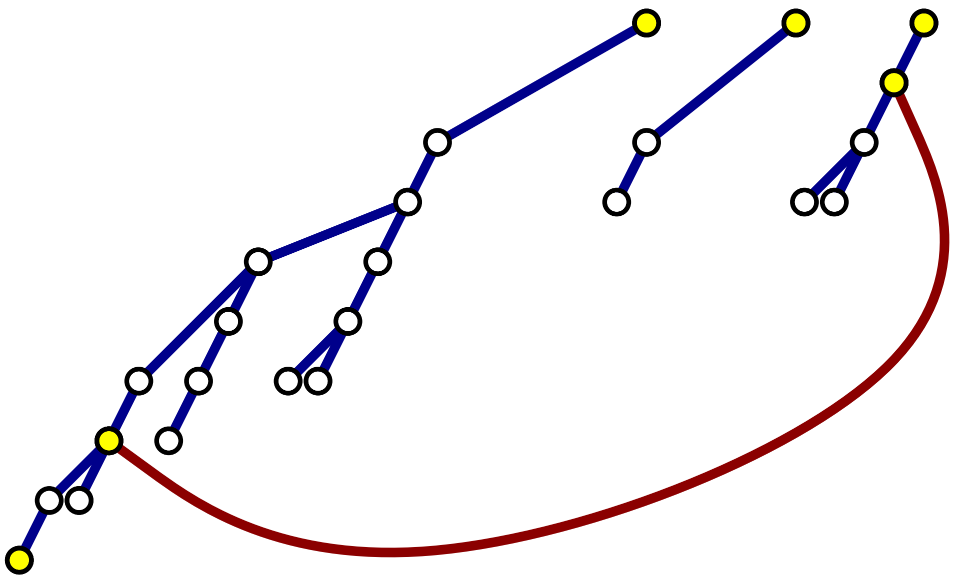

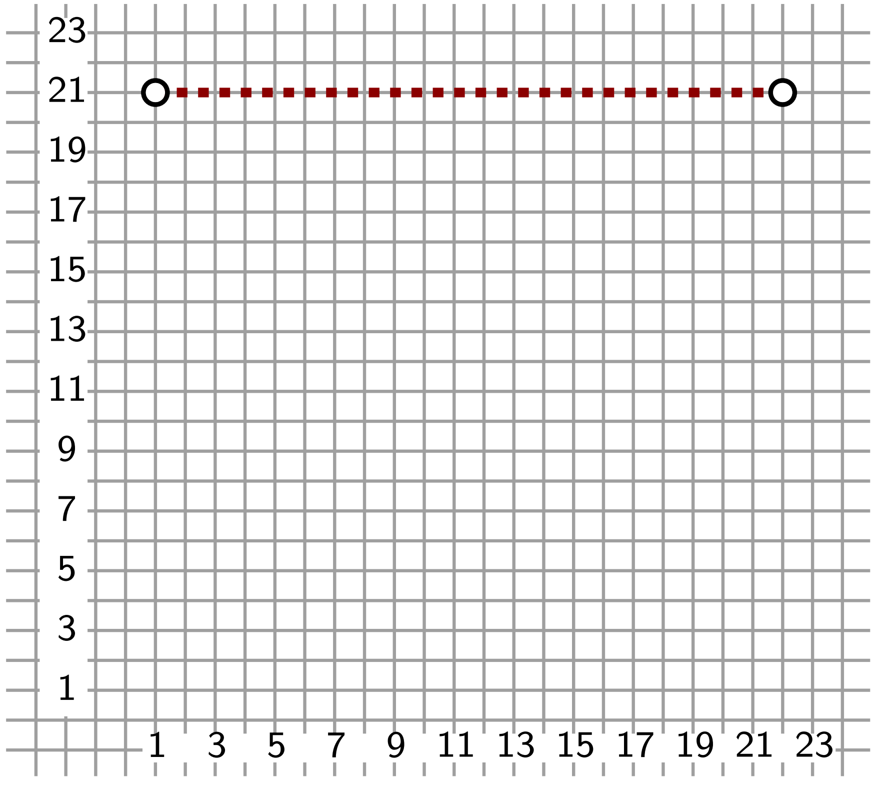

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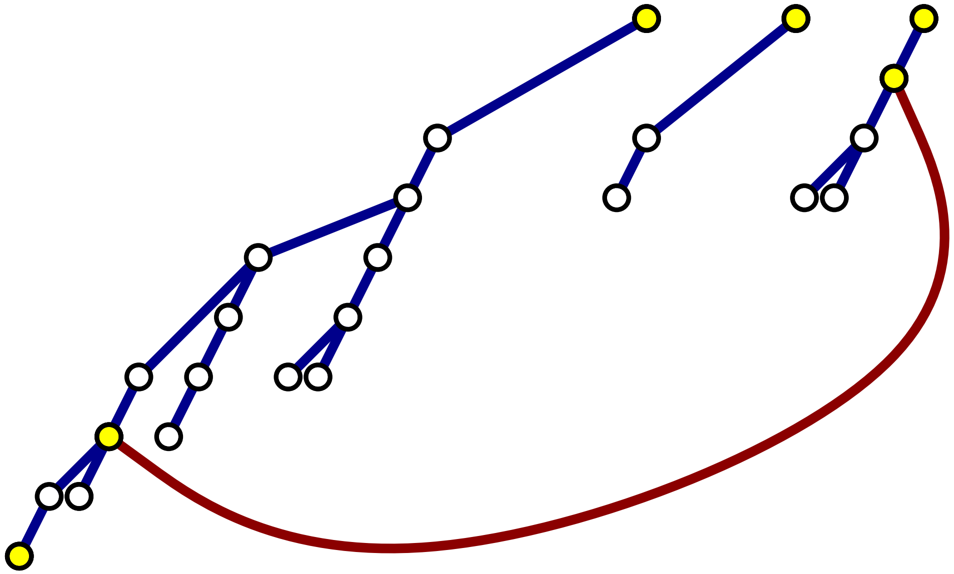

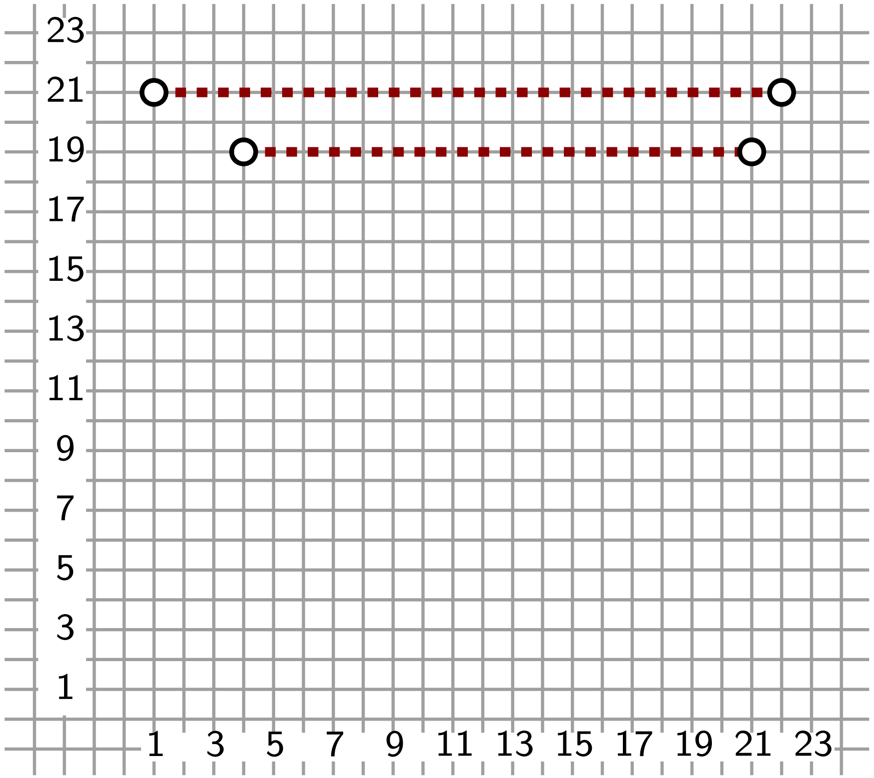

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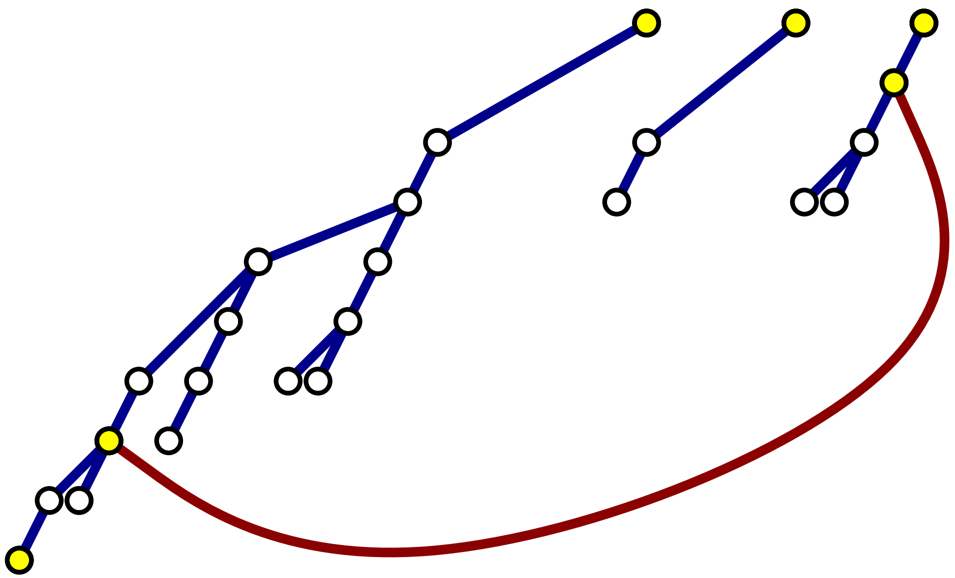

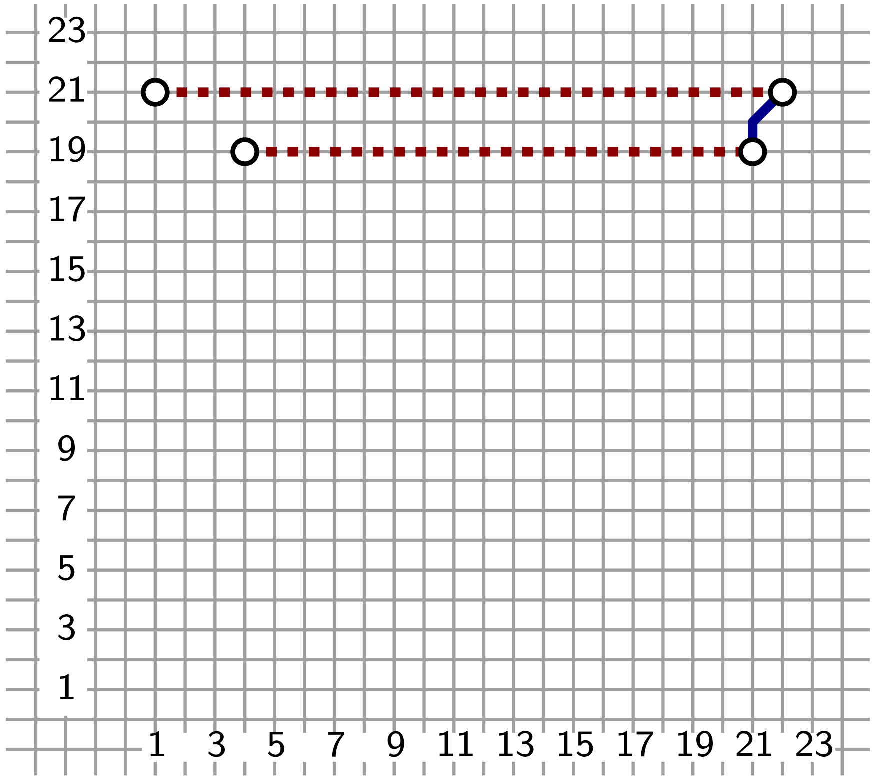

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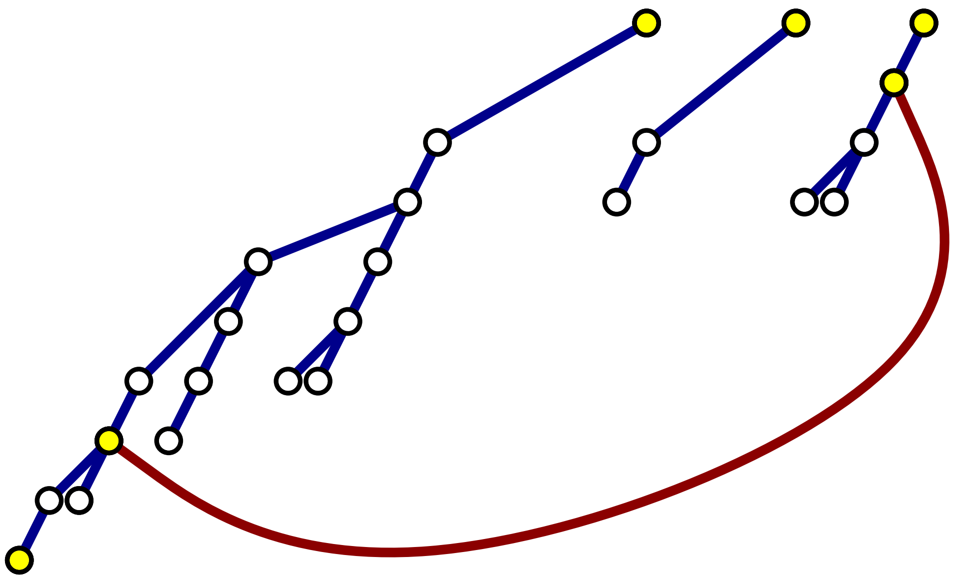

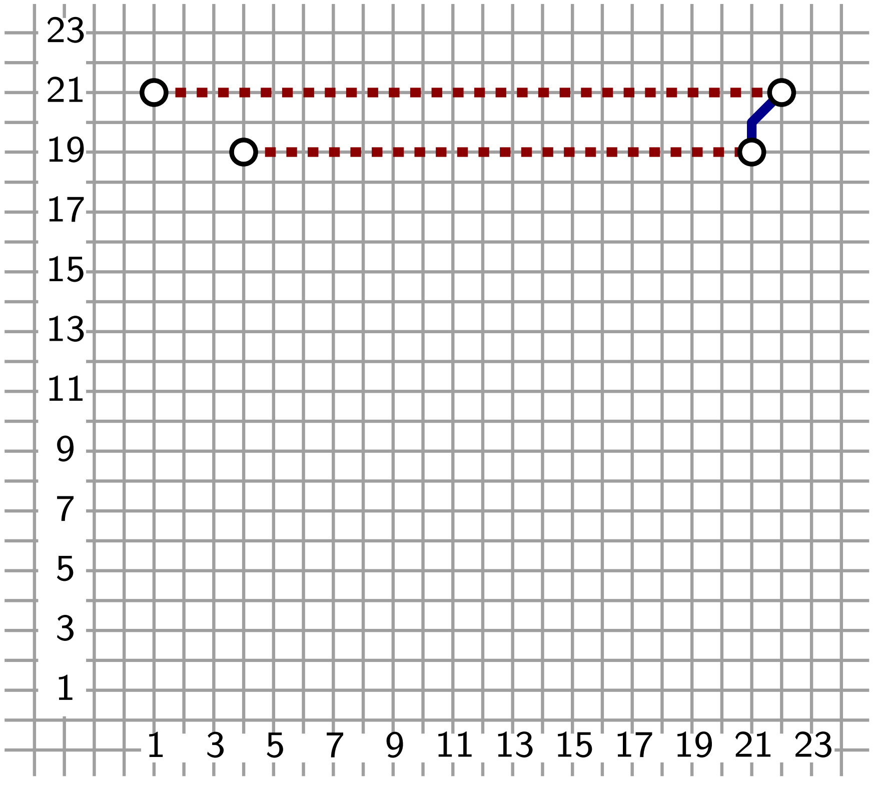

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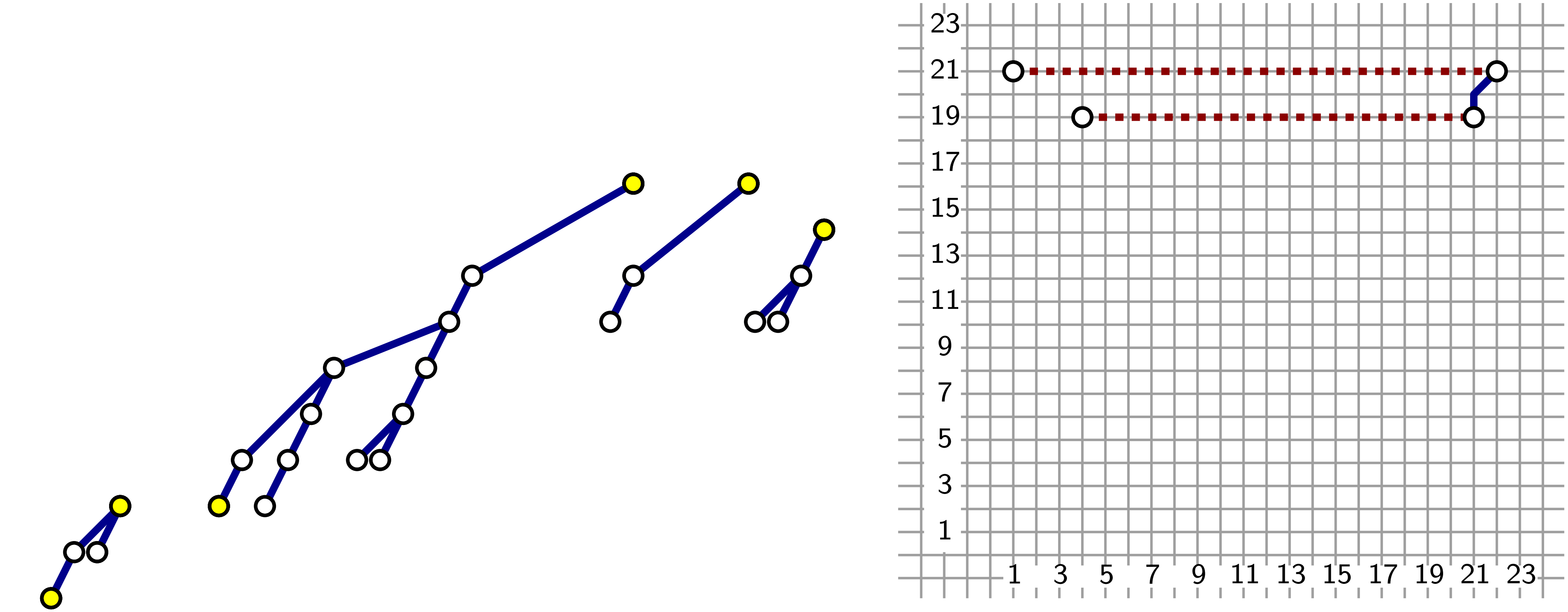

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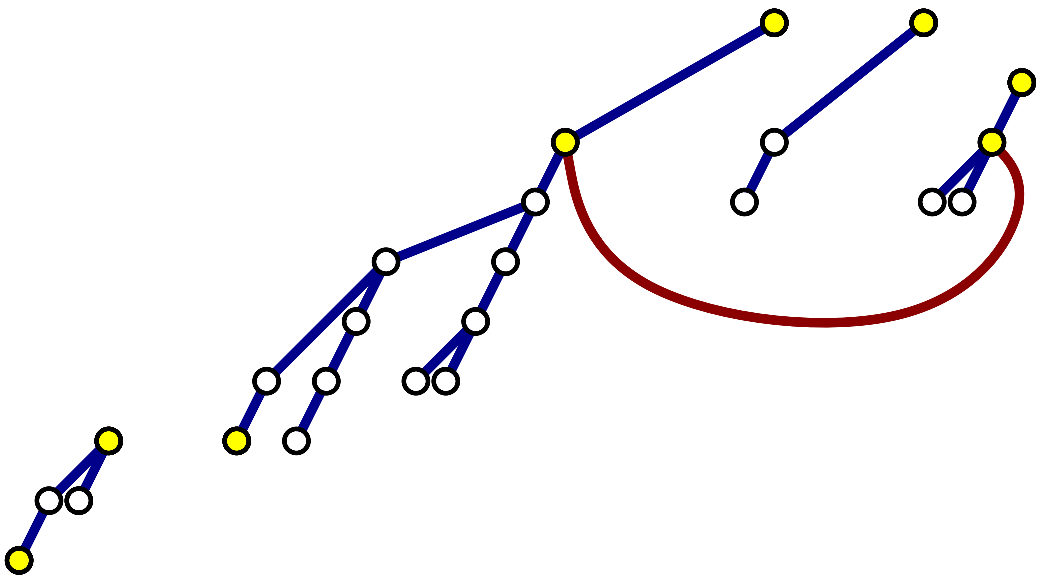

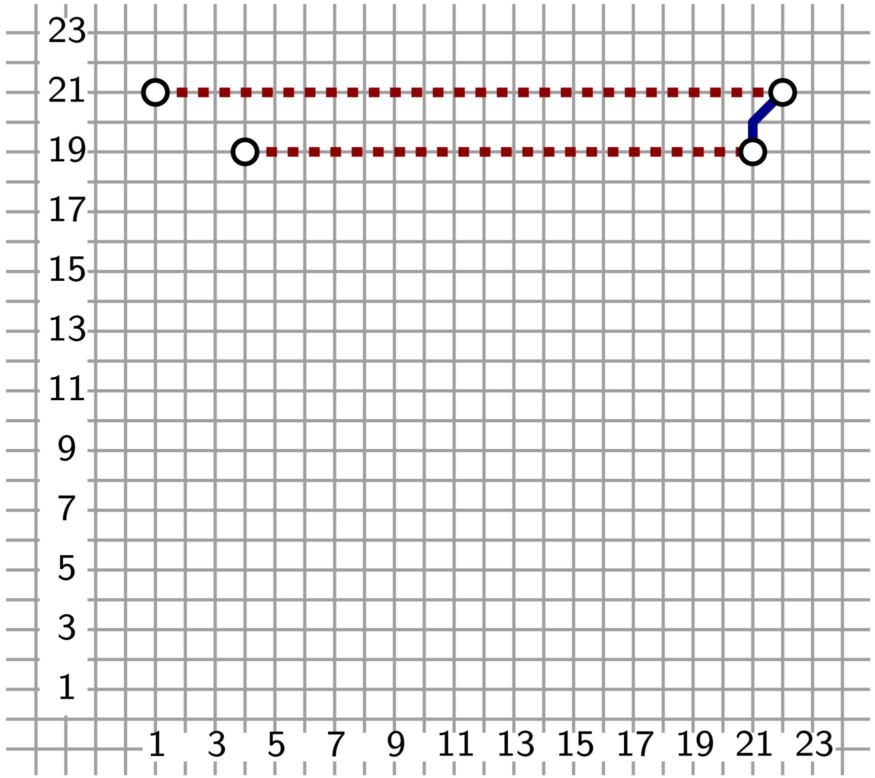

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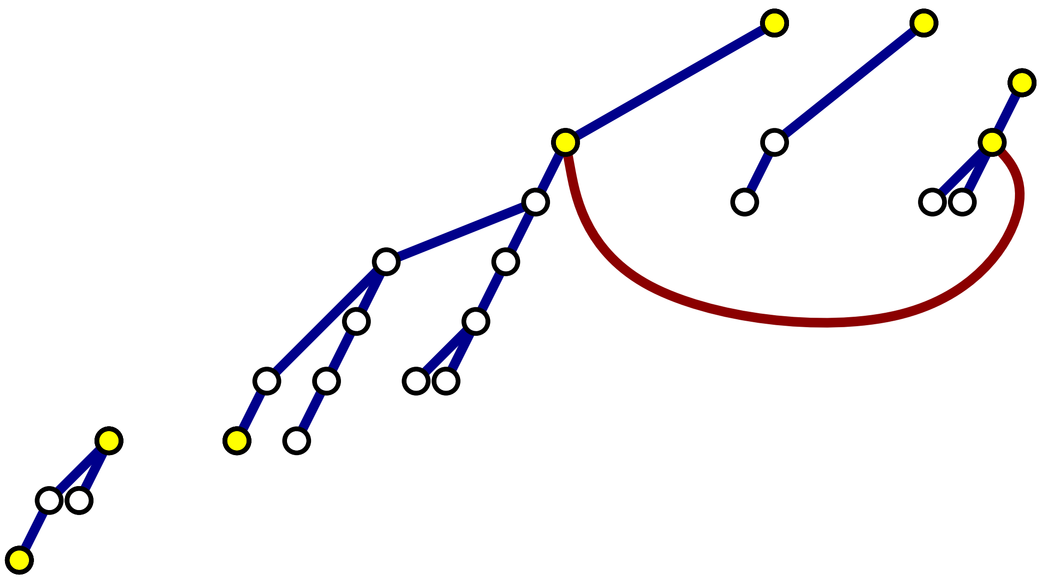

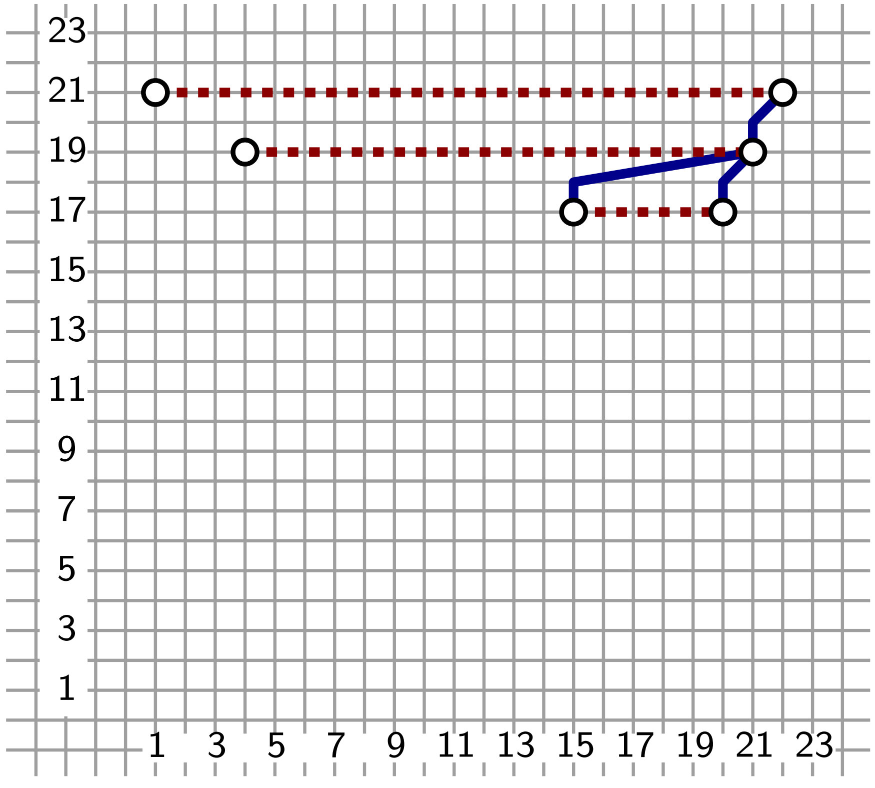

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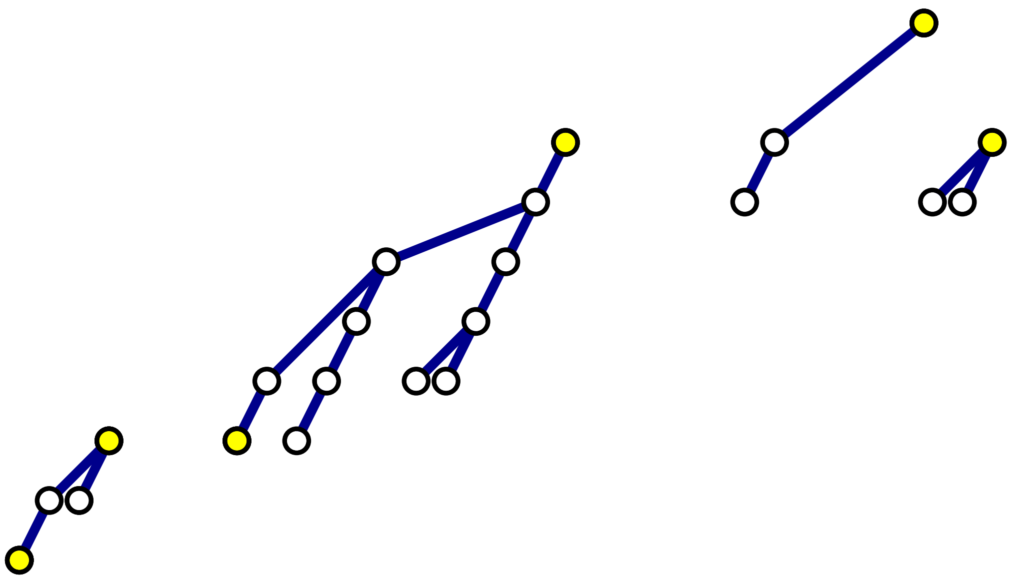

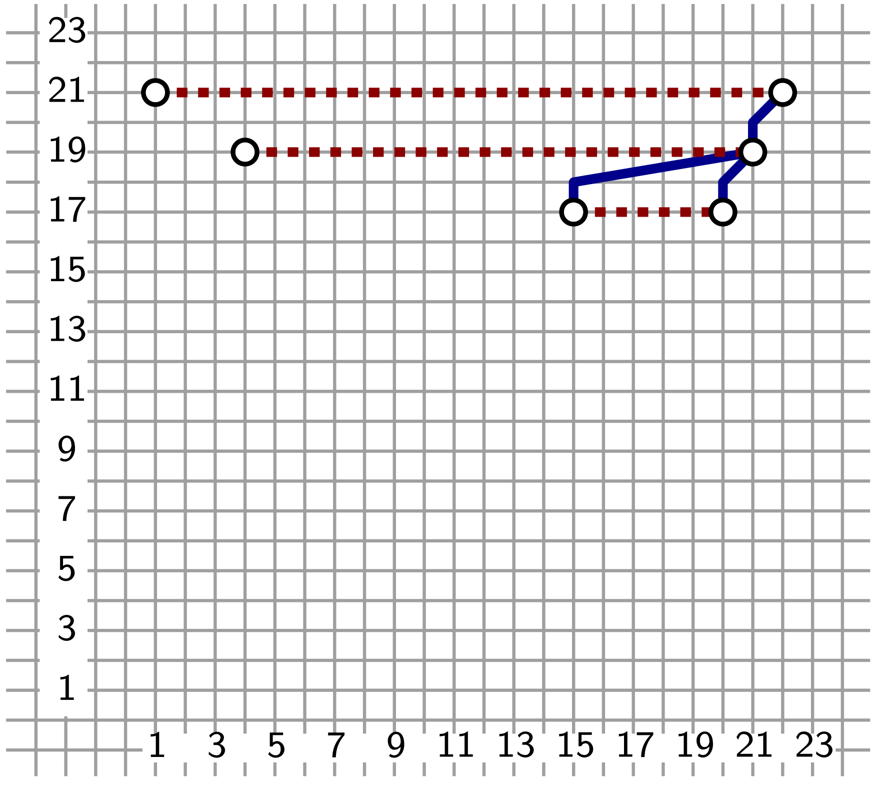

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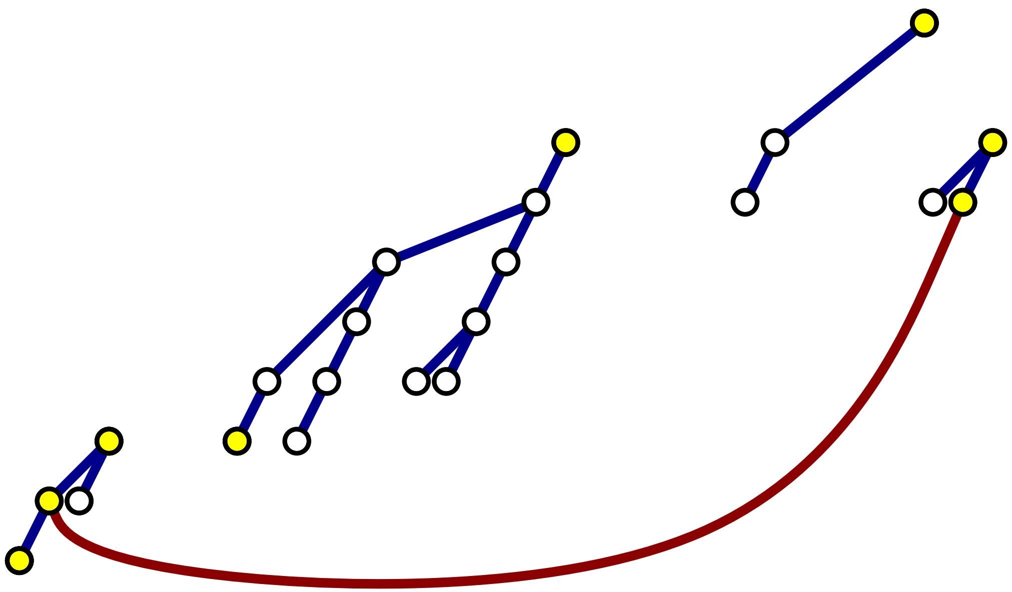

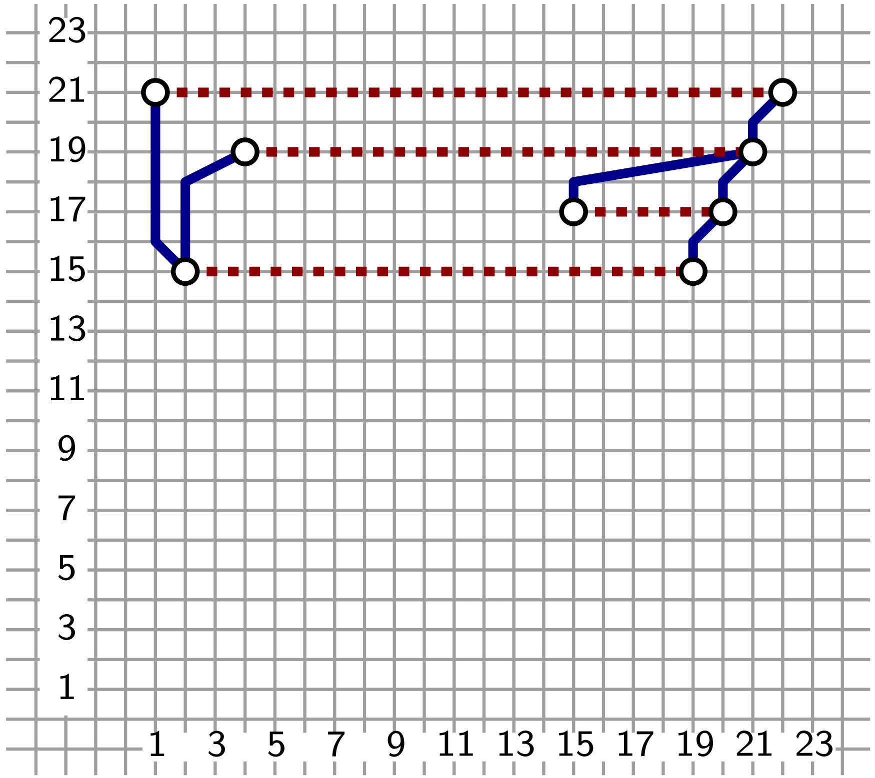

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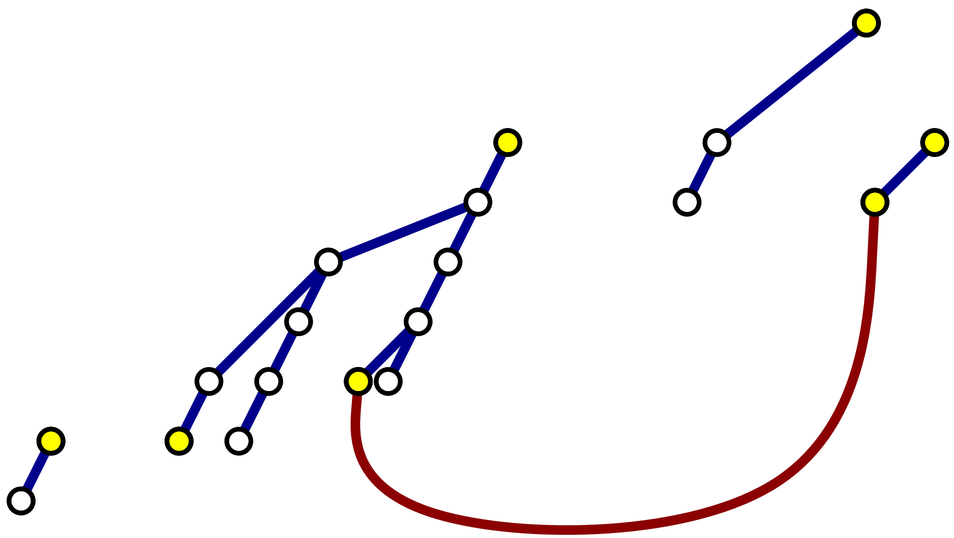

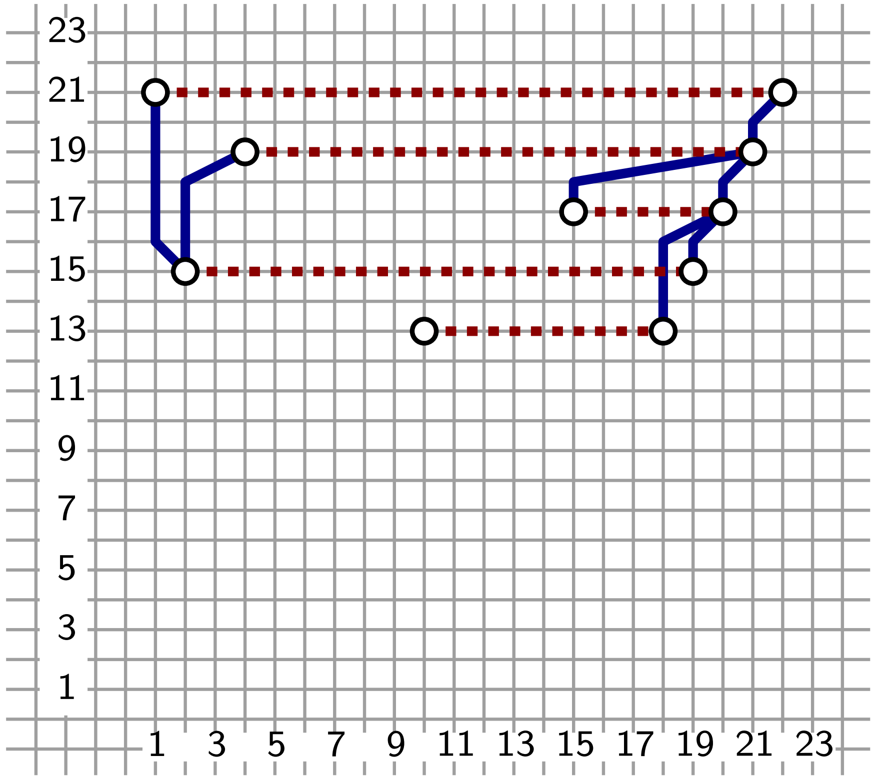

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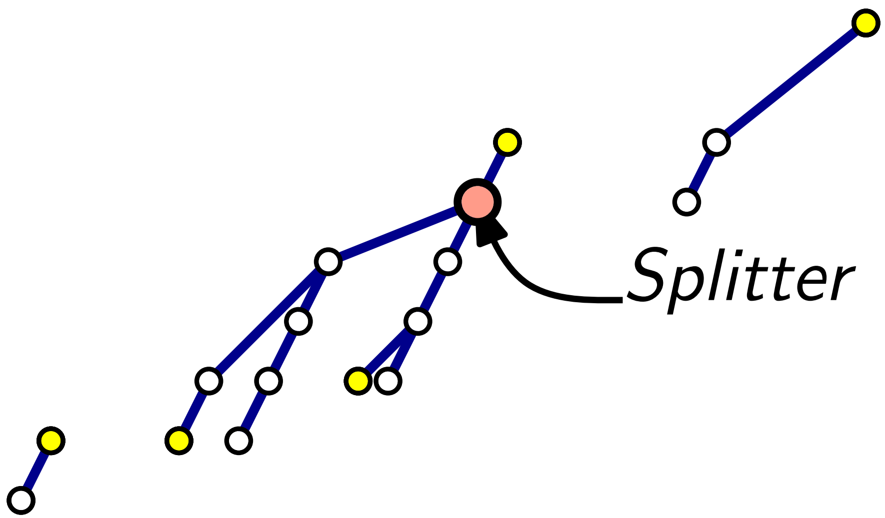

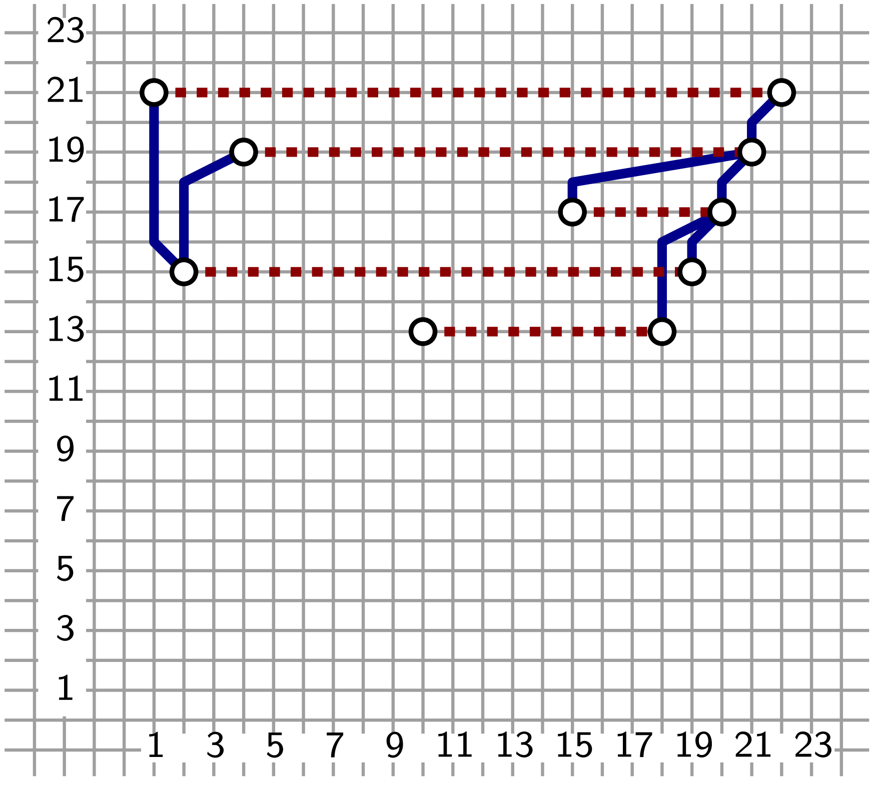

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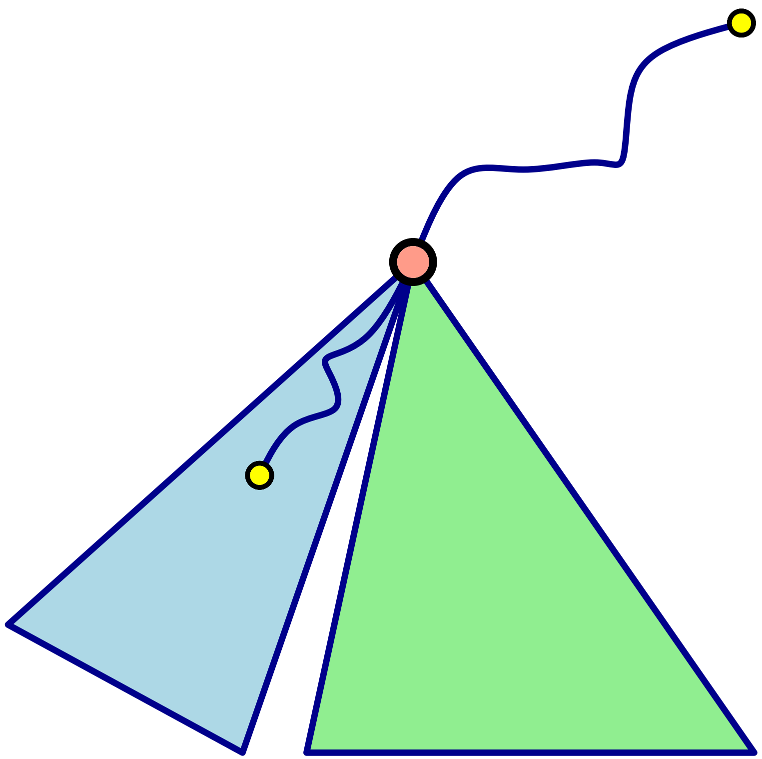

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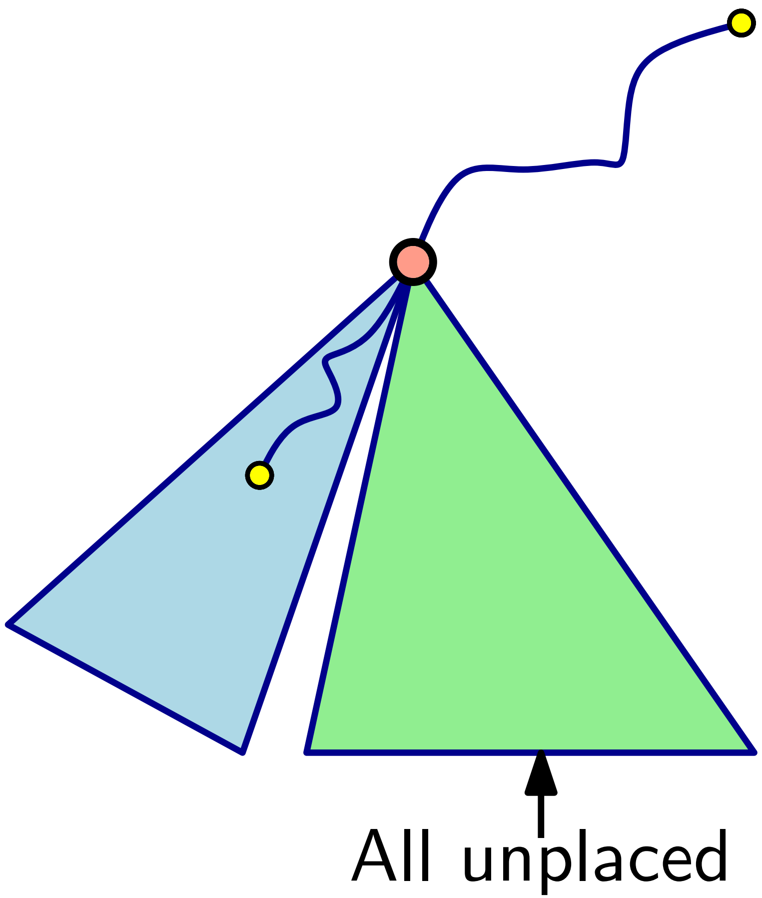

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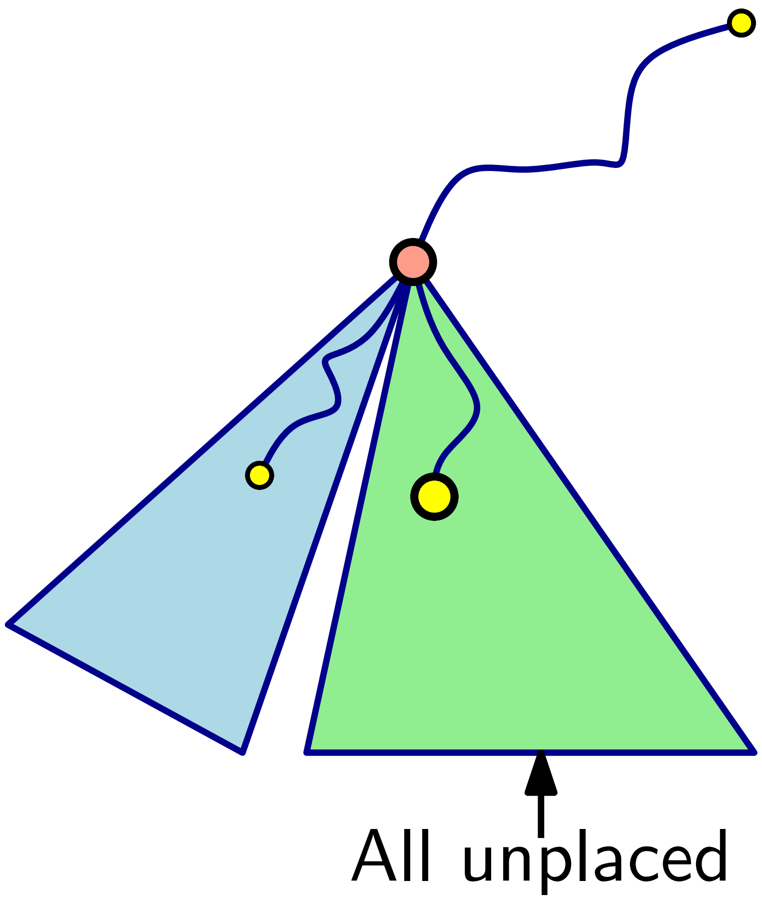

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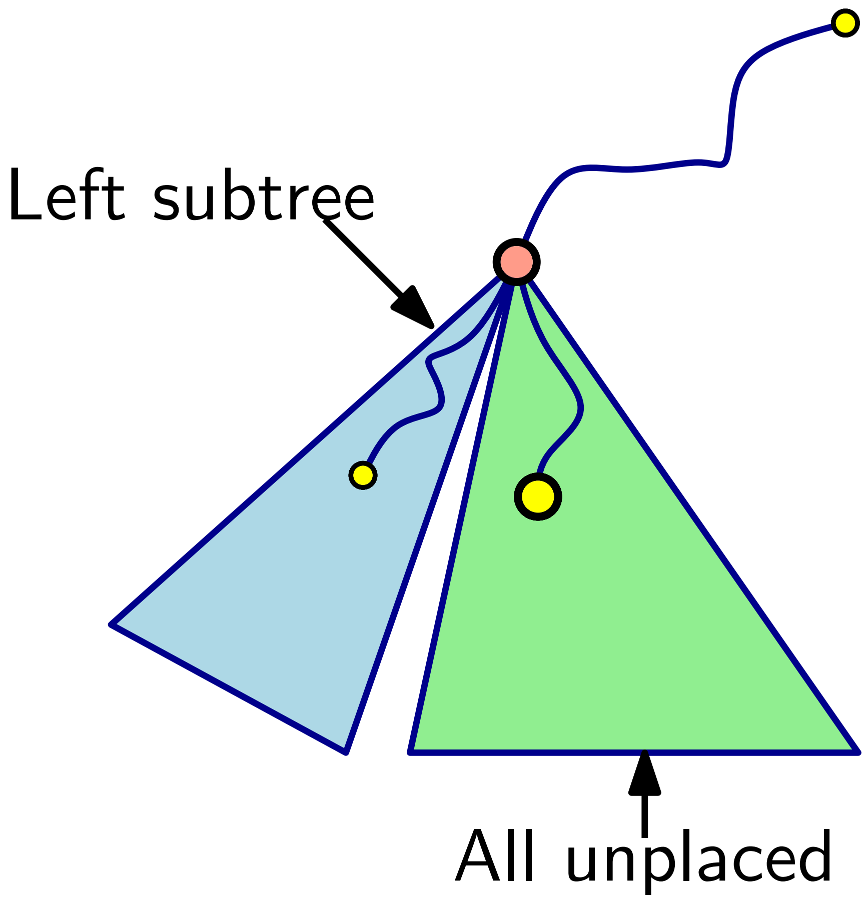

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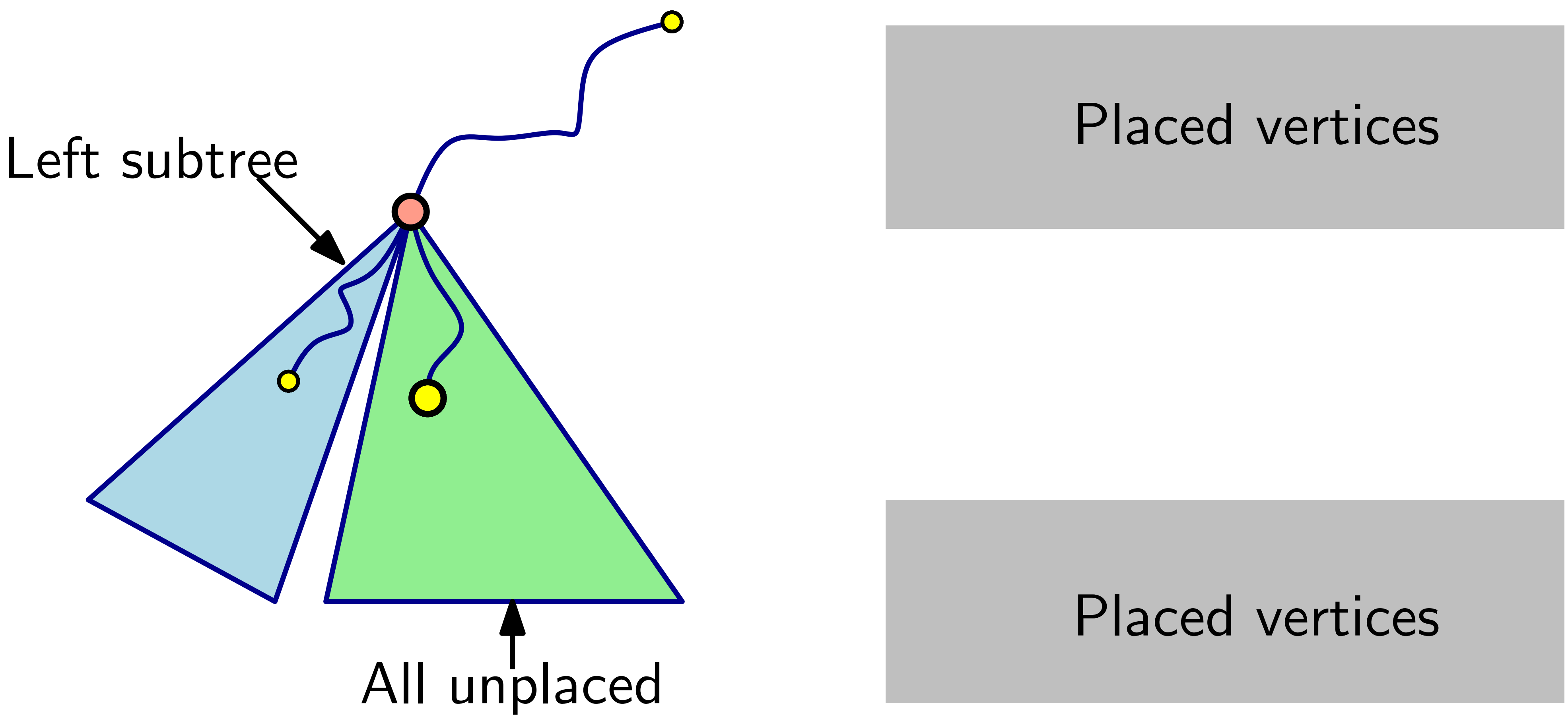

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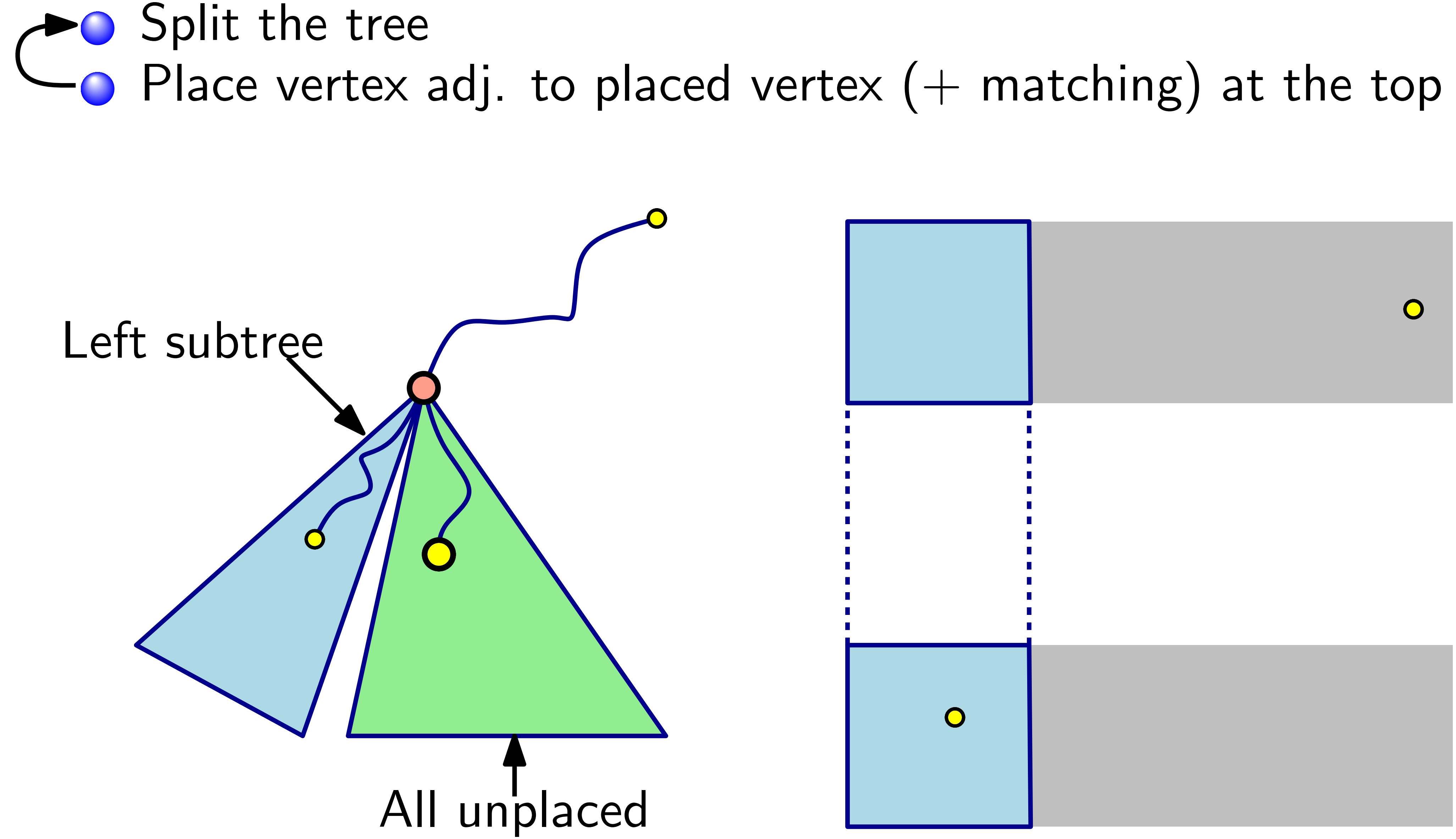

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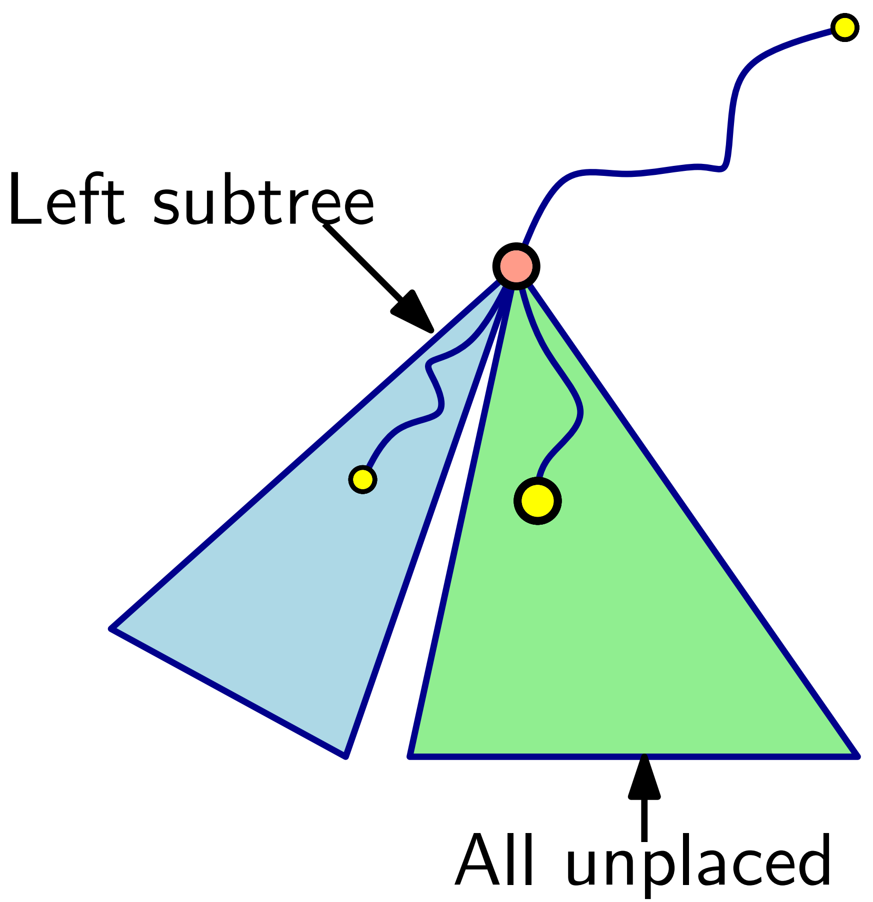
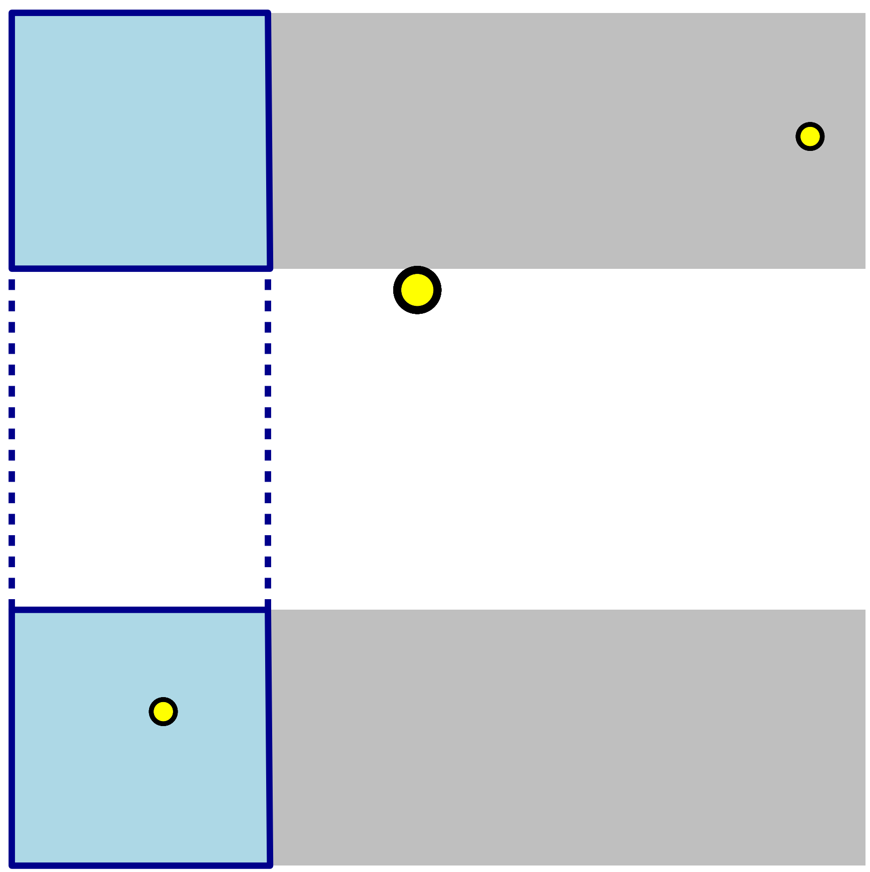
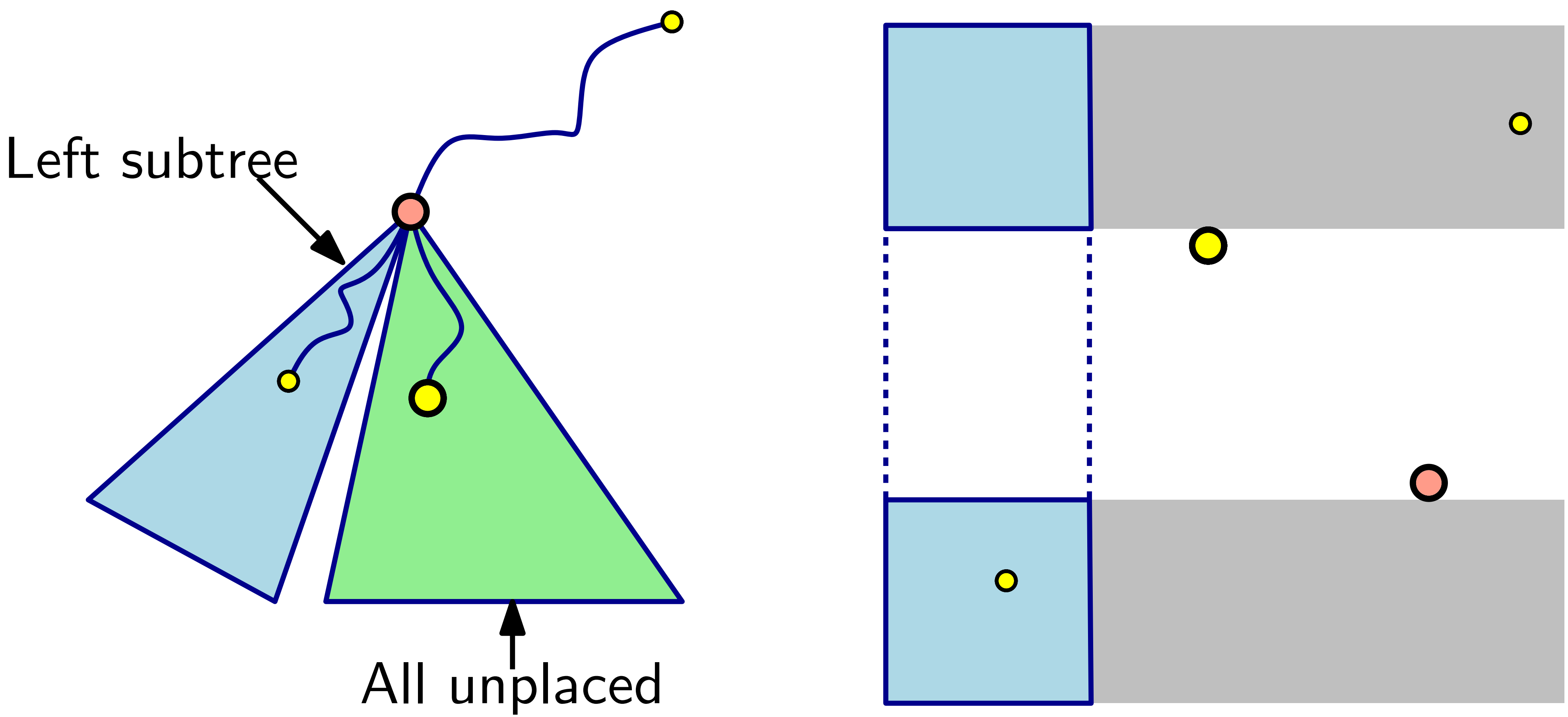




Tree × Matching

- Place root + matching at the top
- Split the tree
- Place vertex adj. to placed vertex (+ matching) at the top



Tree × Matching

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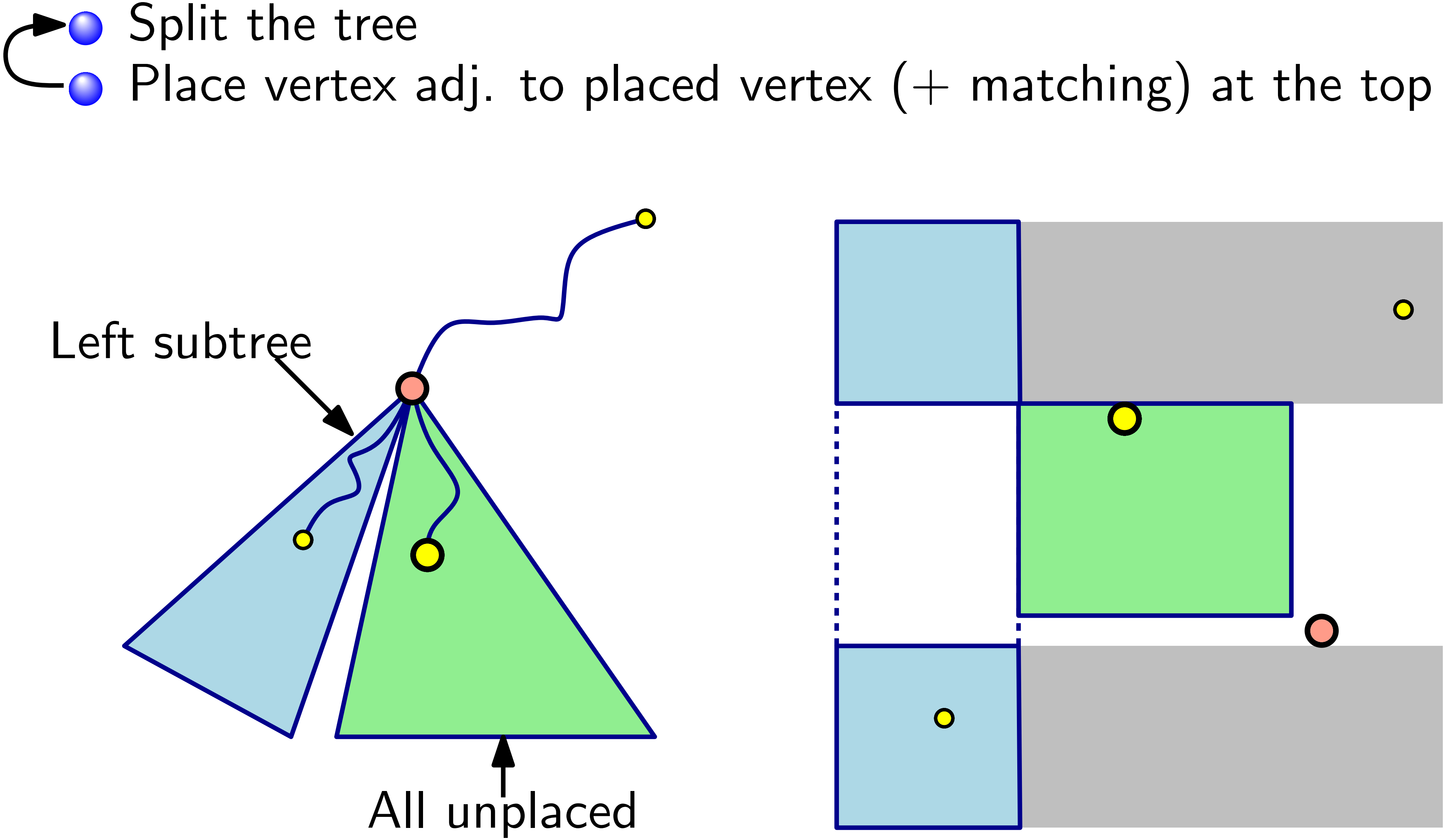

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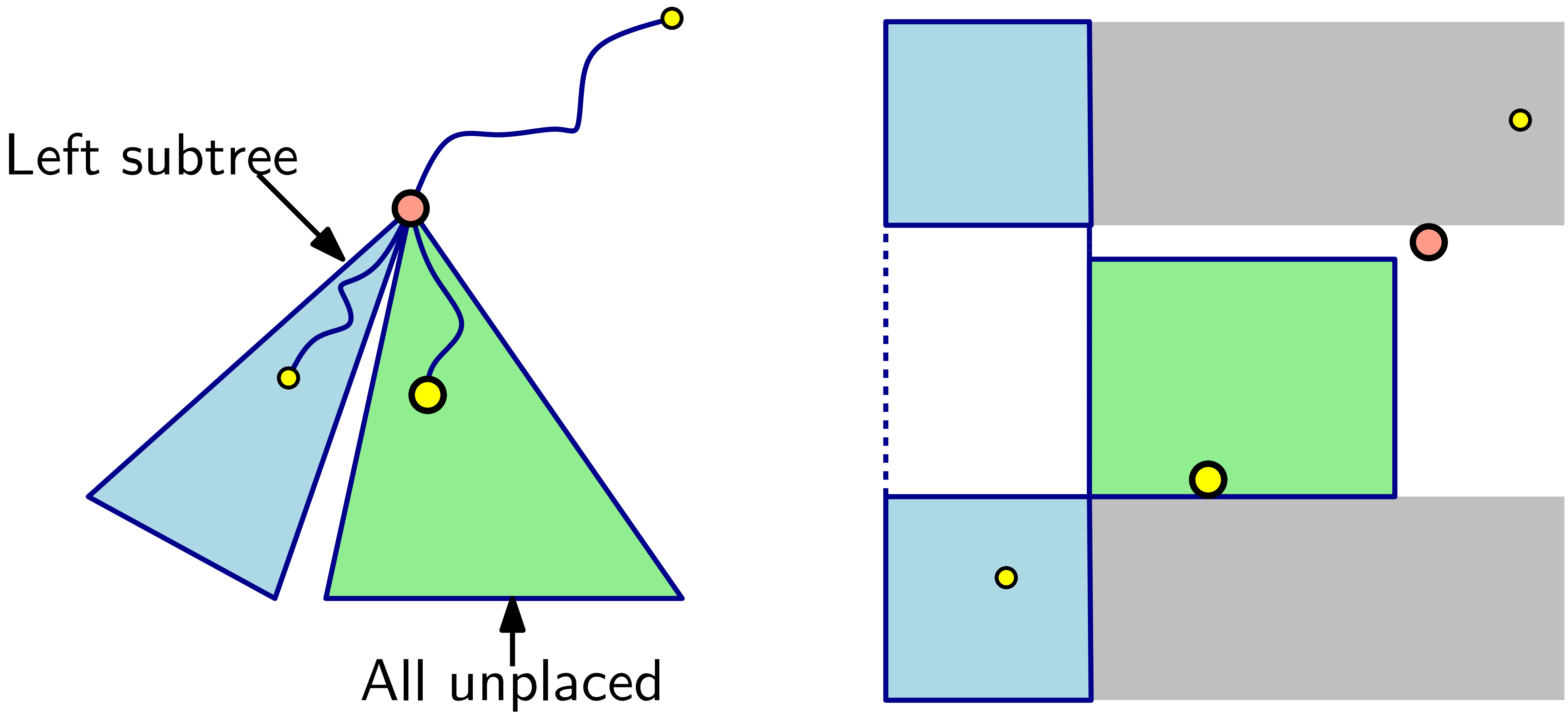
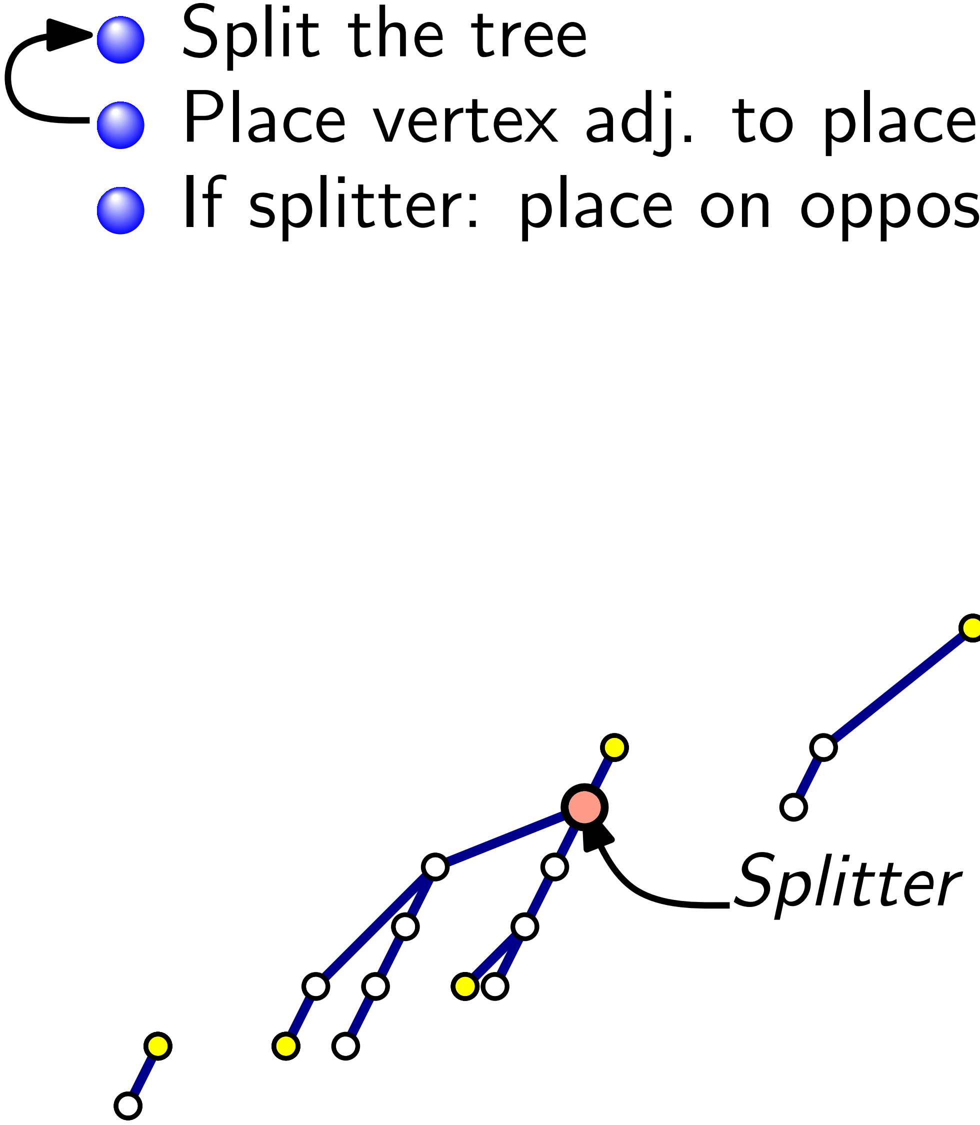
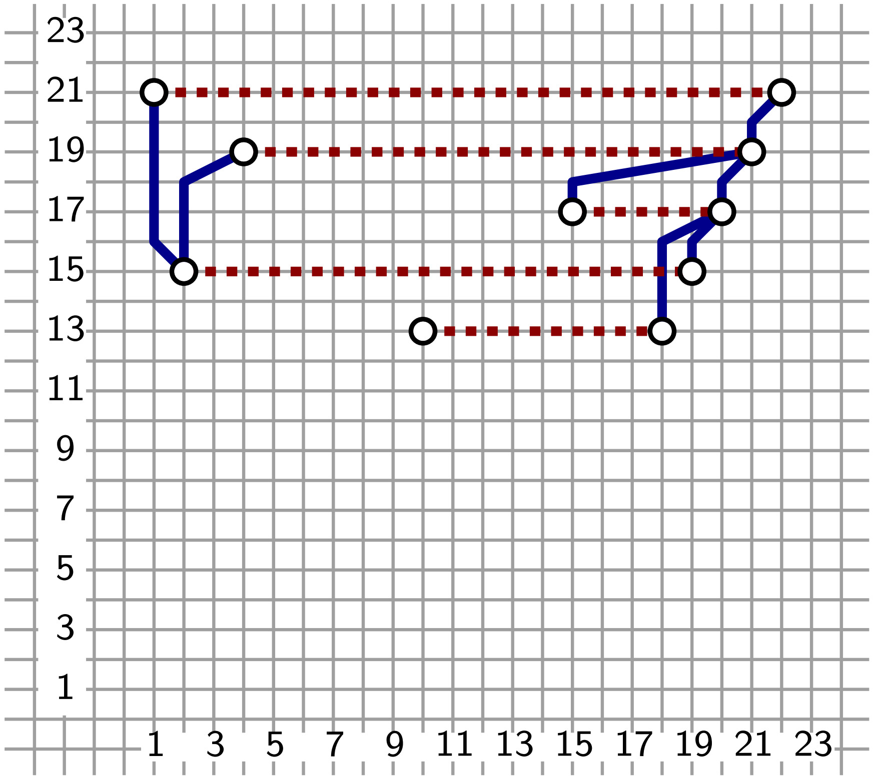


Tree × Matching

- Place root + matching at the top
- Split the tree
- Place vertex adj. to placed vertex (+ matching) at the top
- If splitter: place on opposite side





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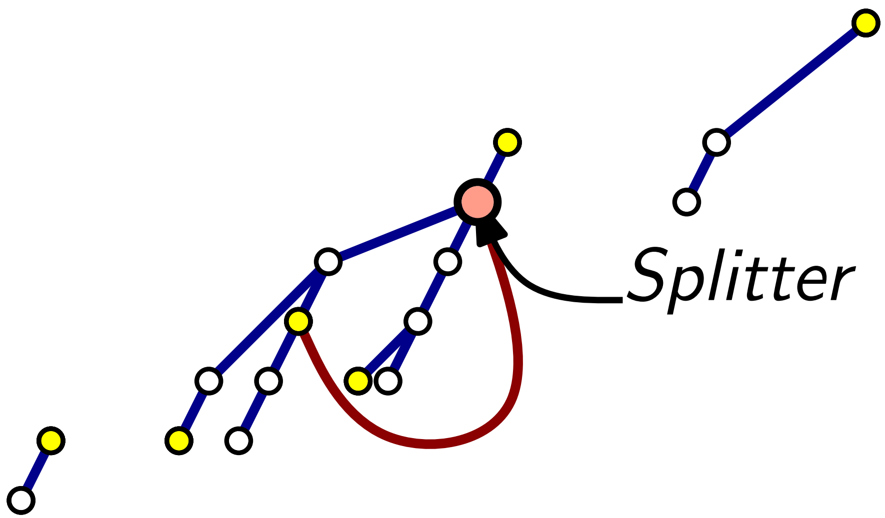

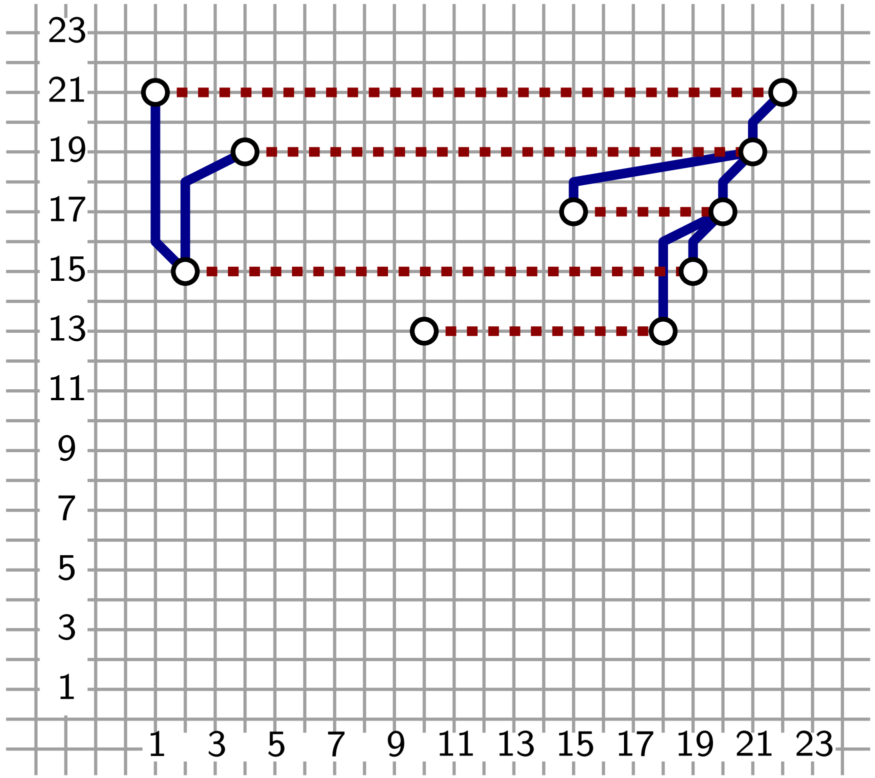

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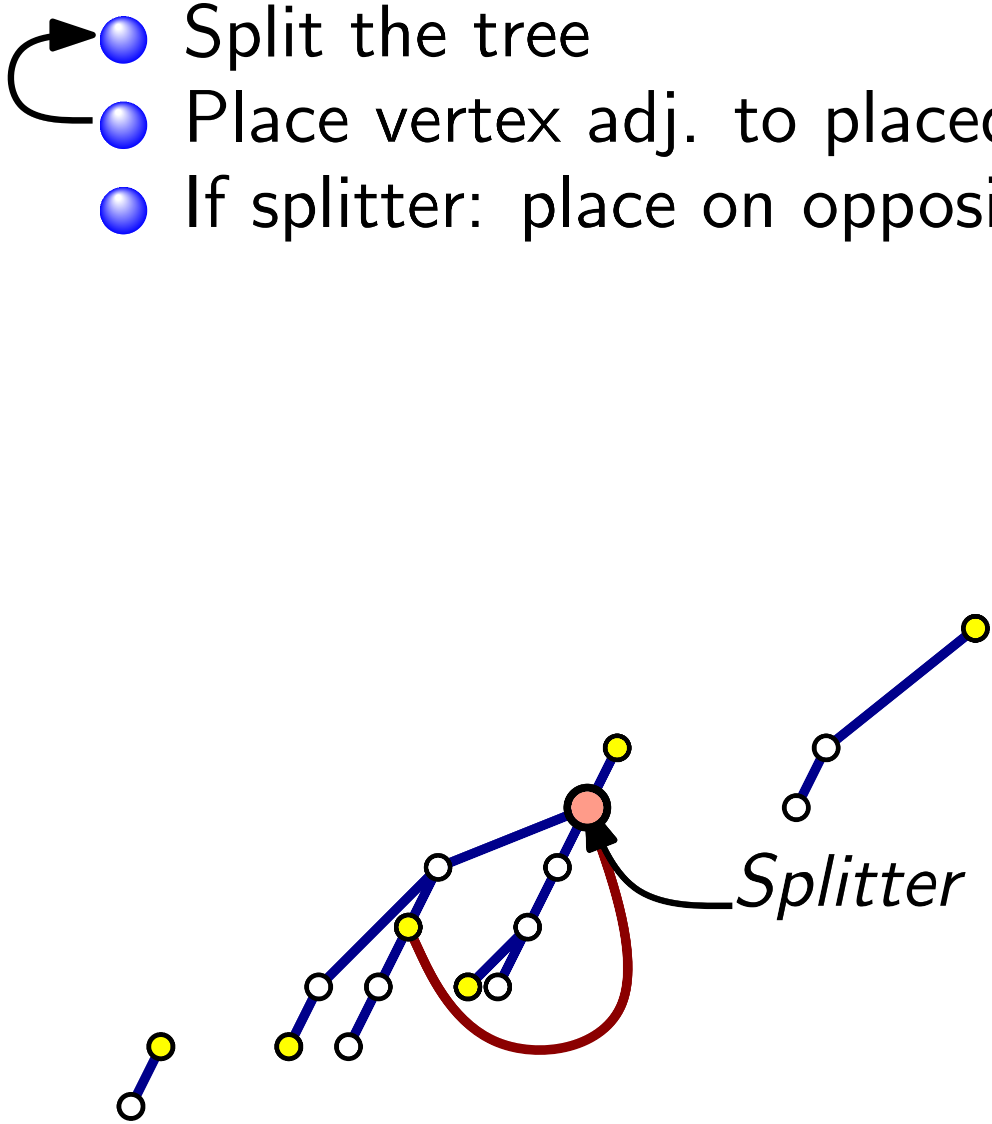

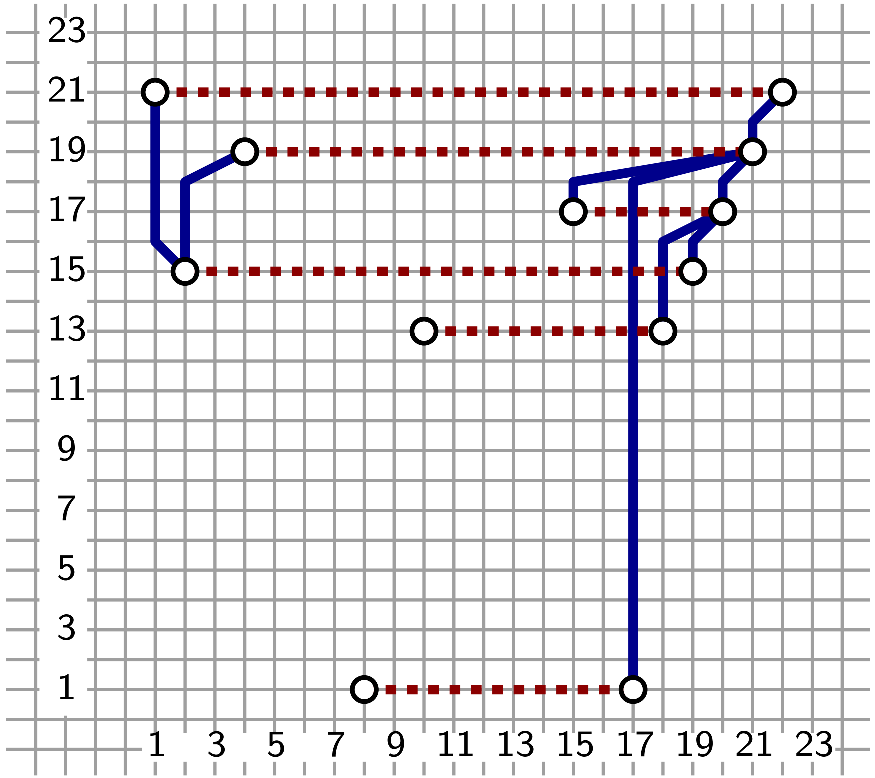

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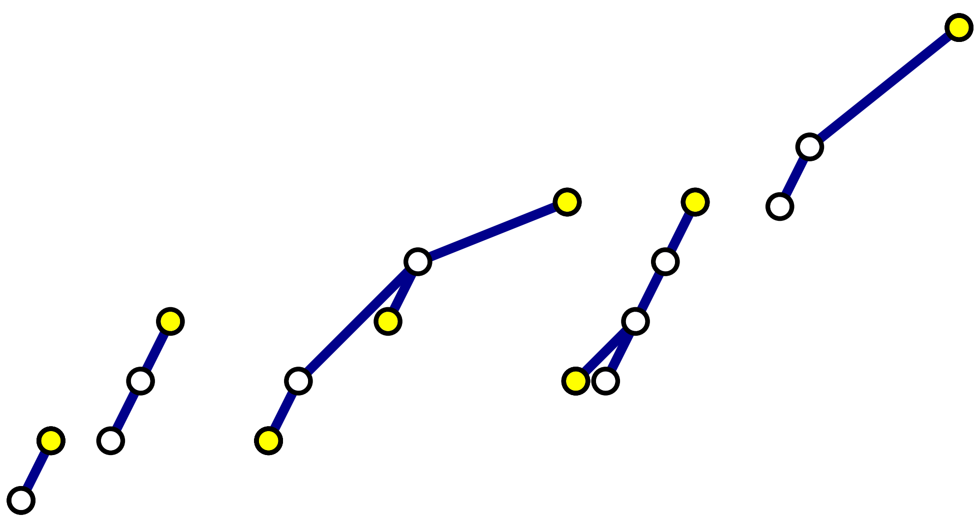

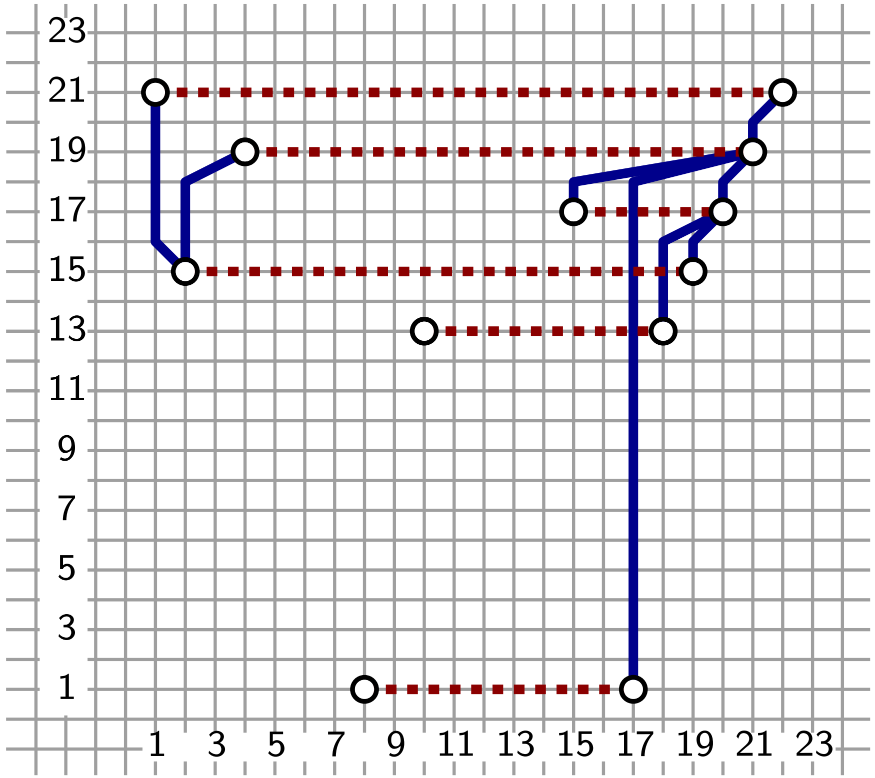

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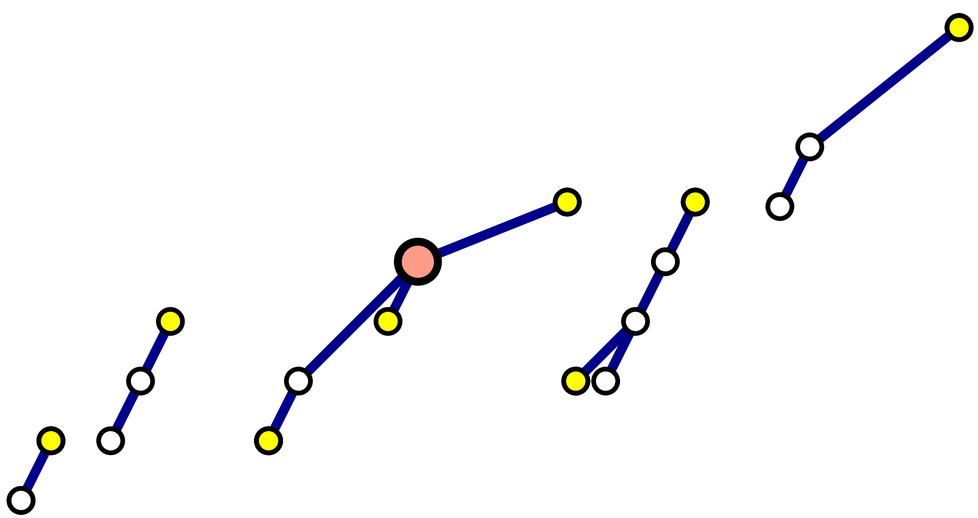

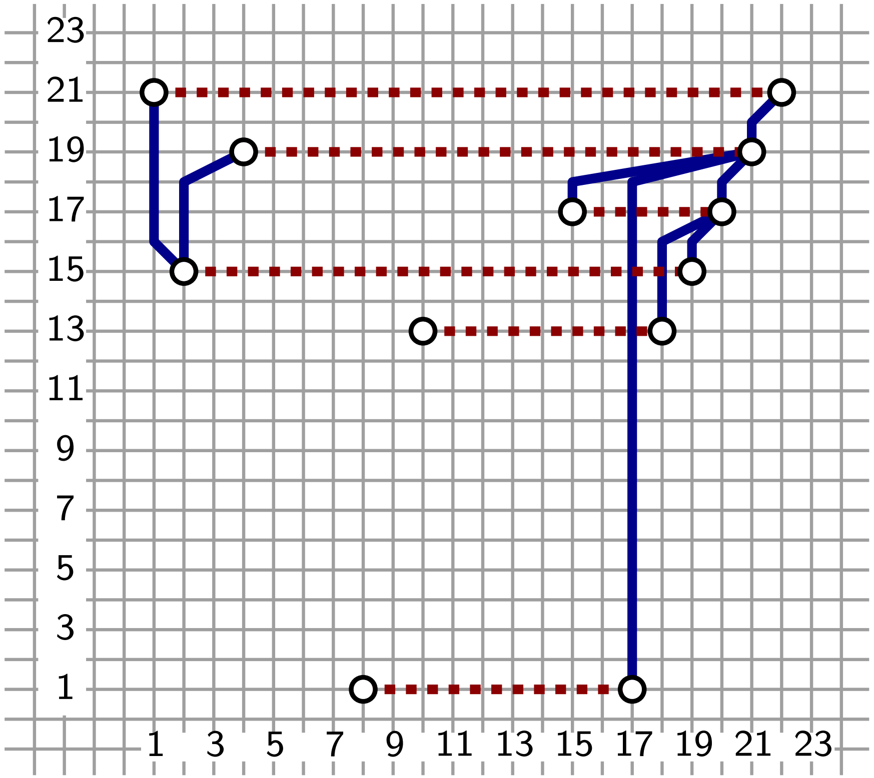

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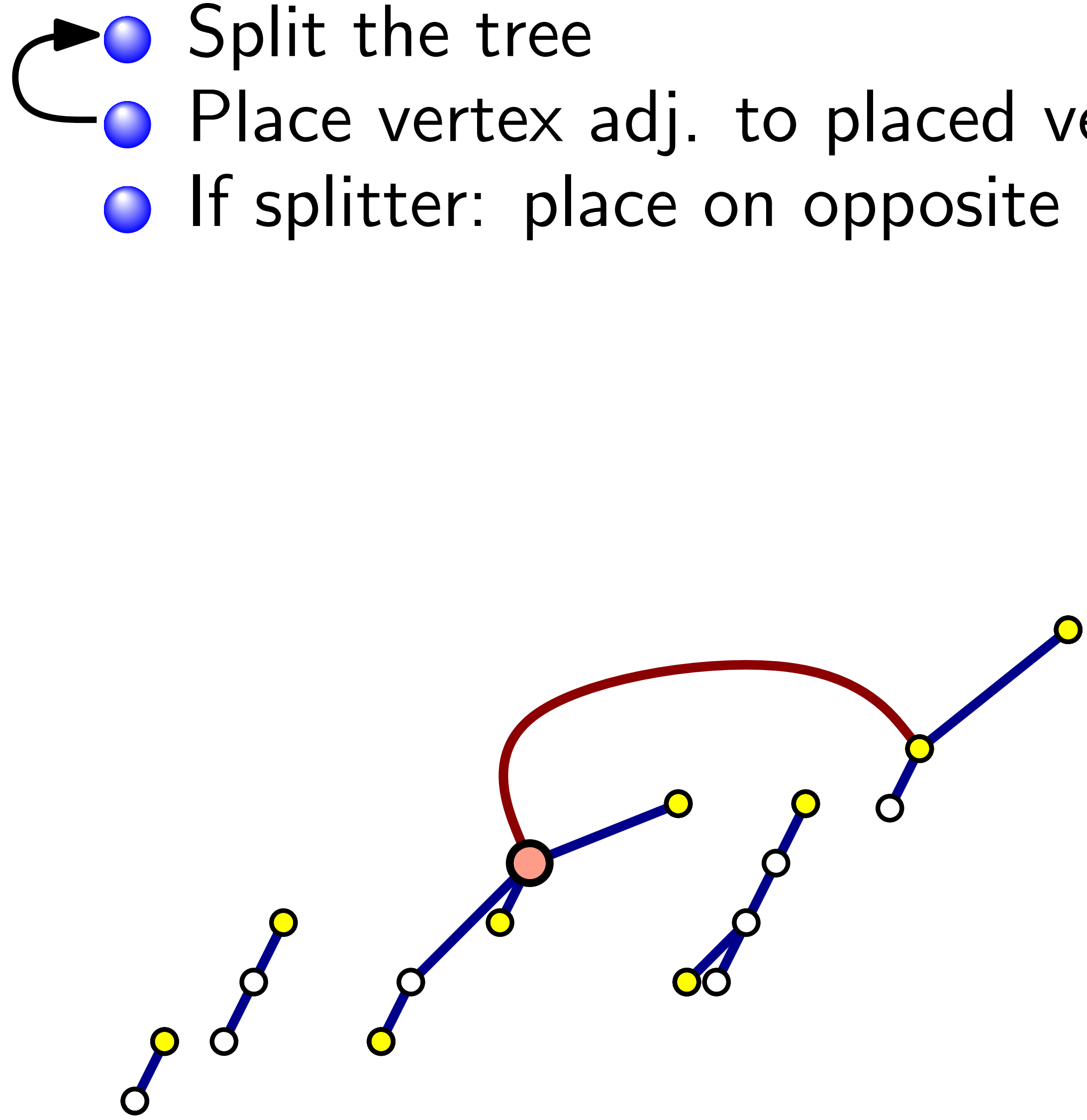

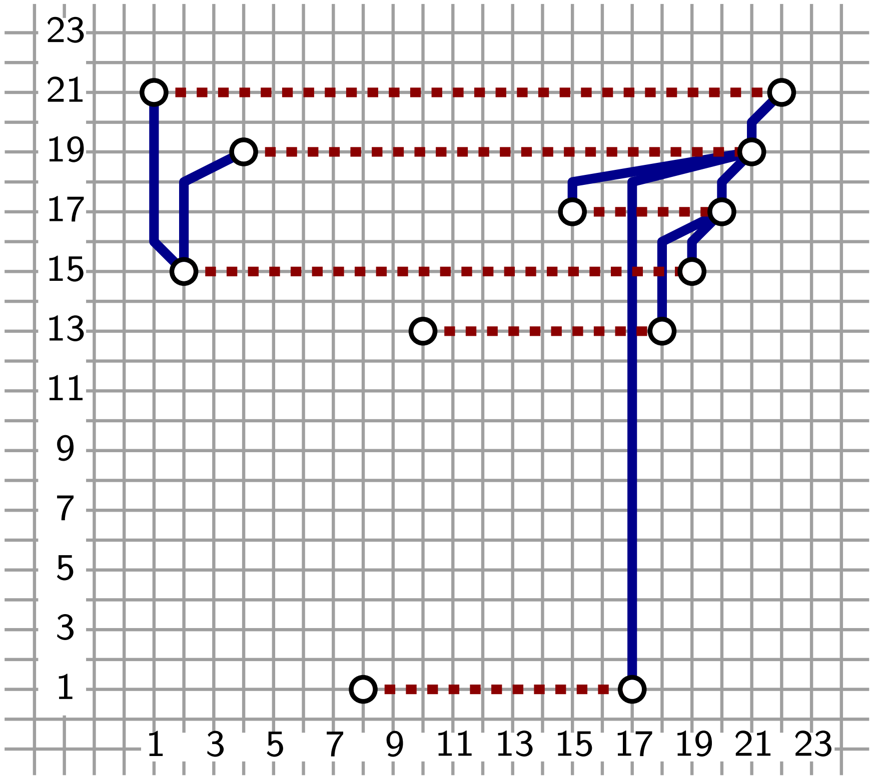

Tree × Matching

- Place root + matching at the top
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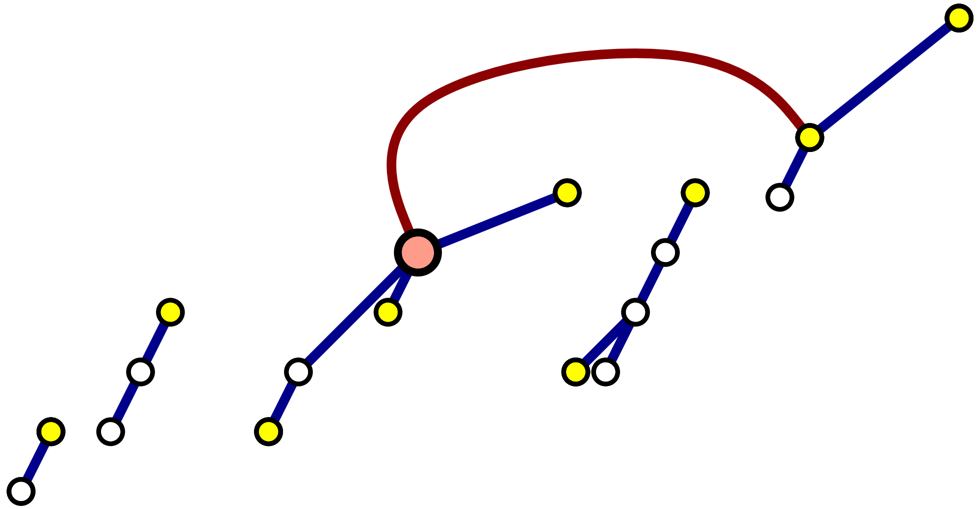

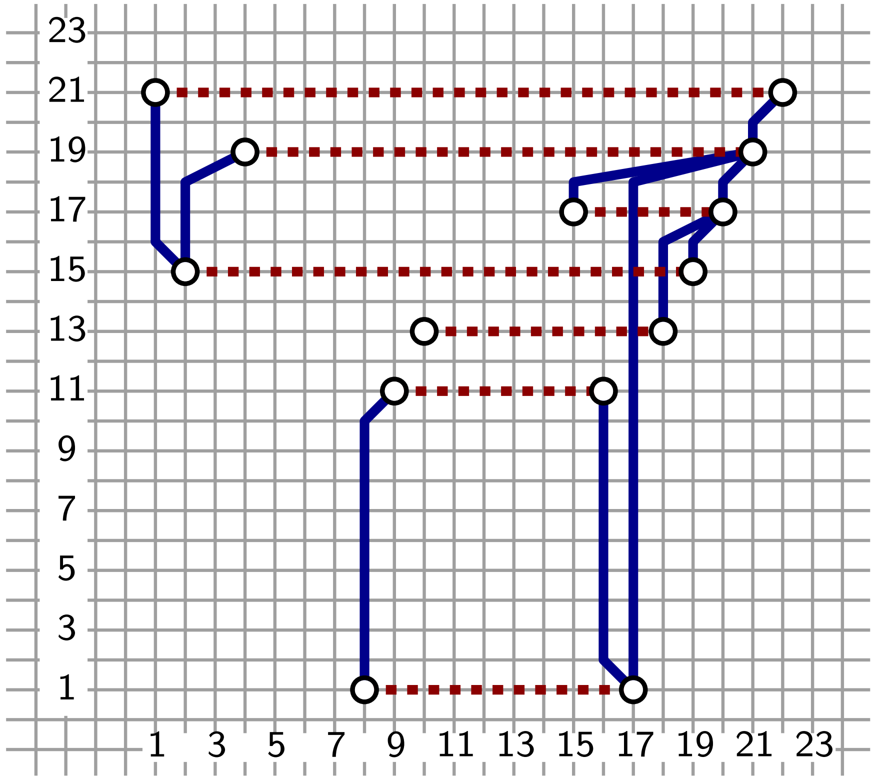

Tree × Matching

- Place root + matching at the top
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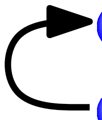
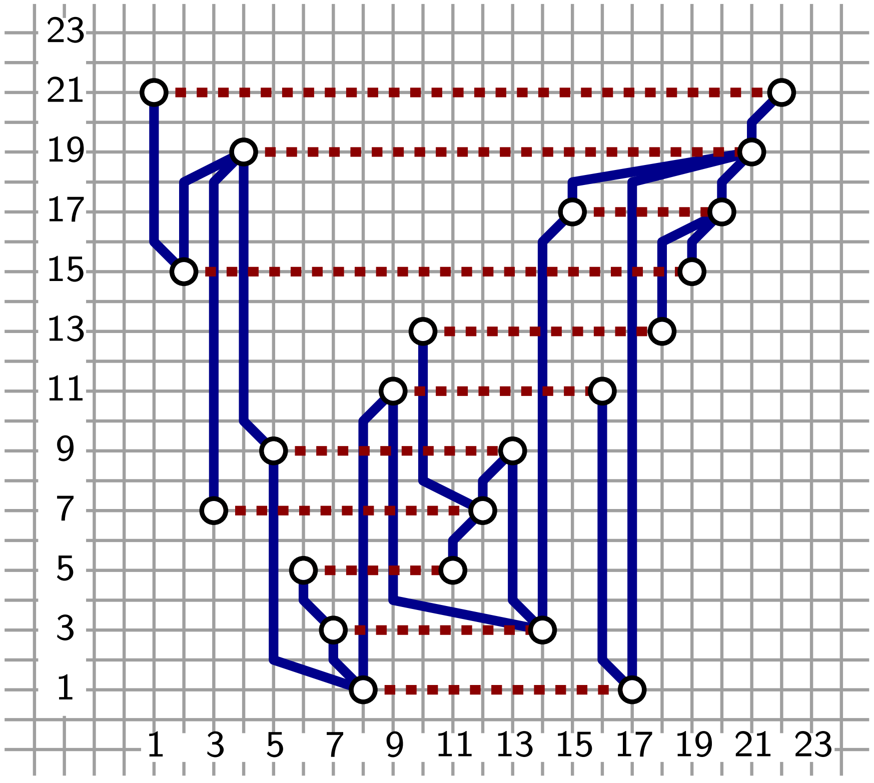
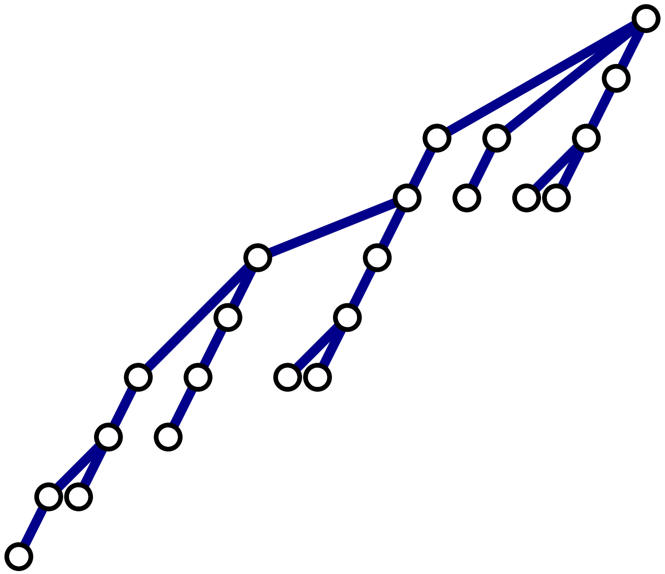


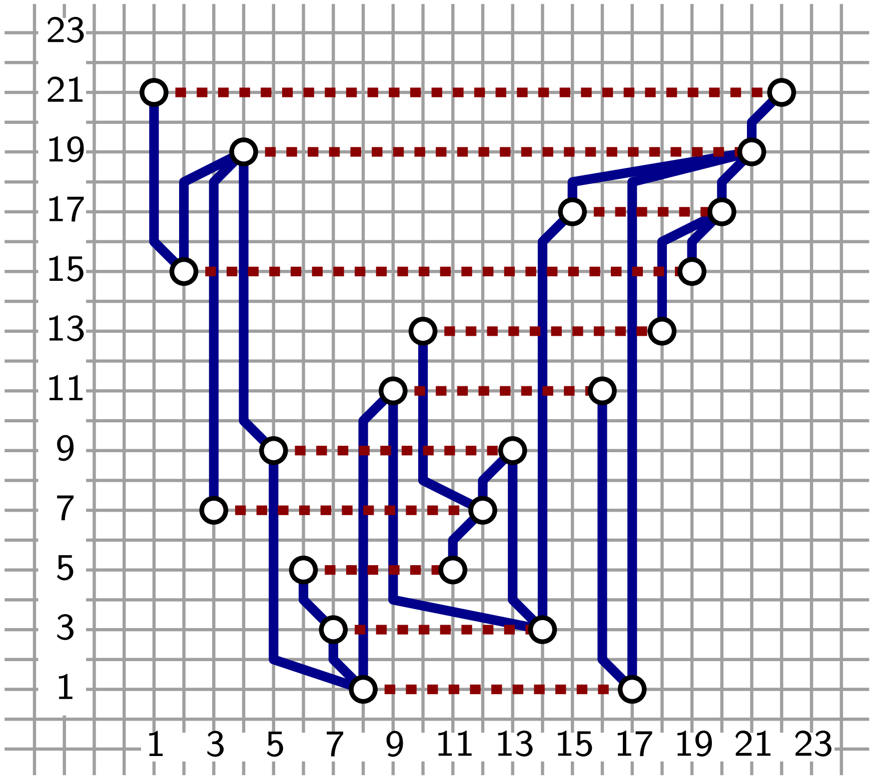


Tree × Matching

- Place root + matching at the top
- Split the tree
- Place vertex adj. to placed vertex (+ matching) at the top
- If splitter: place on opposite side

Bends:	1×0
Grid size:	$n \times (n-1)$

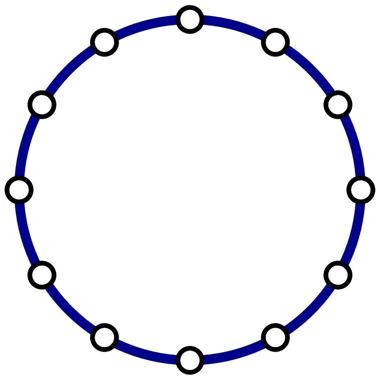




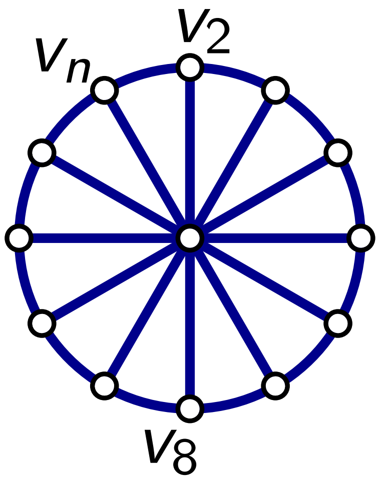
Overview

Graph classes			Number of bends
Cycle	×	Cycle	1×1
Caterpillar	×	Cycle	1×1
Four Matchings			$1 \times 1 \times 1 \times 1$
Tree	×	Matching	1×0 ✓
Wheel	×	Matching	2×0
Outerpath	×	Matching	2×1
Outerplanar	×	Outerplanar	3×3
2-page book emb.	×	2-page book emb.	4×4
Planar	×	Planar	6×6

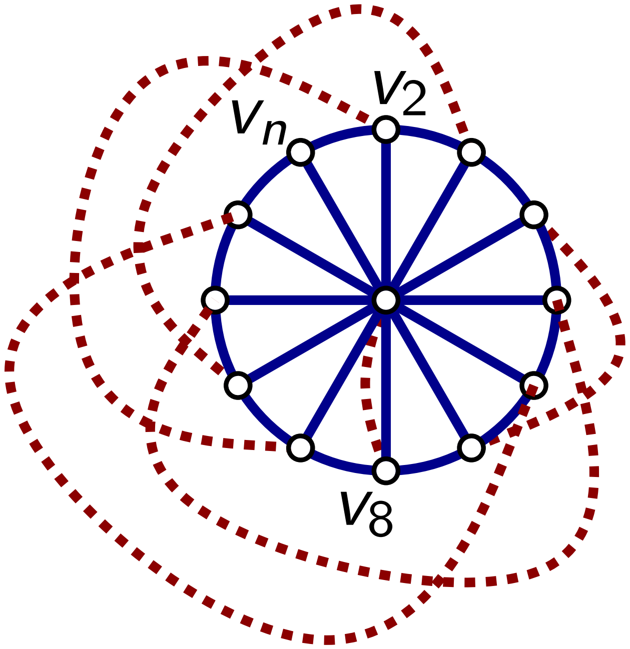
Wheel $\times$ Matching



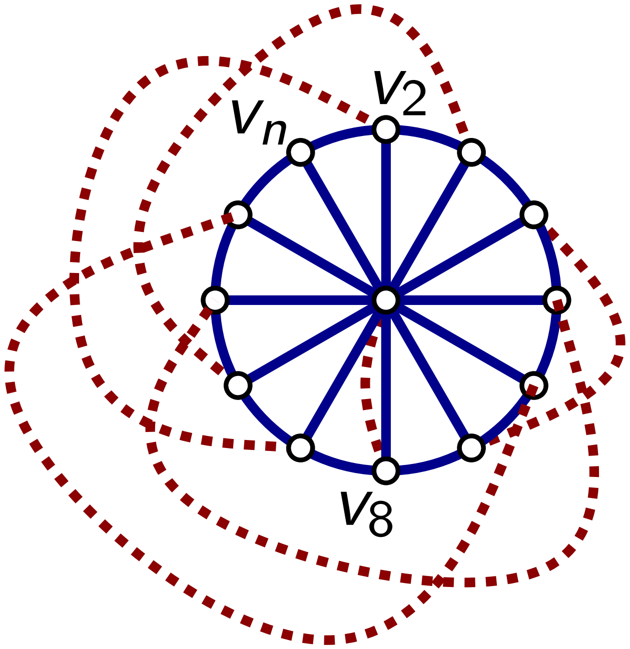
Wheel × Matching

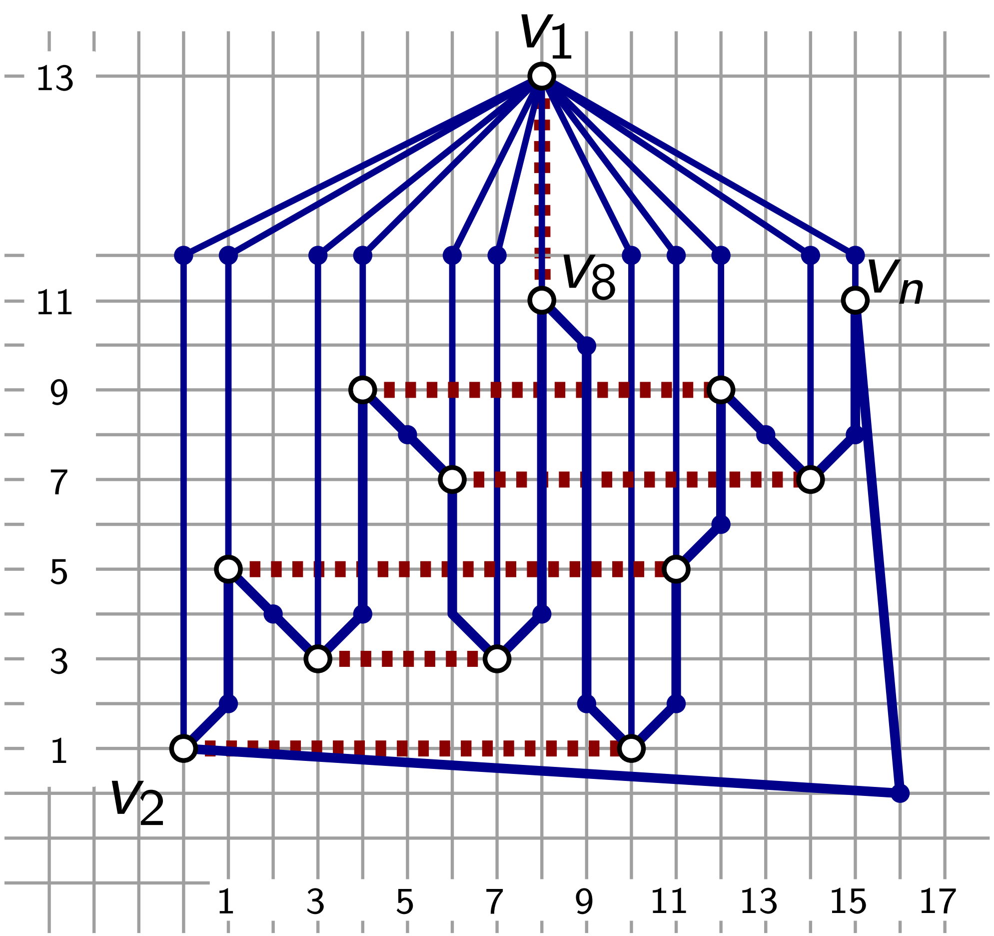


Wheel × Matching

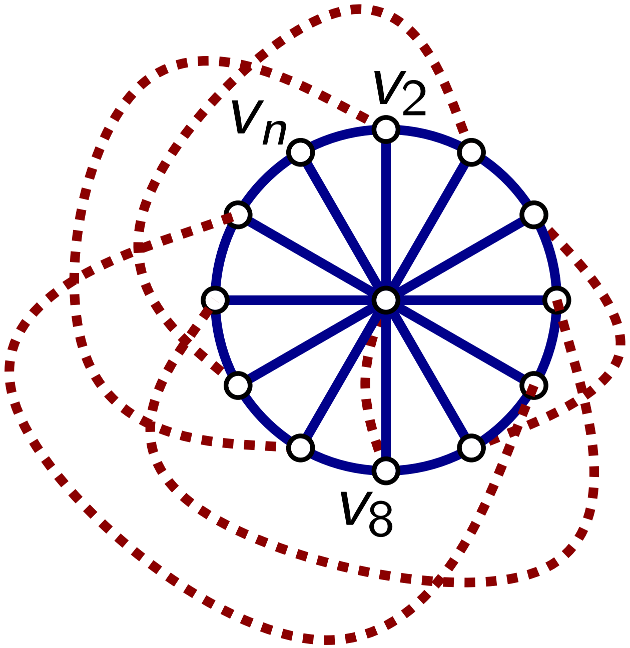


Wheel × Matching

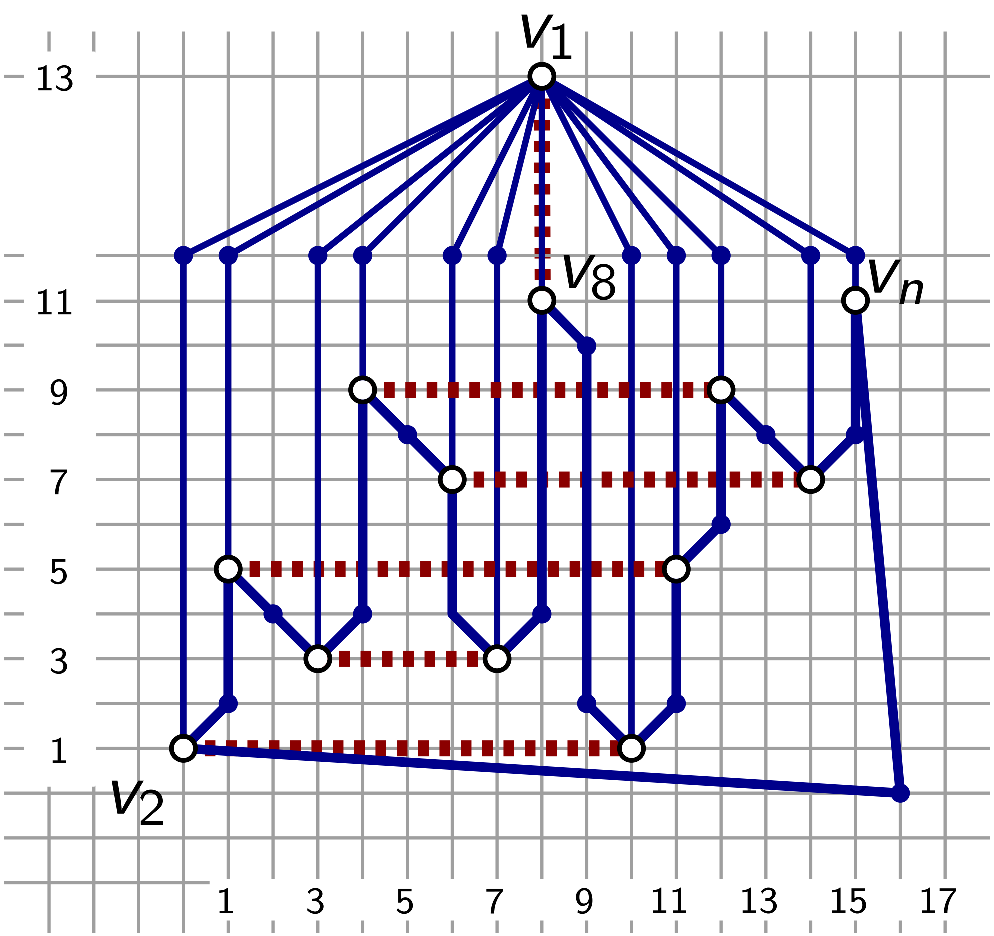




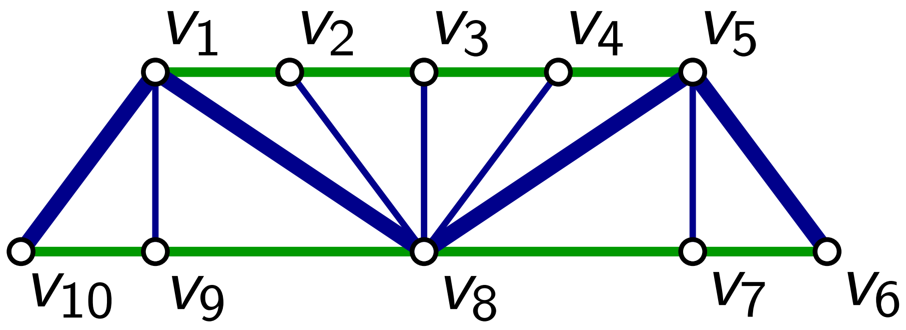
Wheel × Matching



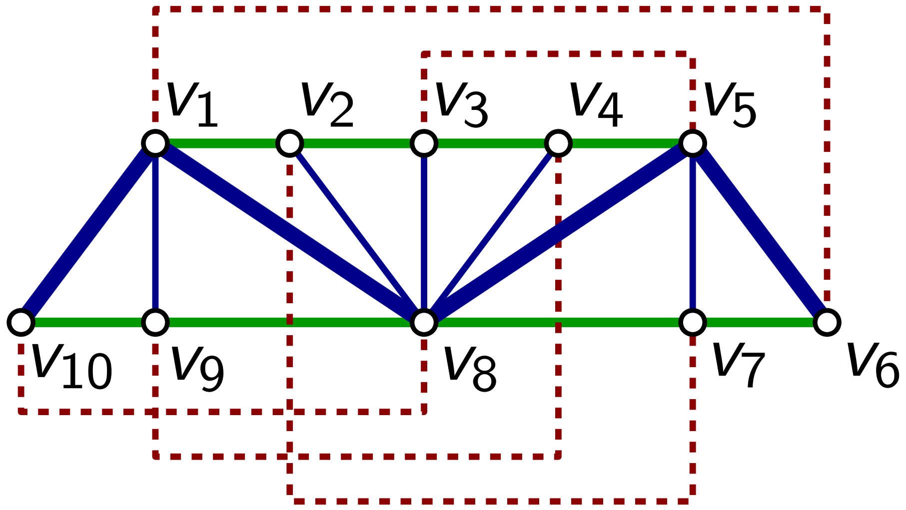
Bends: 2×0
Grid size: $(1.5n - 1) \times (n + 2)$



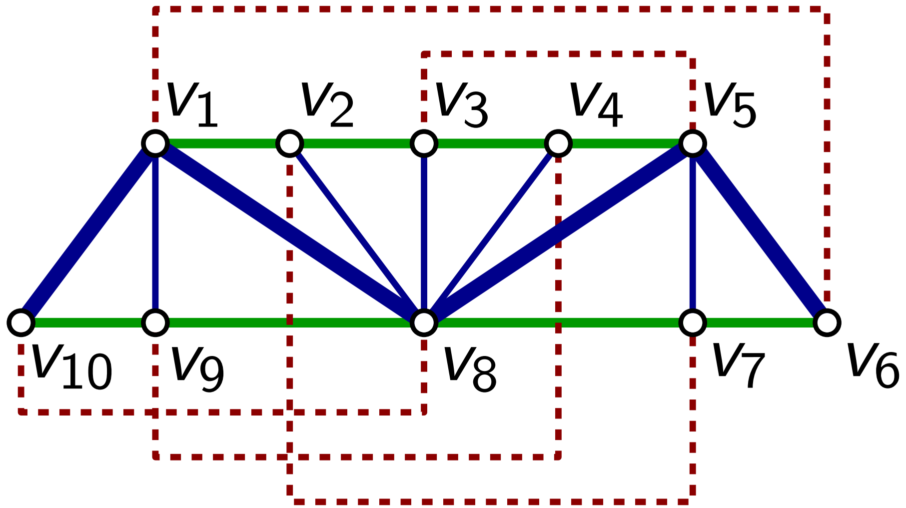
Outerpath × Matching

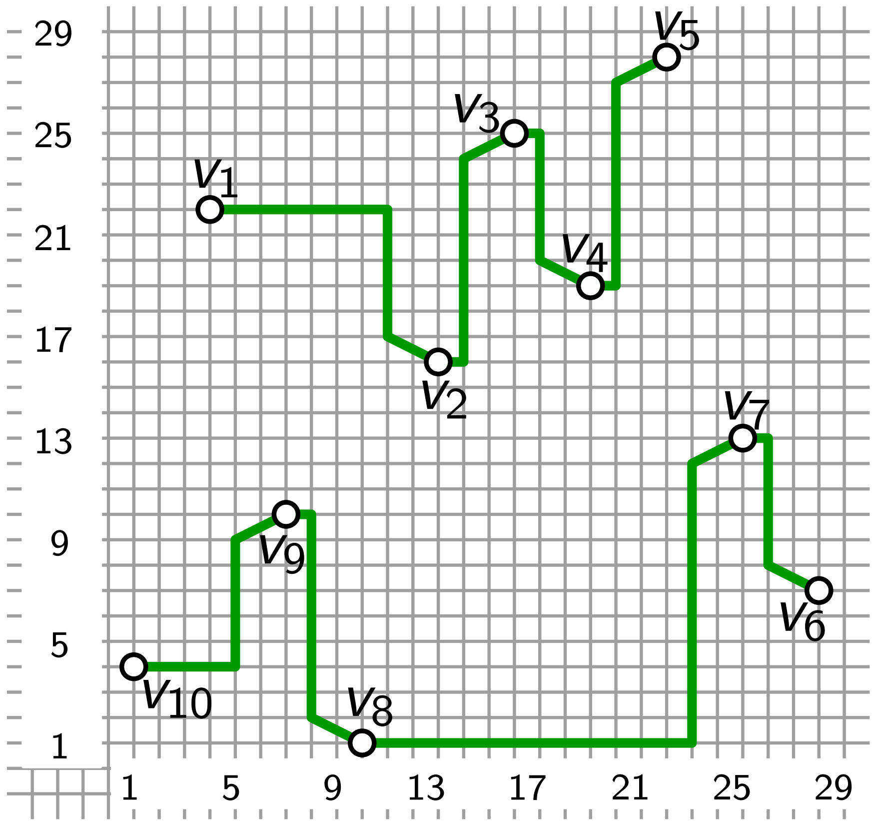


Outerpath × Matching

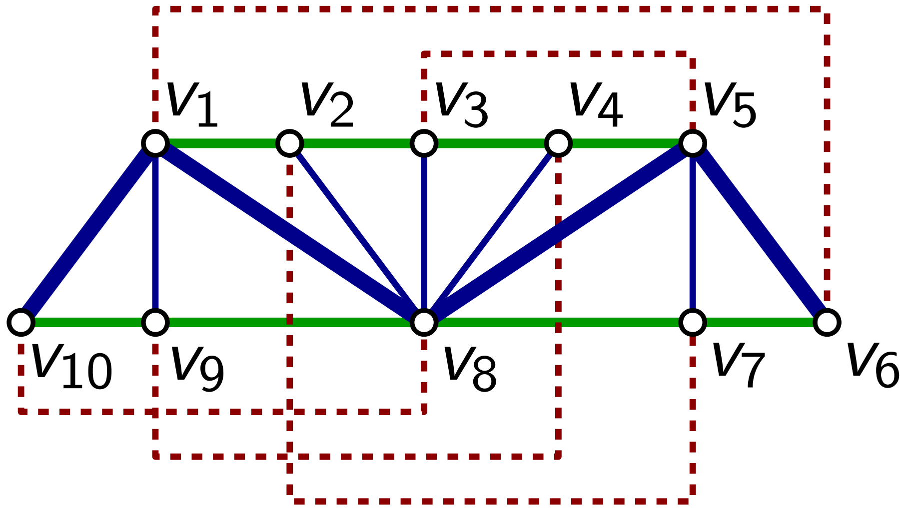


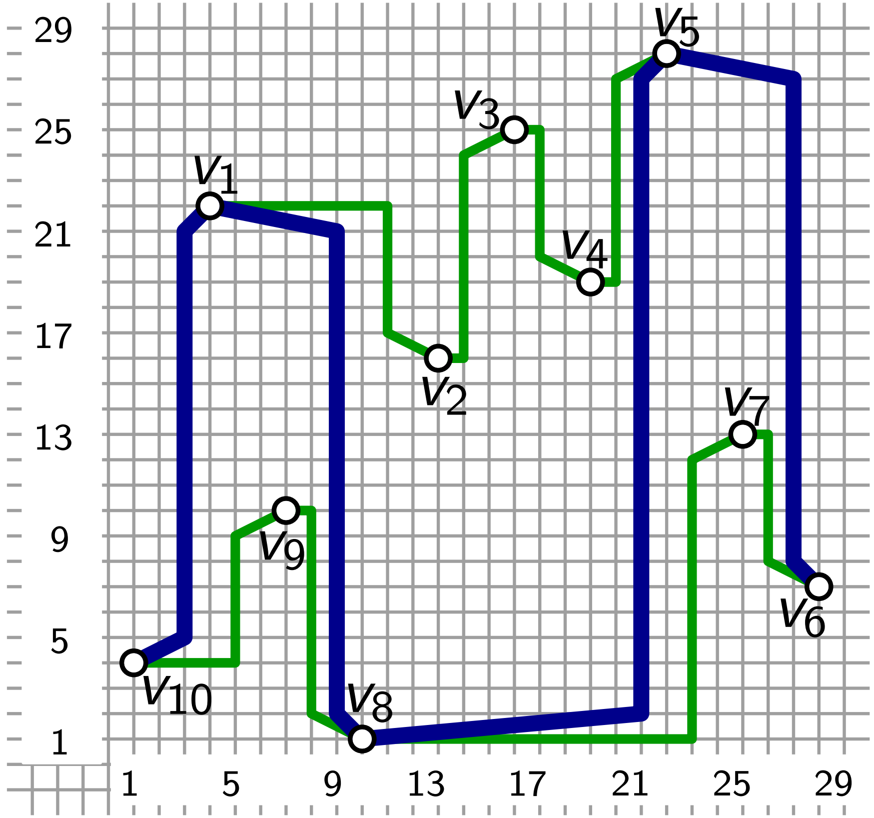
Outerpath × Matching



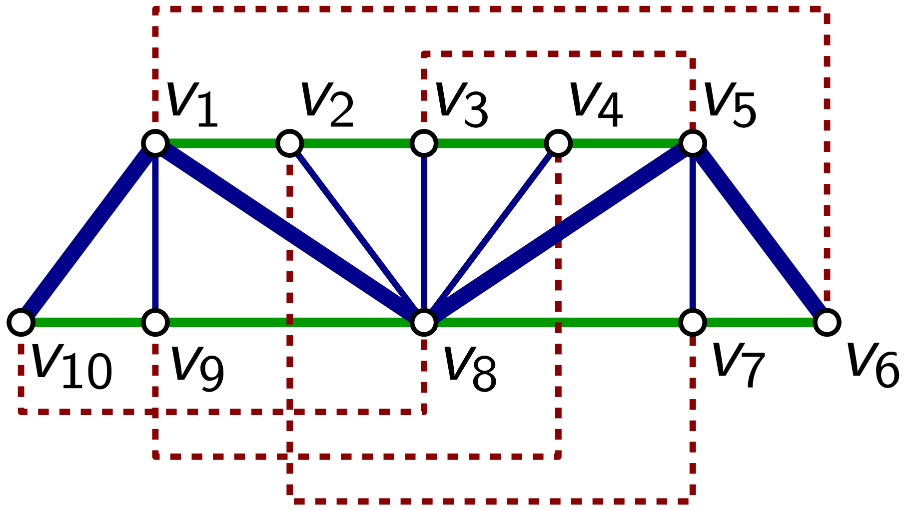


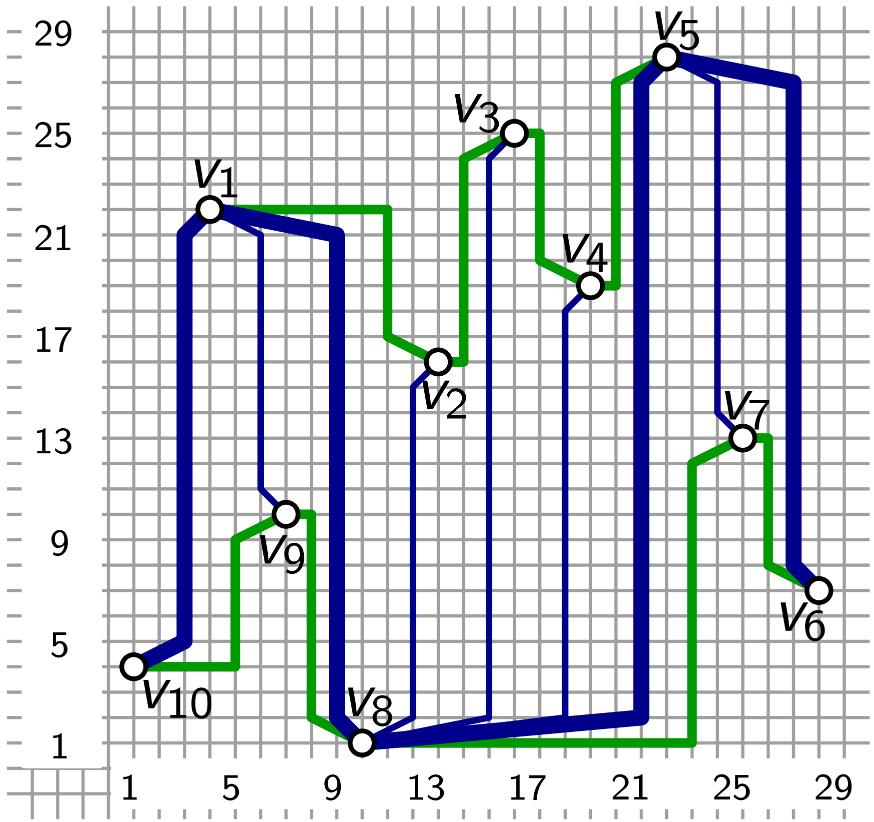
Outerpath × Matching



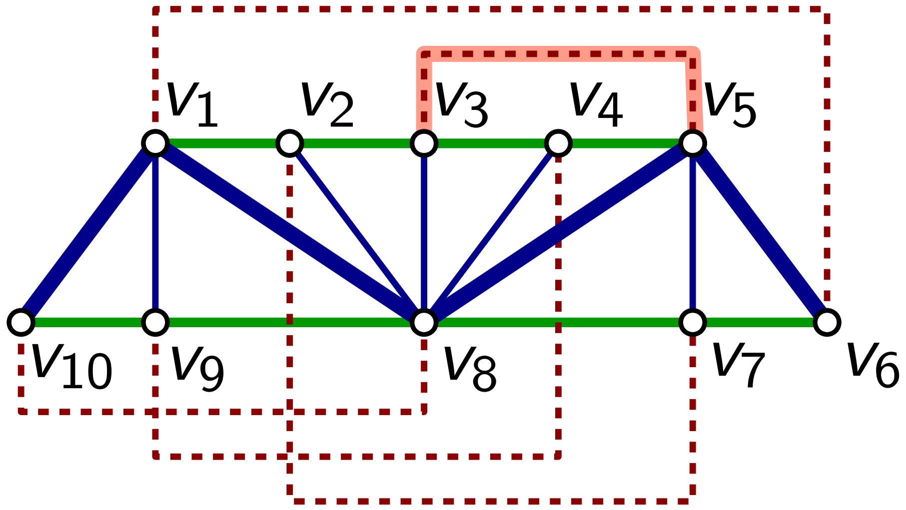


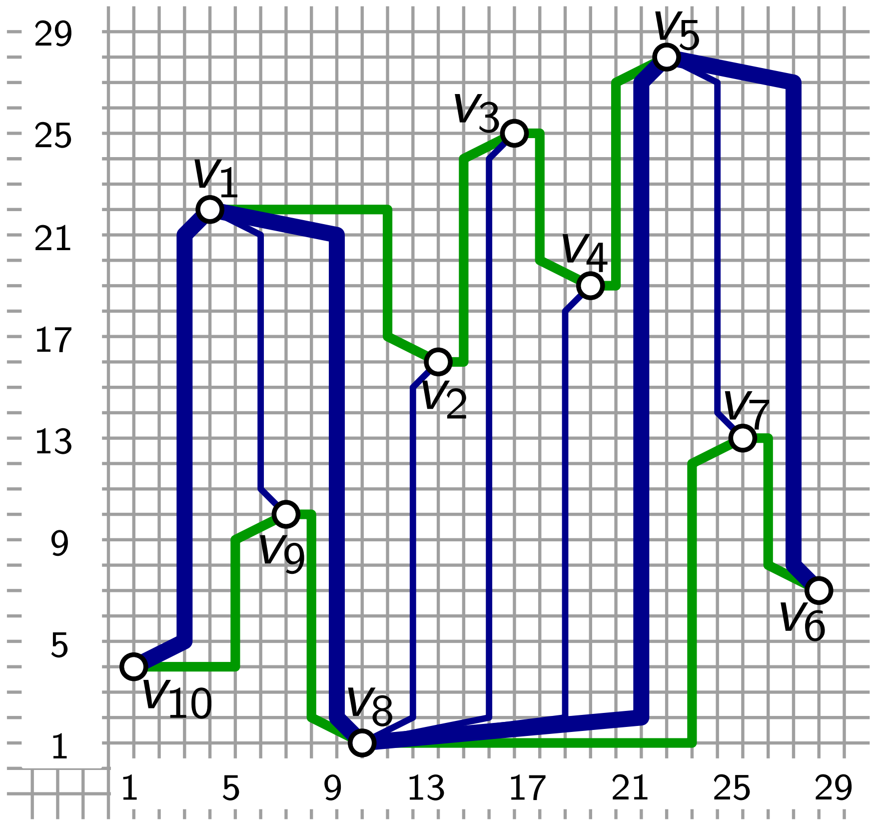
Outerpath × Matching



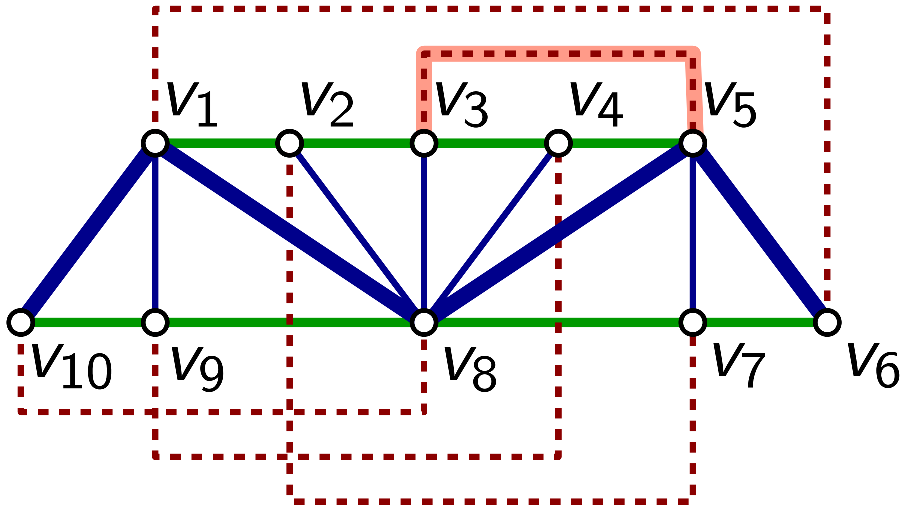


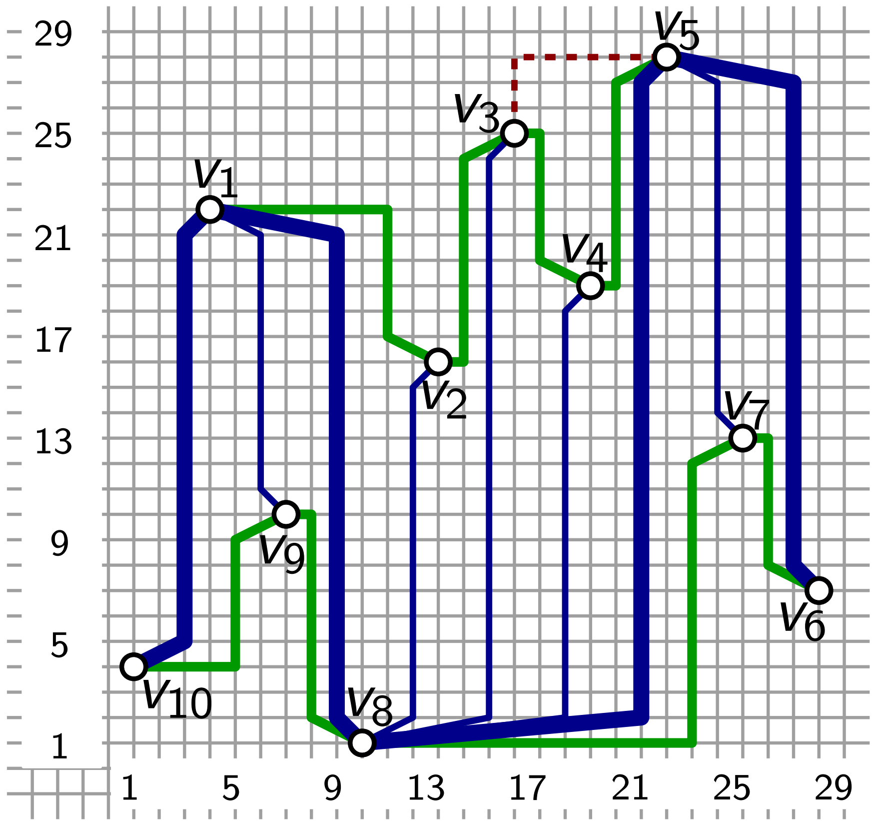
Outerpath × Matching



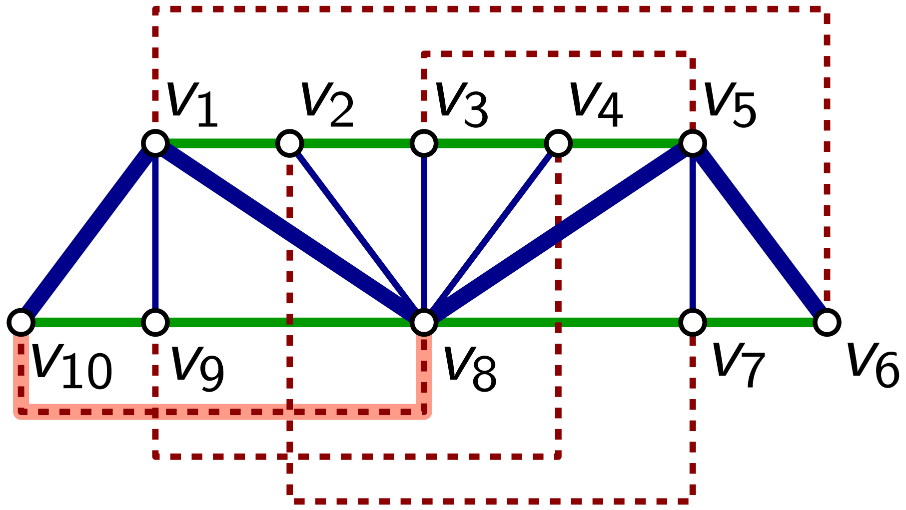


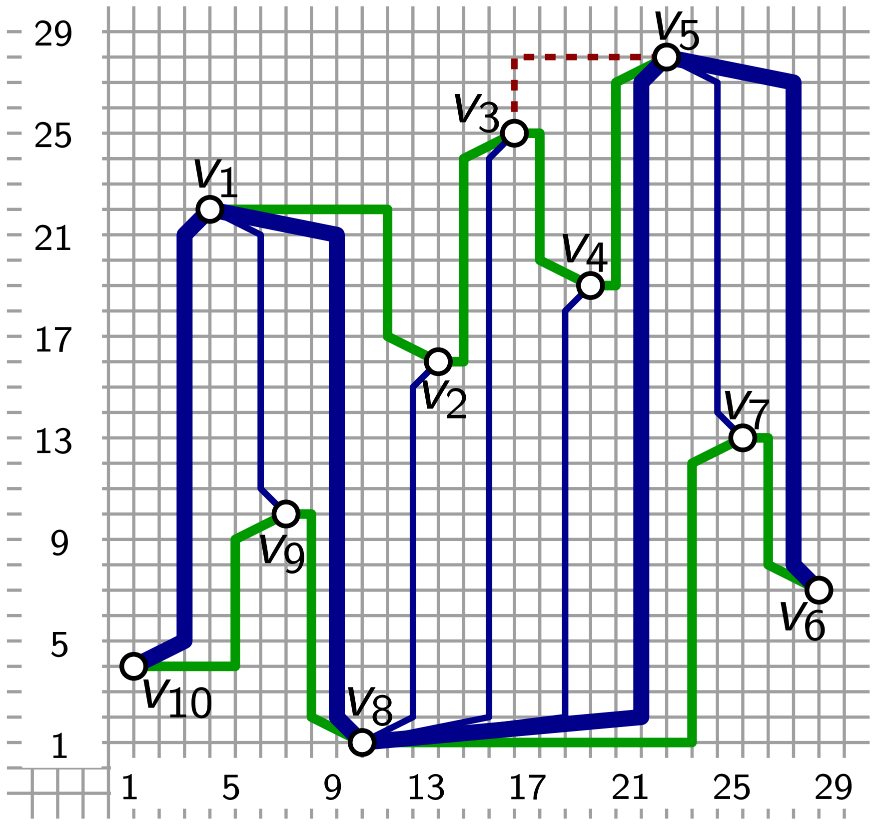
Outerpath × Matching



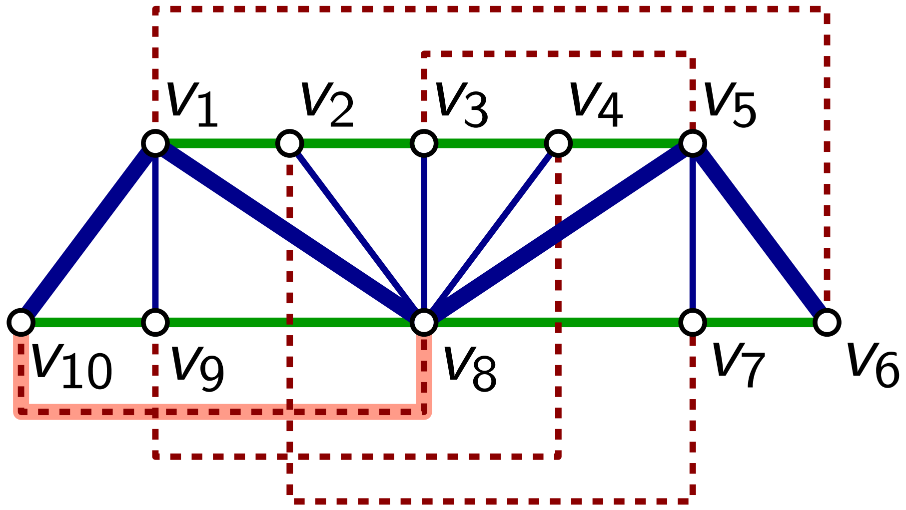


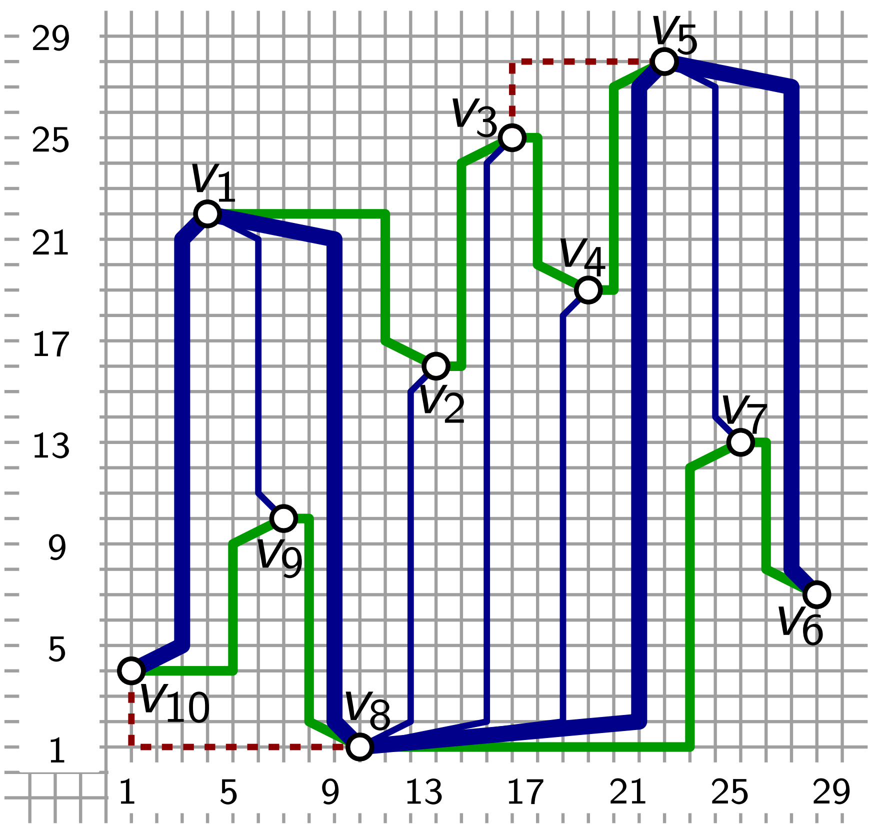
Outerpath × Matching



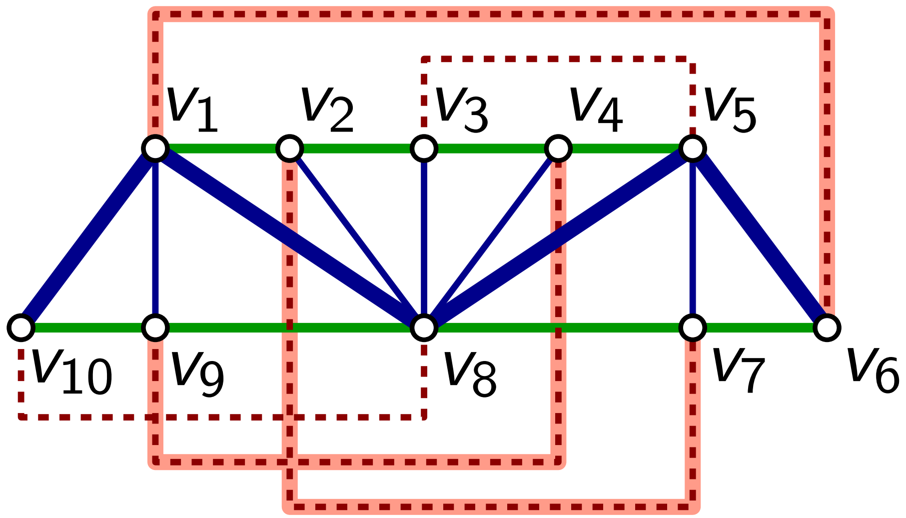


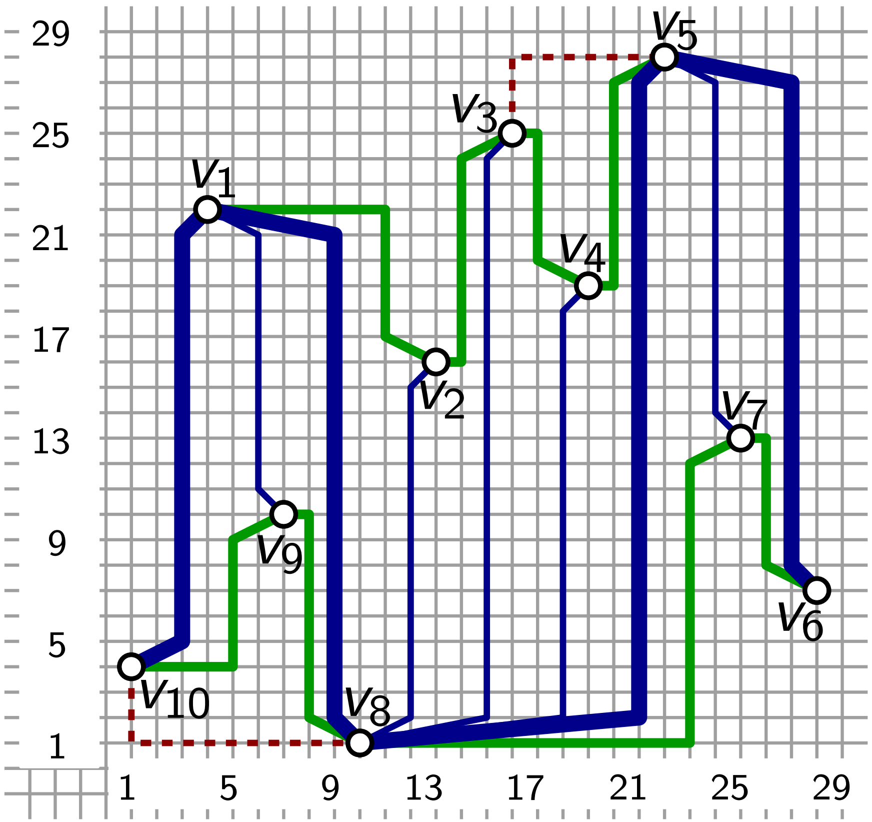
Outerpath × Matching



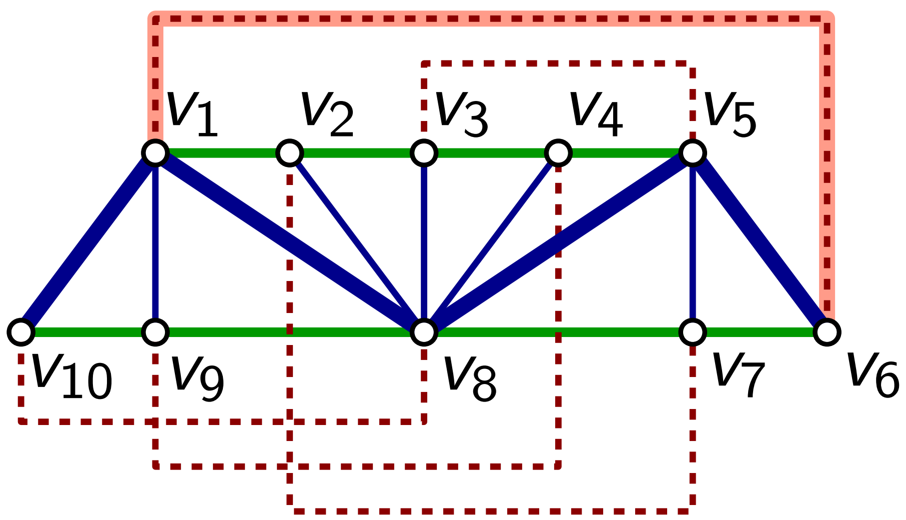


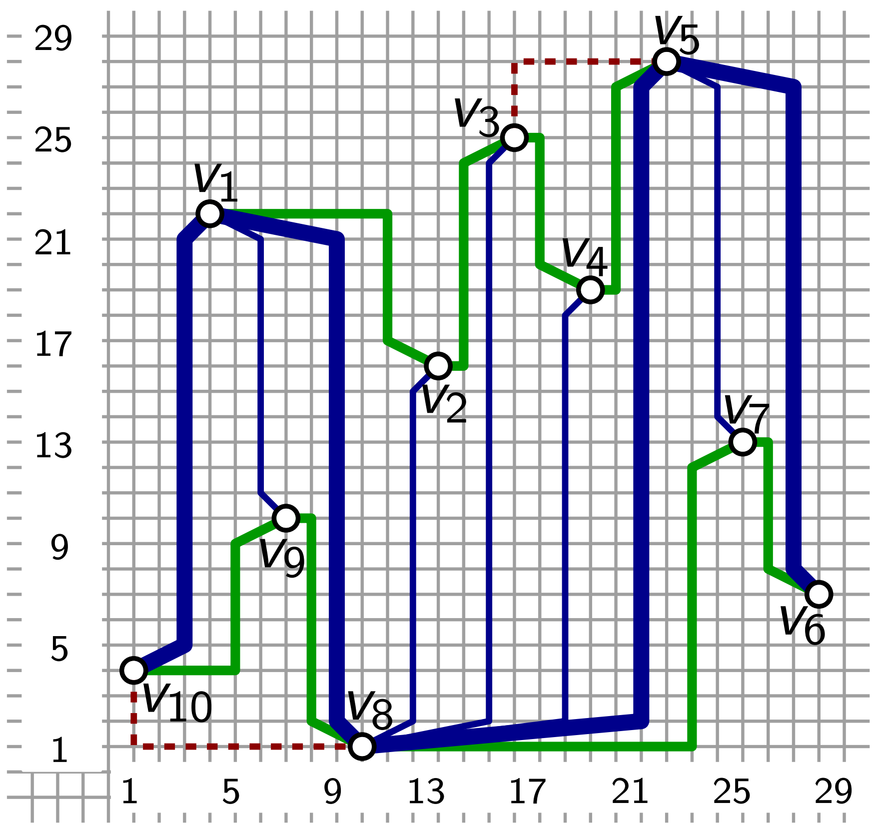
Outerpath × Matching



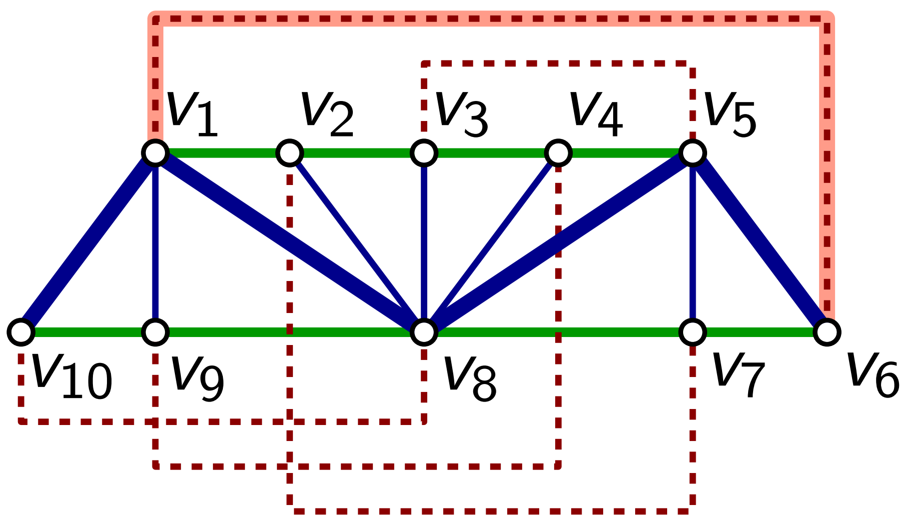


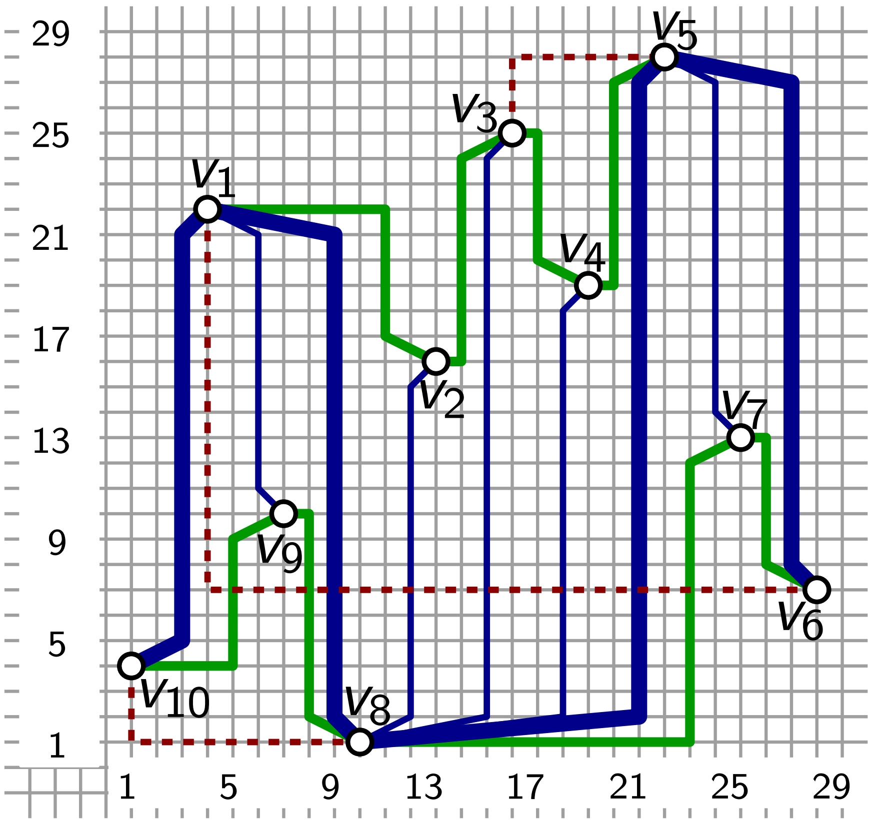
Outerpath × Matching



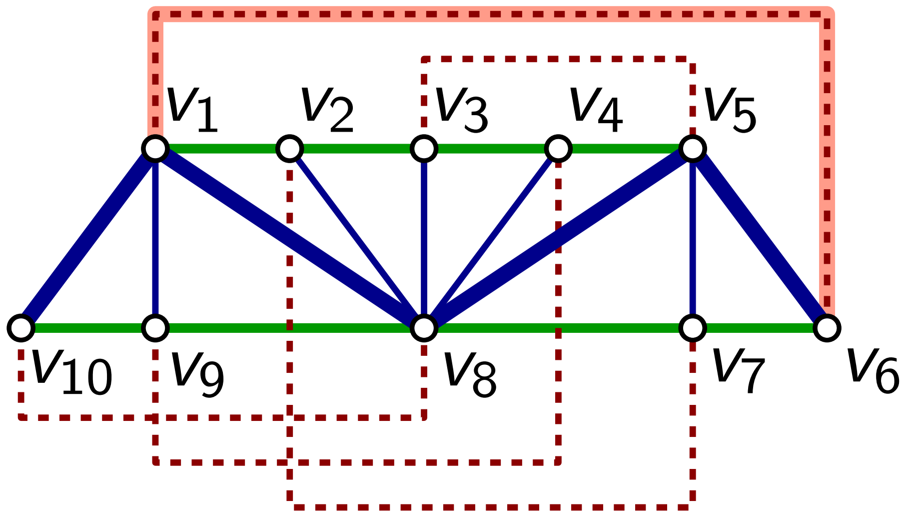


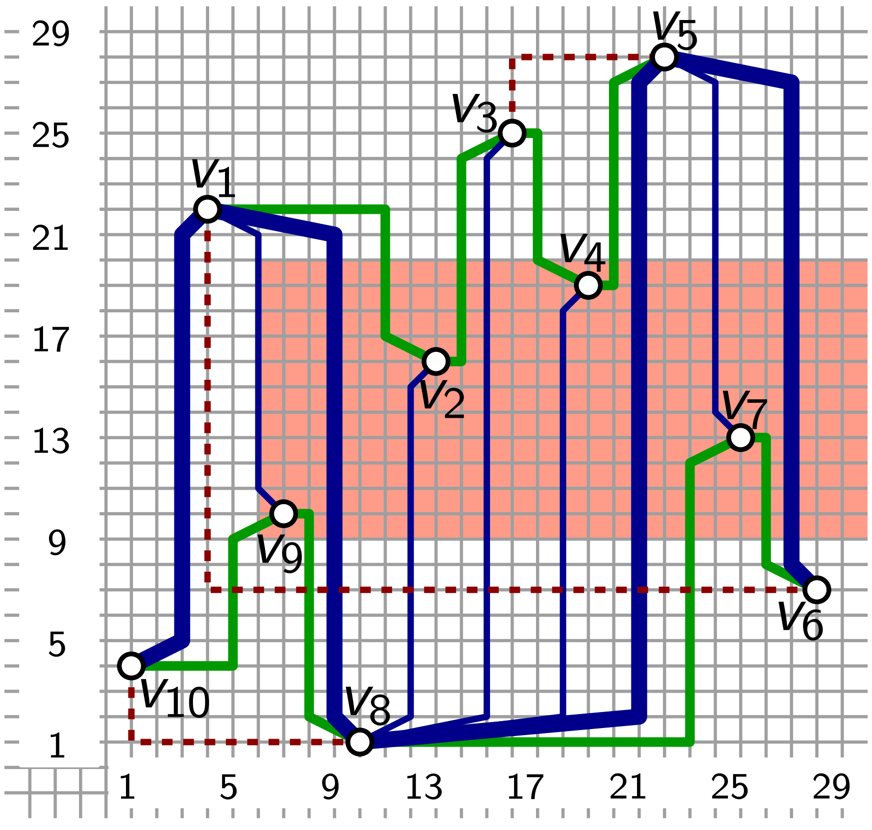
Outerpath × Matching





Outerpath × Matching





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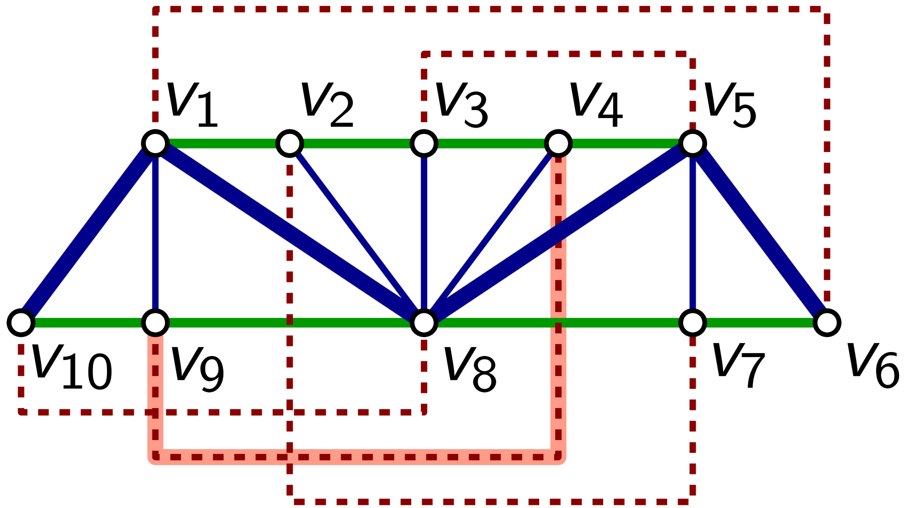

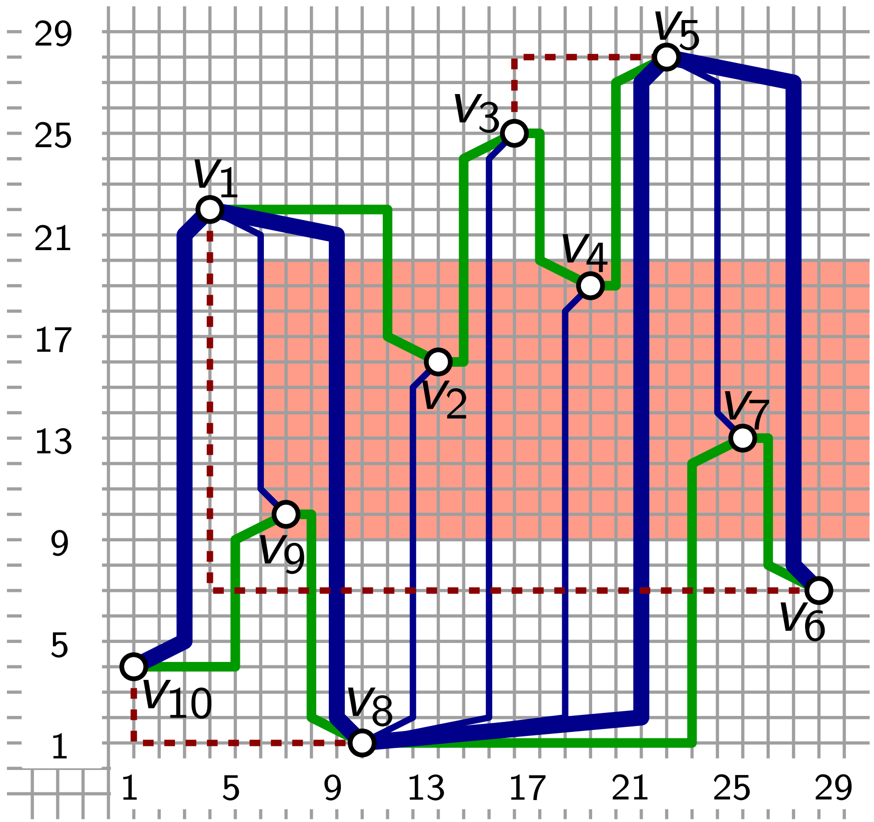

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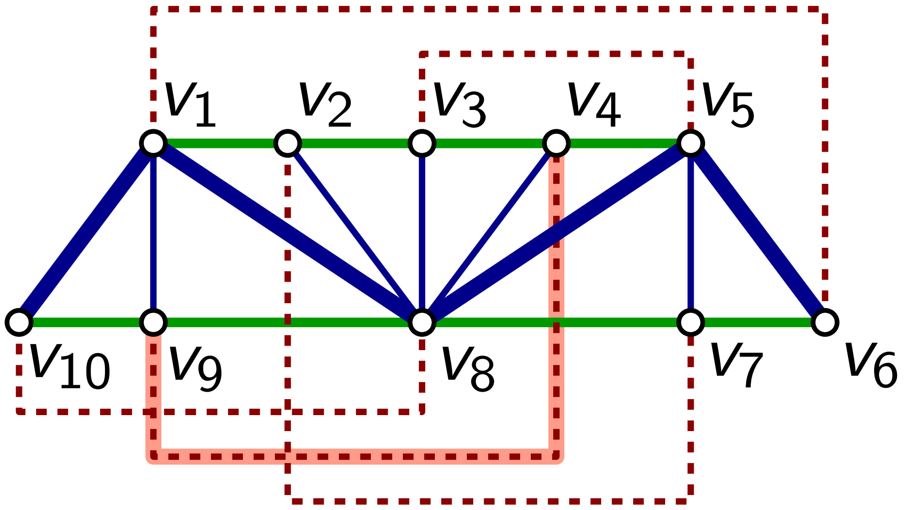
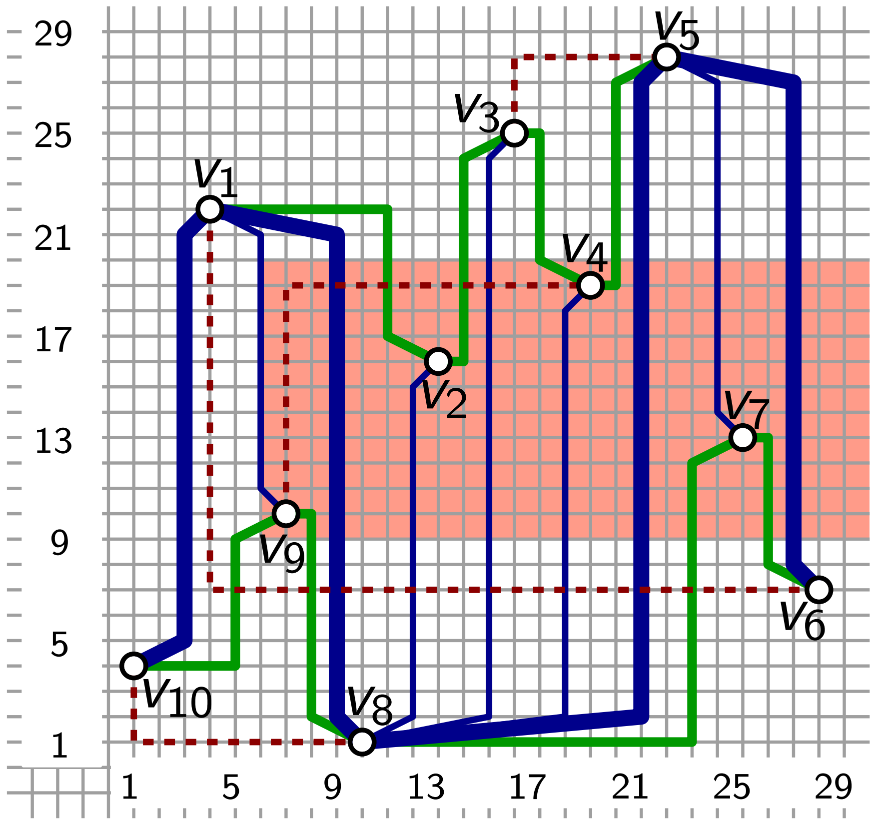
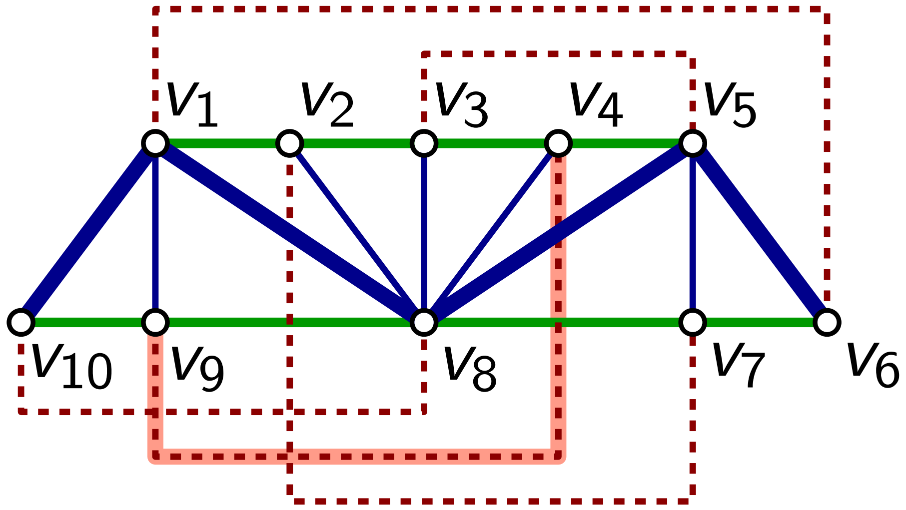
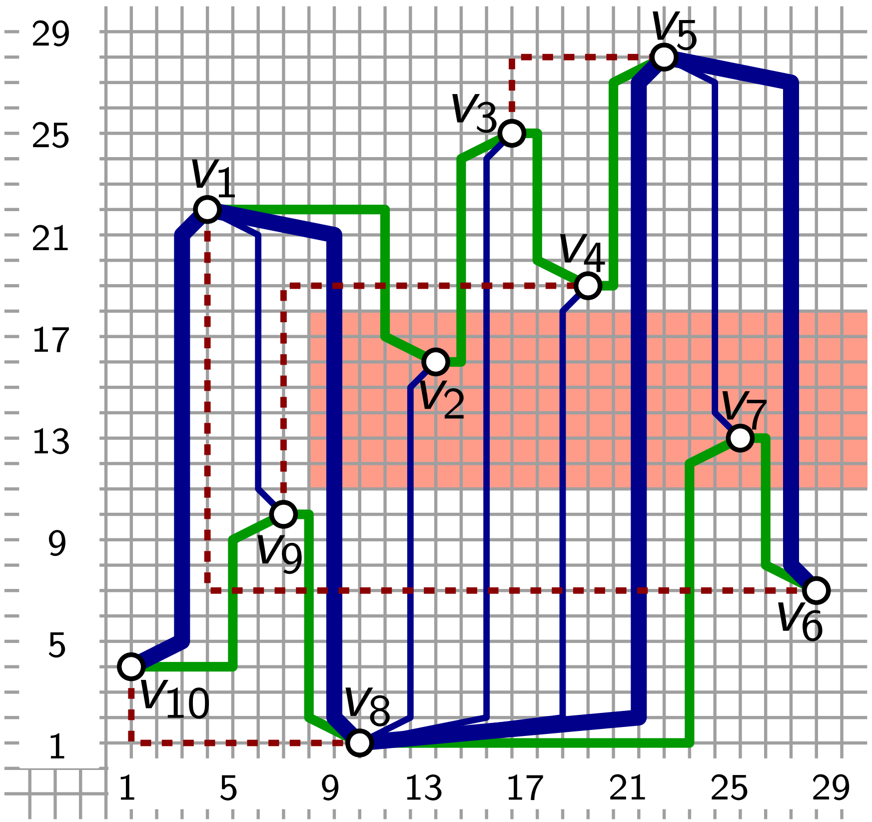


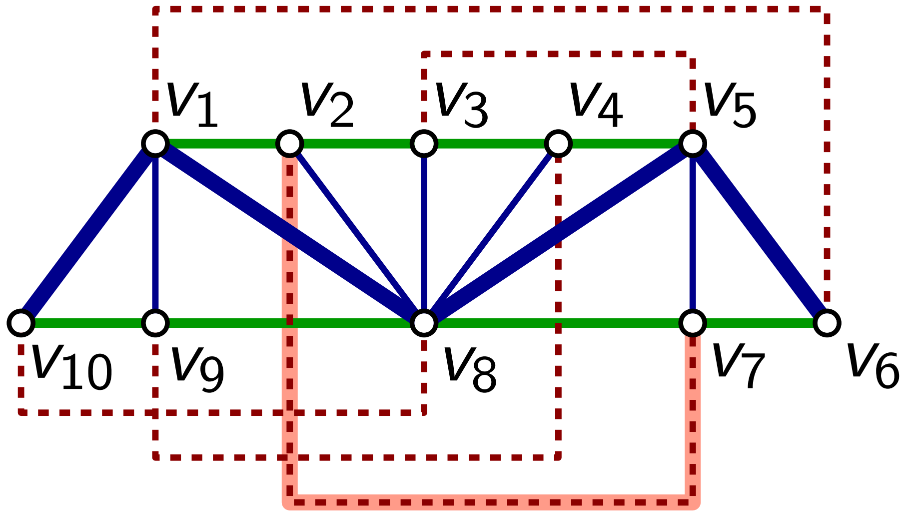


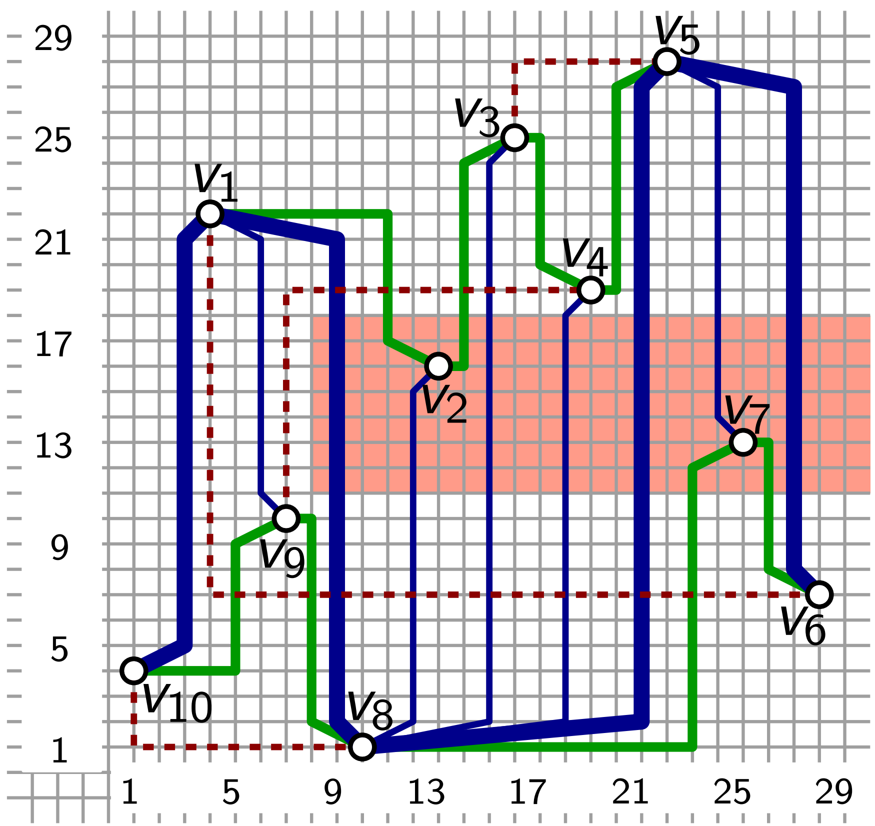
Outerpath × Matching



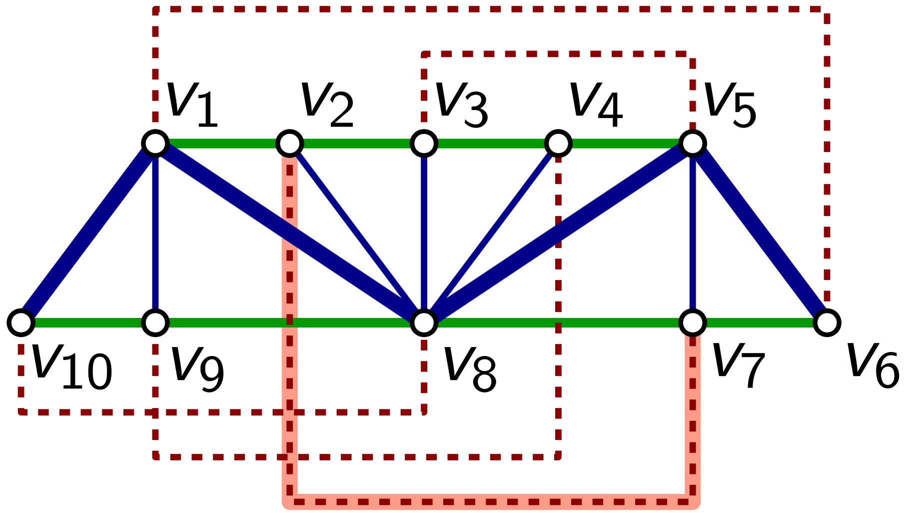


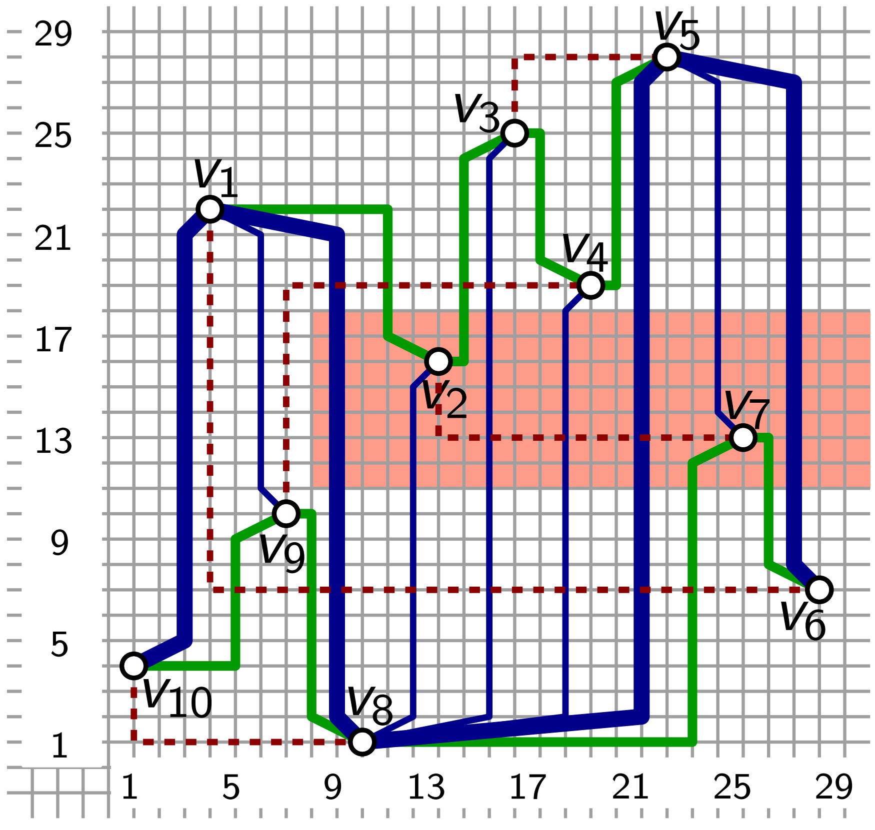
Outerpath × Matching



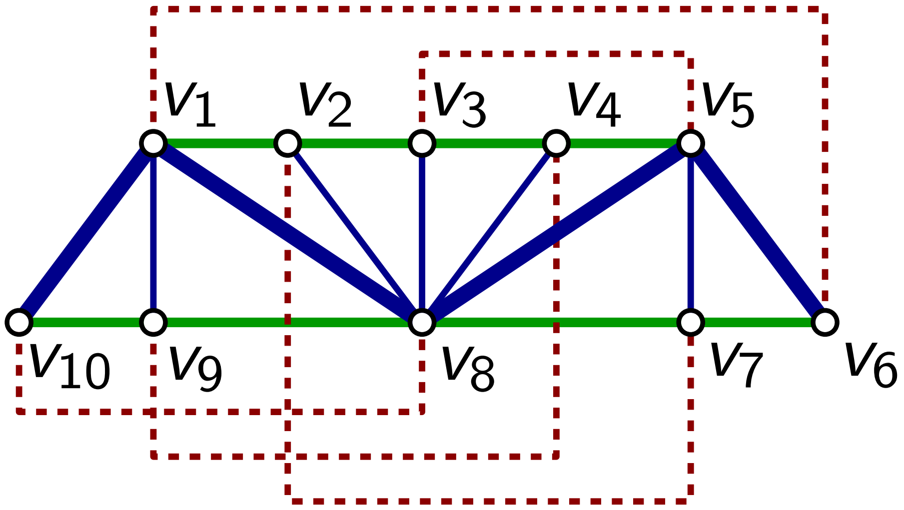


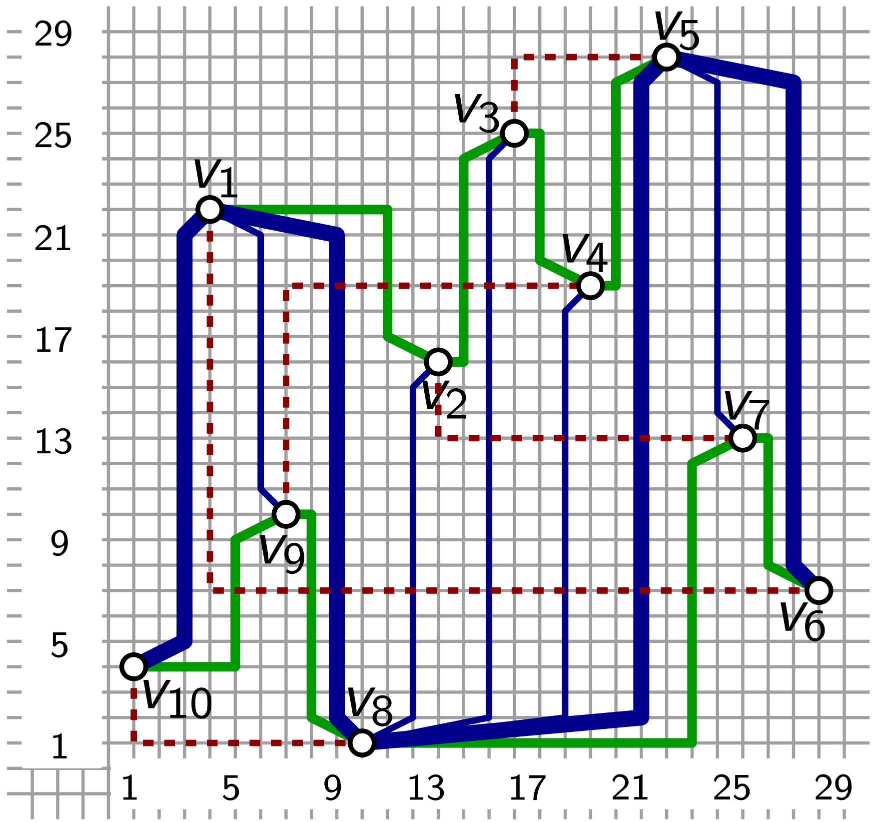
Outerpath × Matching



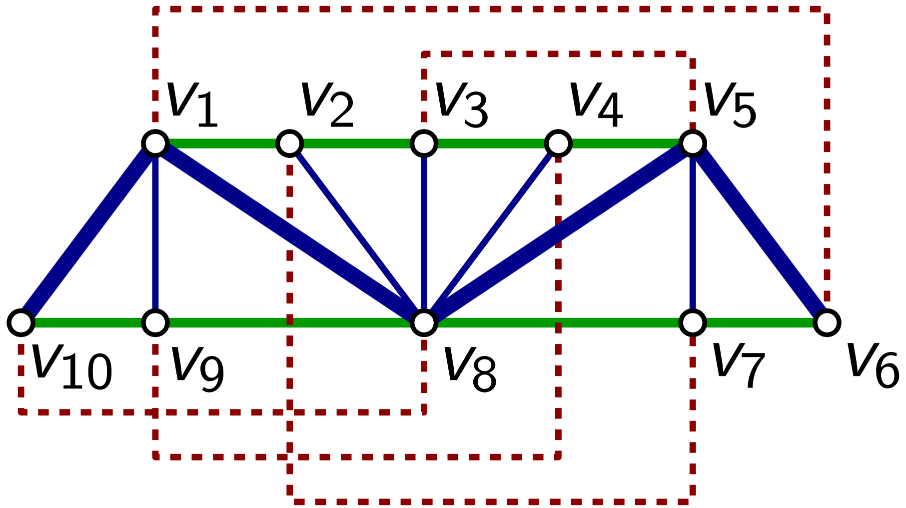


Outerpath × Matching

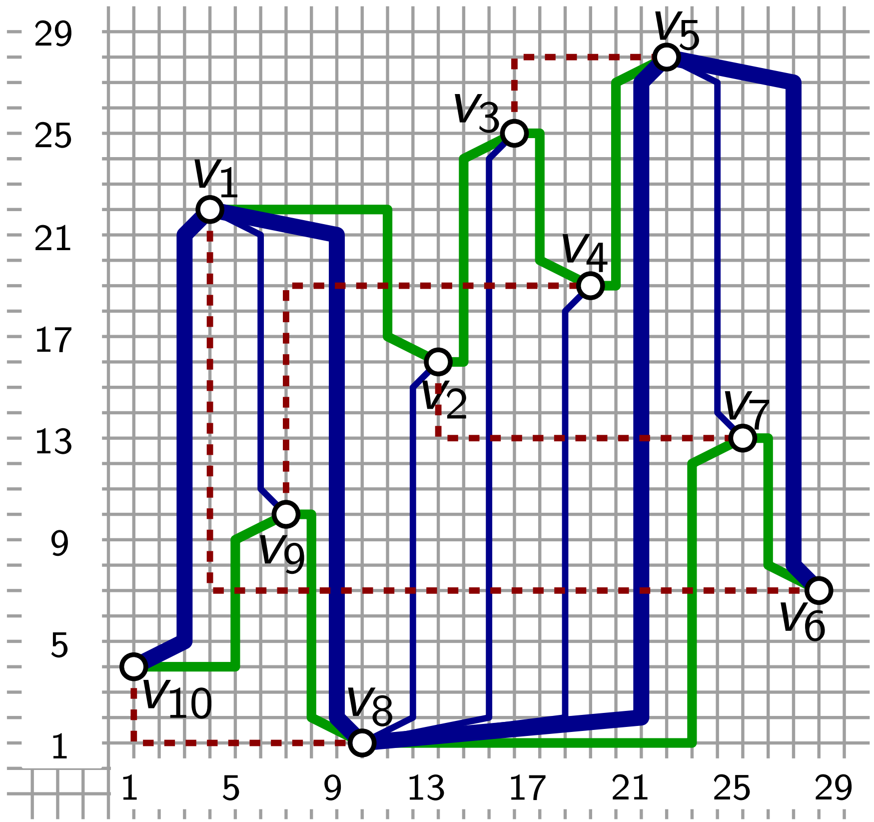




Outerpath × Matching



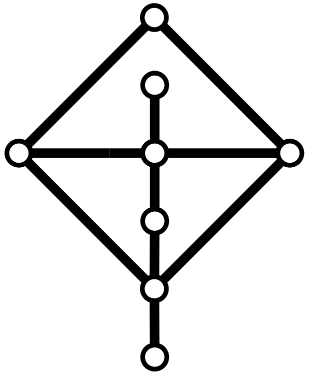
Bends: 2×1
Grid size: $(3n-2)^2$



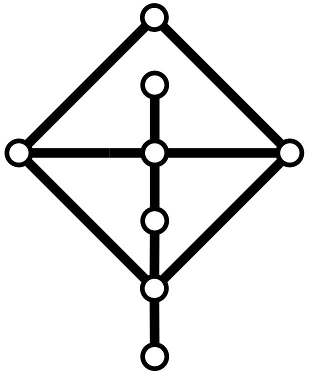
Overview

Graph classes			Number of bends	
Cycle	×	Cycle	1 × 1	
Caterpillar	×	Cycle	1 × 1	
Four Matchings			1 × 1 × 1 × 1	
Tree	×	Matching	1 × 0	
Wheel	×	Matching	2 × 0	✓
Outerpath	×	Matching	2 × 1	✓
Outerplanar	×	Outerplanar	3 × 3	
2-page book emb.	×	2-page book emb.	4 × 4	
Planar	×	Planar	6 × 6	

Planar × Planar

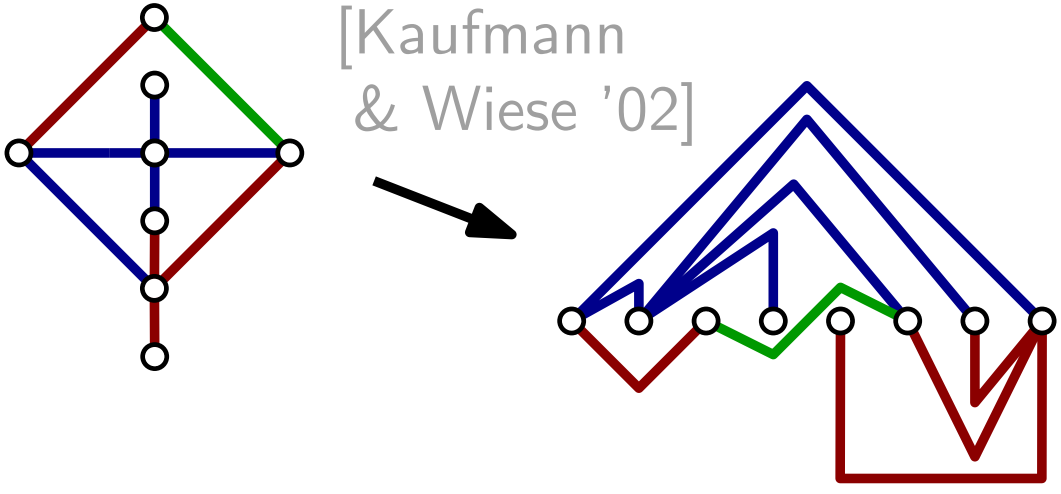


Planar × Planar



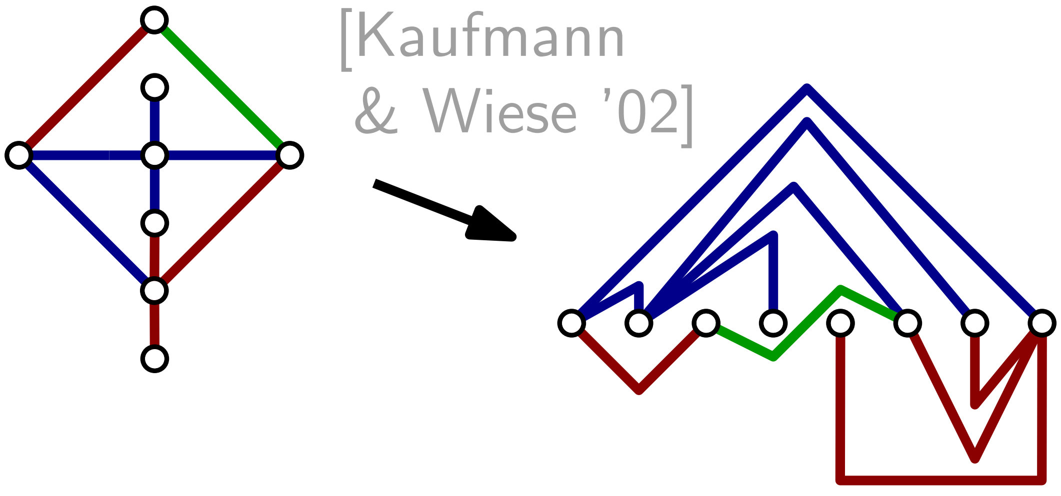
[Kaufmann & Wiese '02]

Planar × Planar



[Kaufmann & Wiese '02]

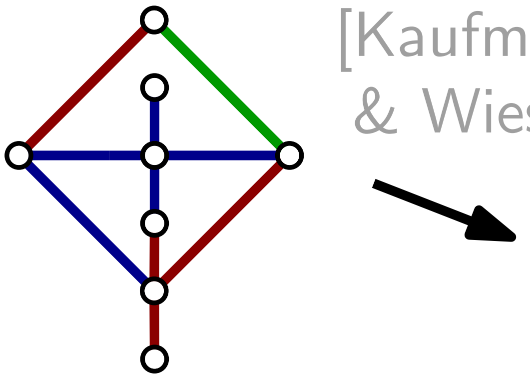
Planar × Planar



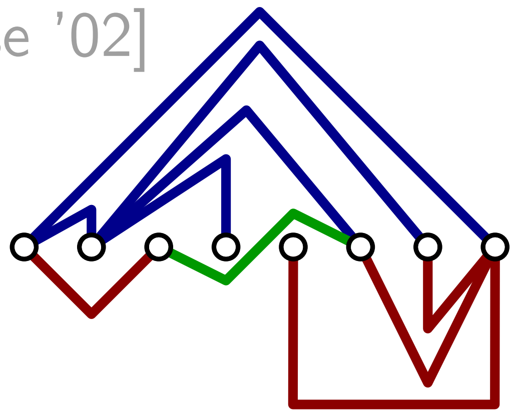
Graph 1: x-coordinates

Graph 2: y-coordinates

Planar × Planar



[Kaufmann & Wiese '02]



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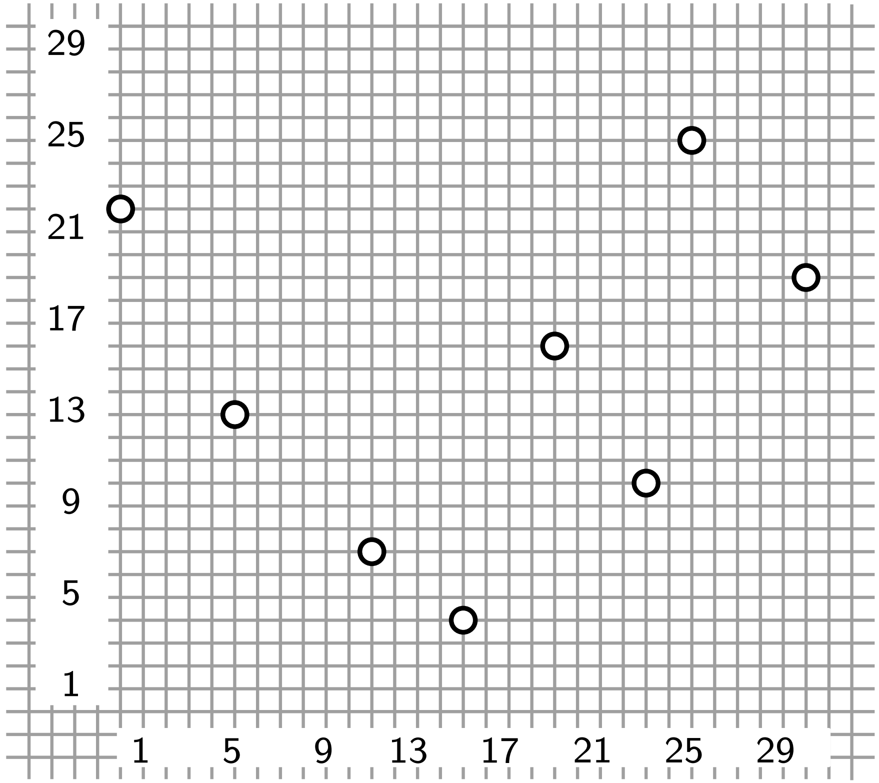
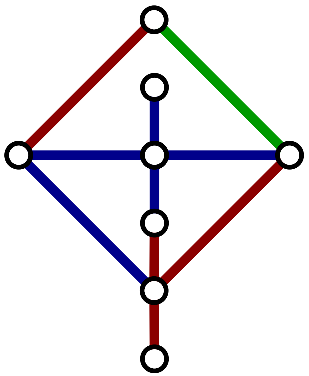


Planar × Planar



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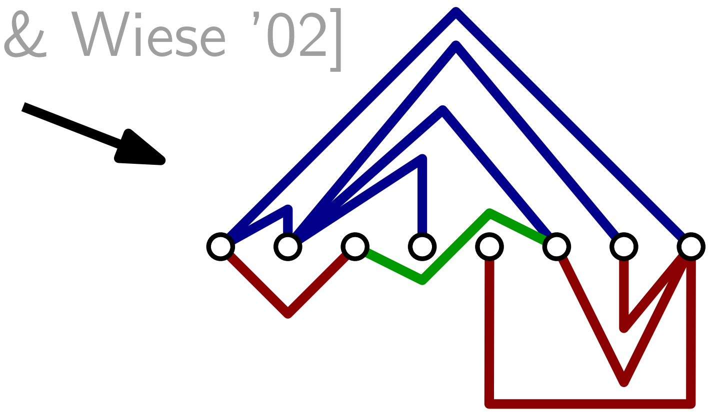

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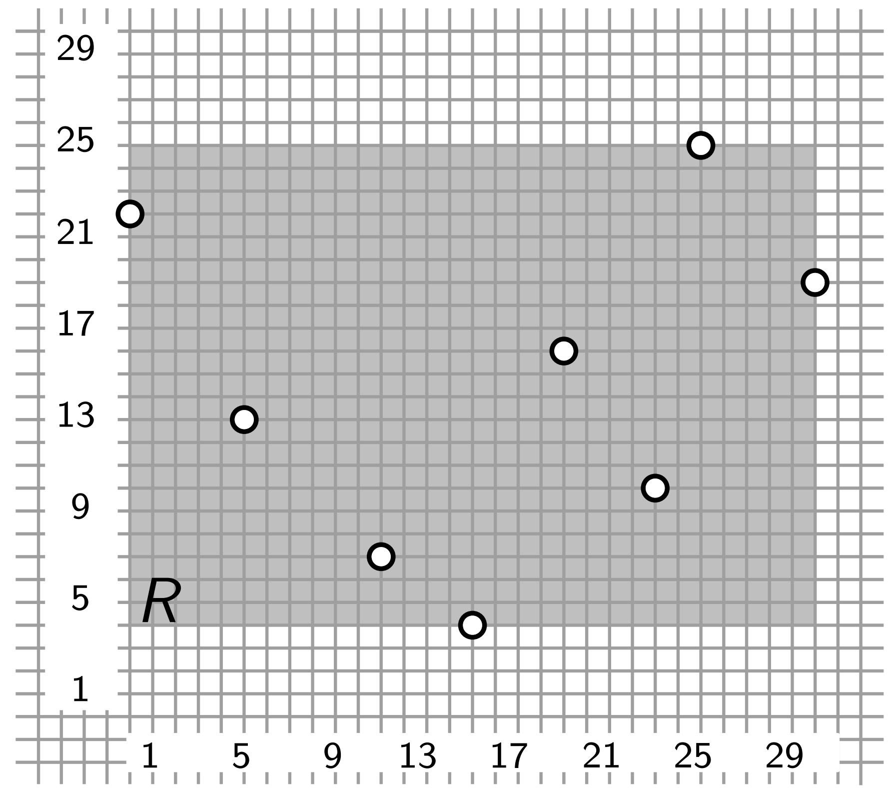

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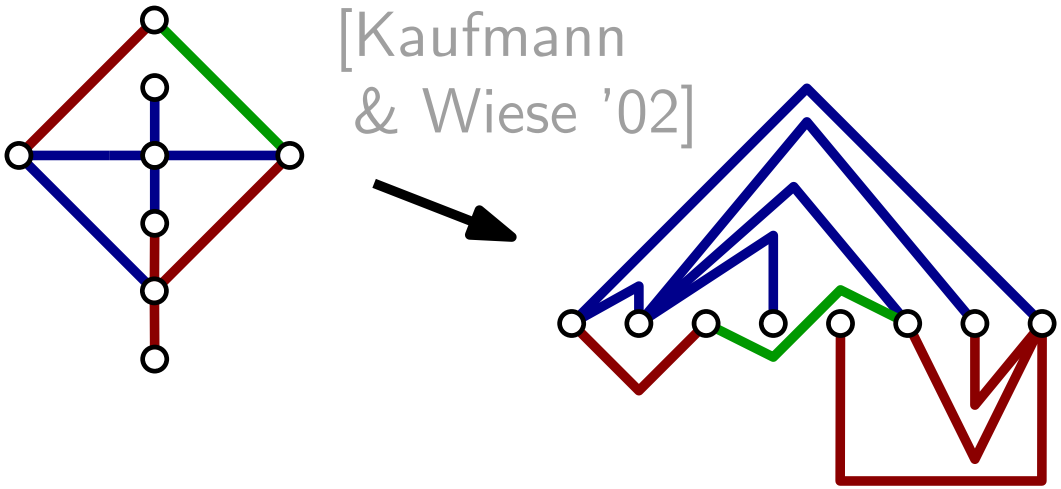

[Kaufmann & Wiese '02]

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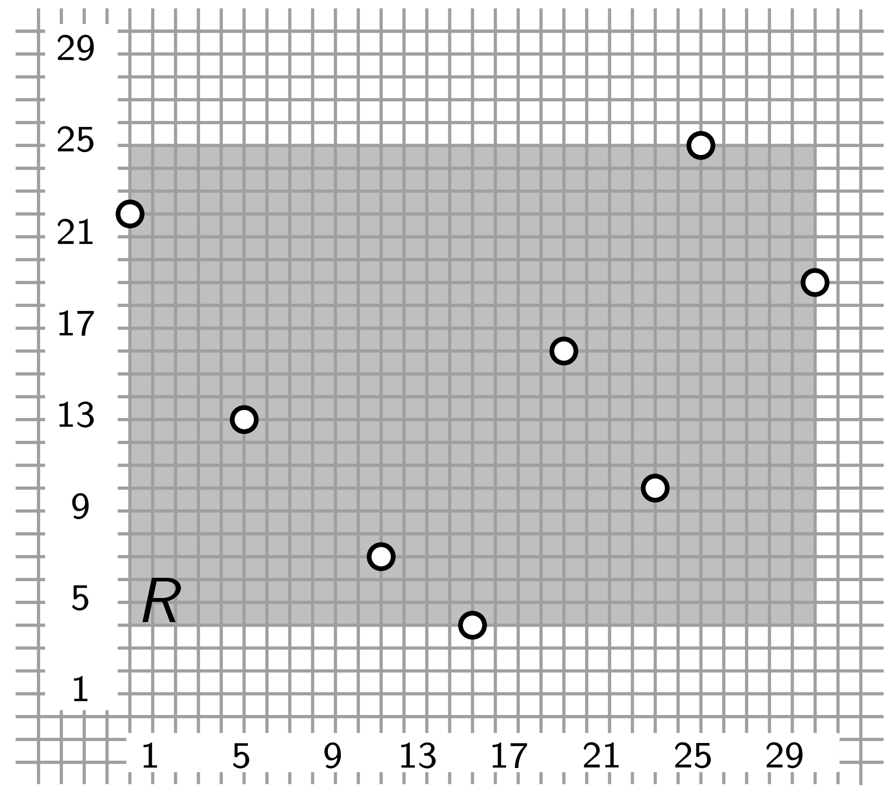

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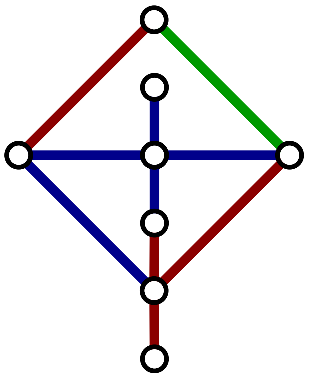

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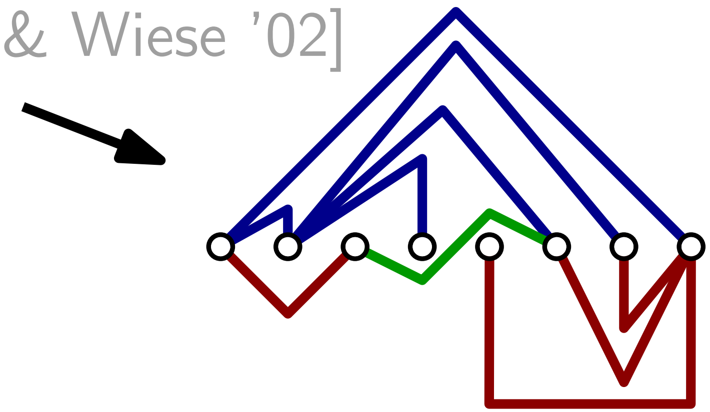
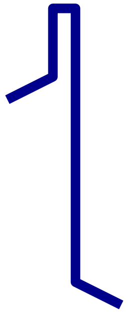


Graph 1: *x*-coordinates

In *R*: All segments vertical or slanted of *y*-length 1.

Graph 2: *y*-coordinates

Edges:



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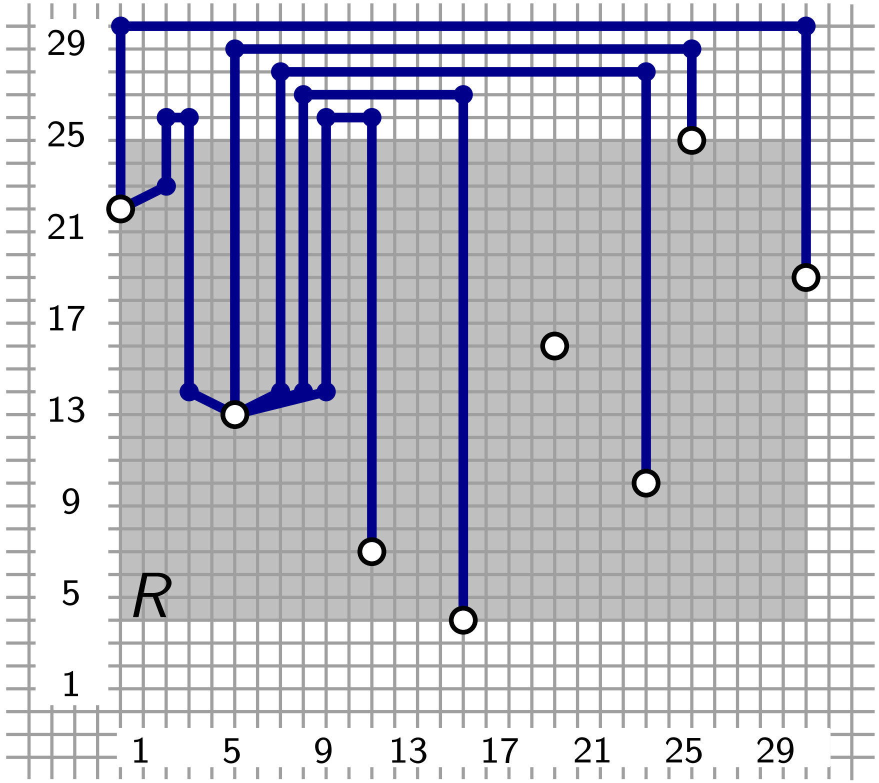
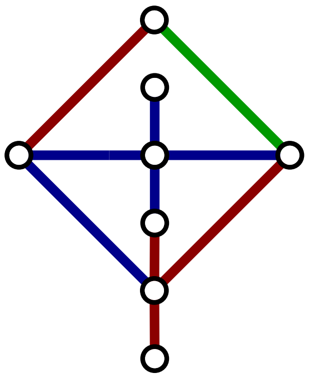


Planar × Planar



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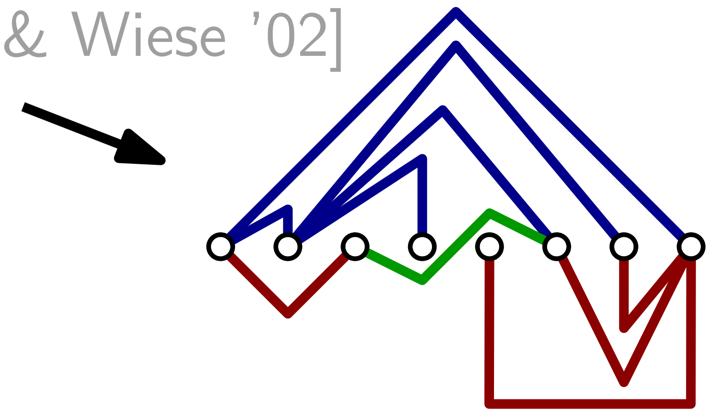

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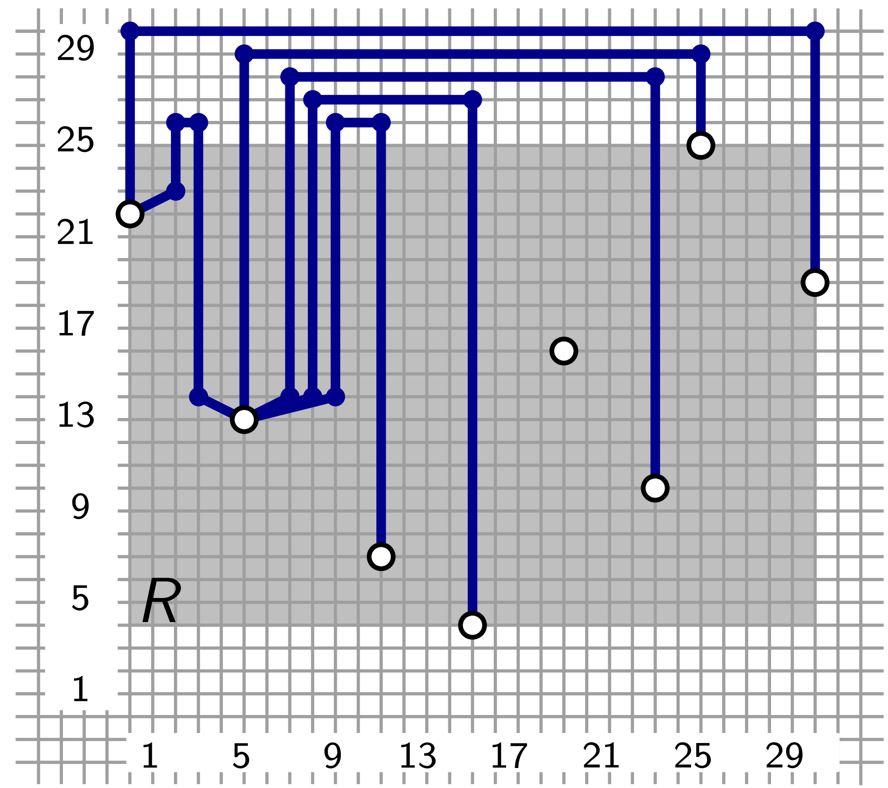
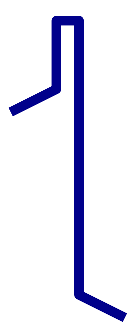
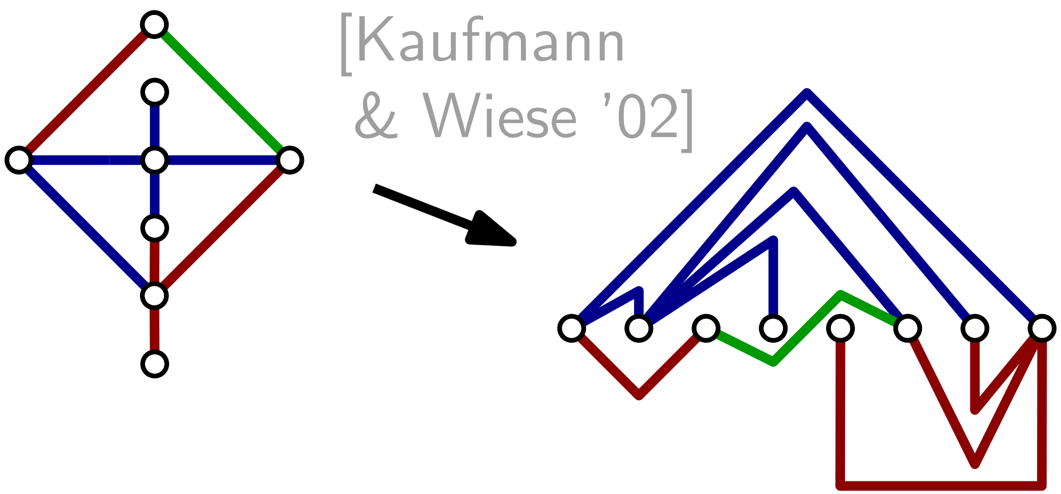


Edges:



4

Planar × Planar



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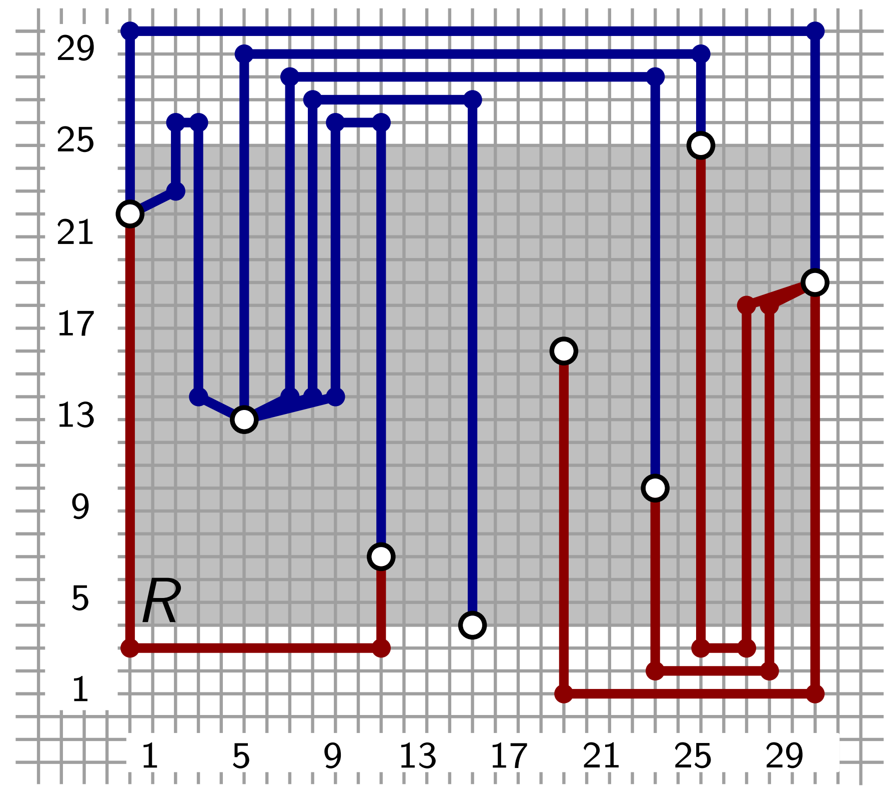
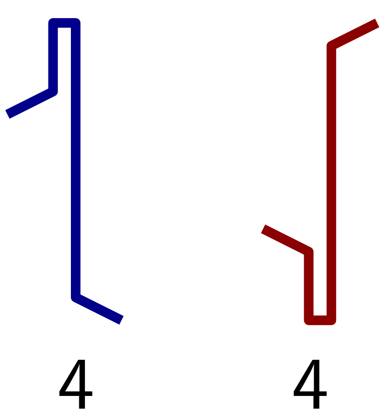
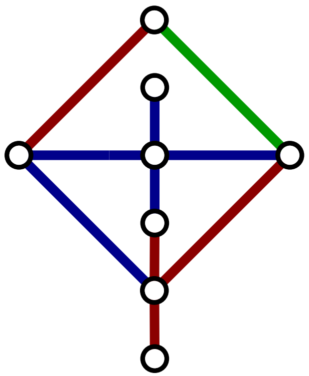


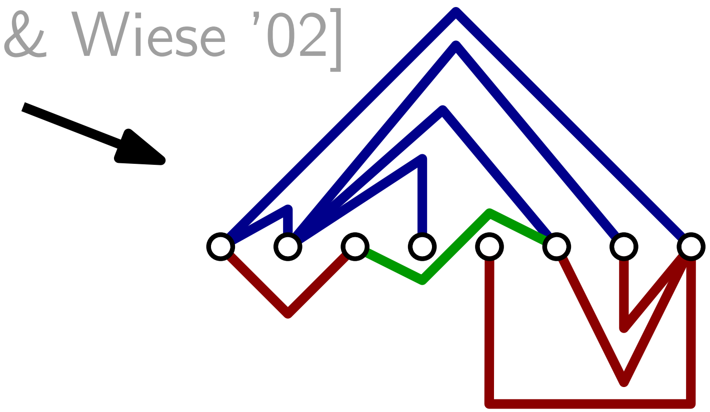
Edges:



Planar × Planar



[Kaufmann & Wiese '02]



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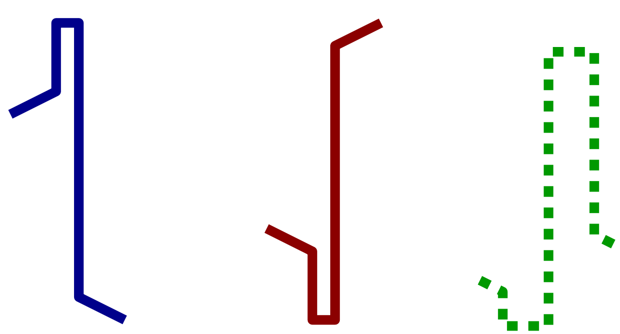
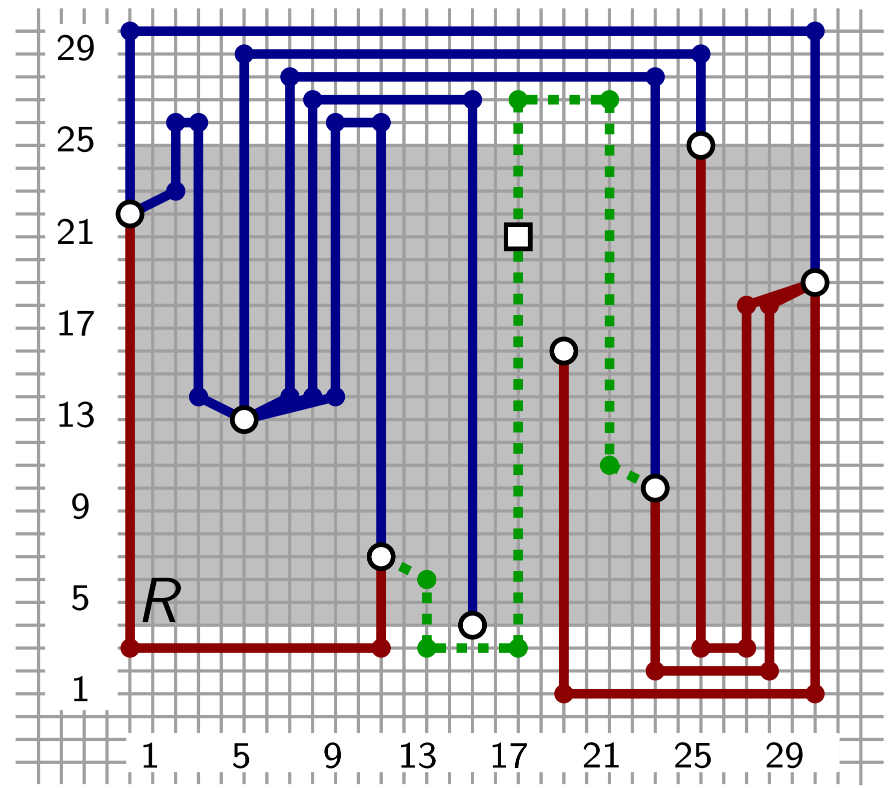
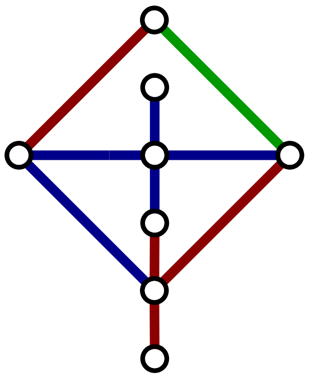




Planar × Planar



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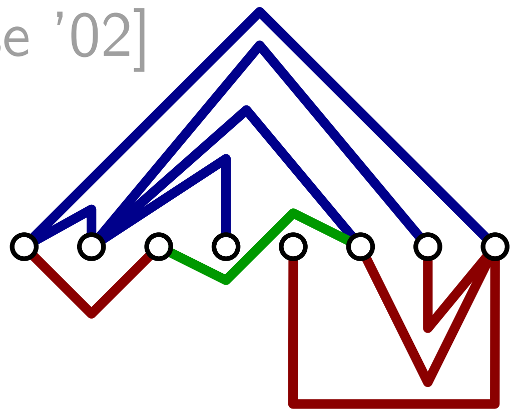

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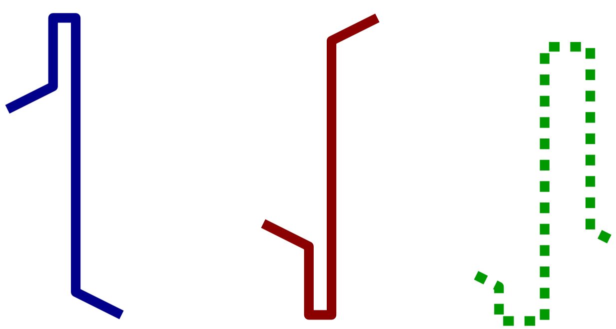
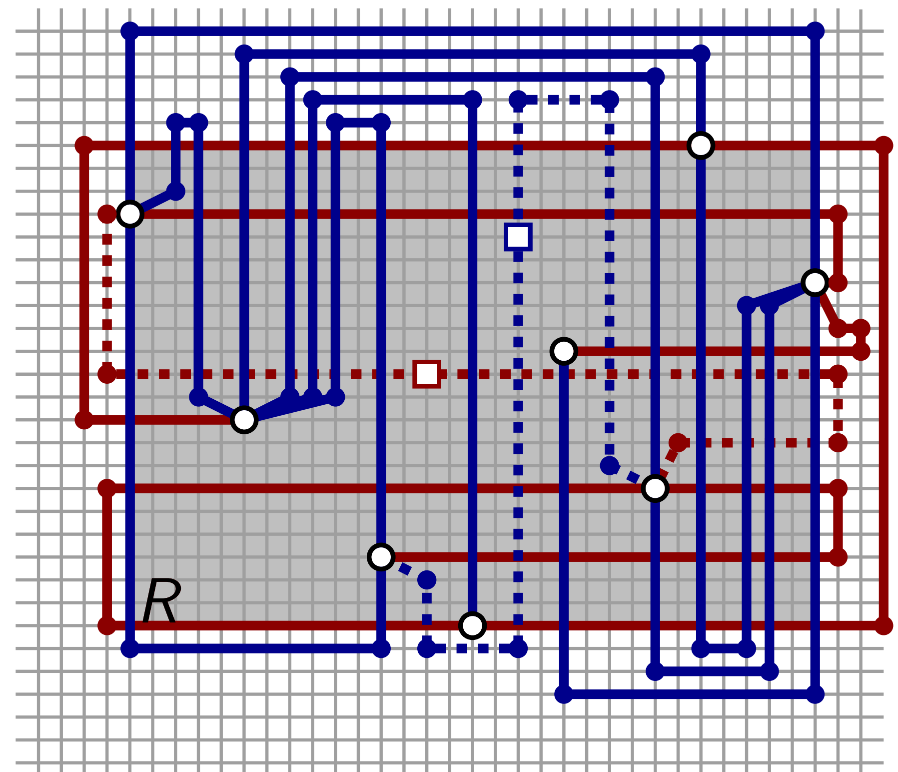


4 4 6



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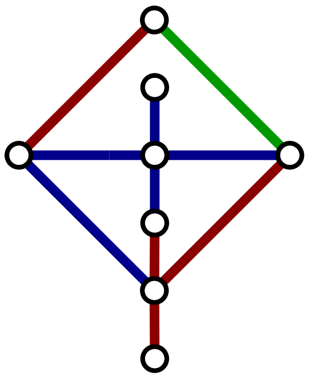

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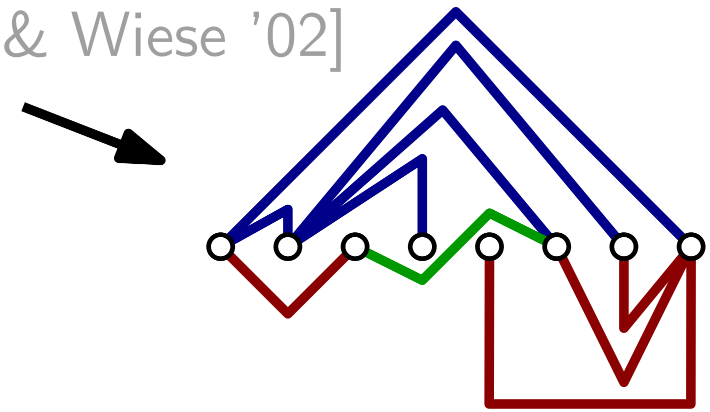
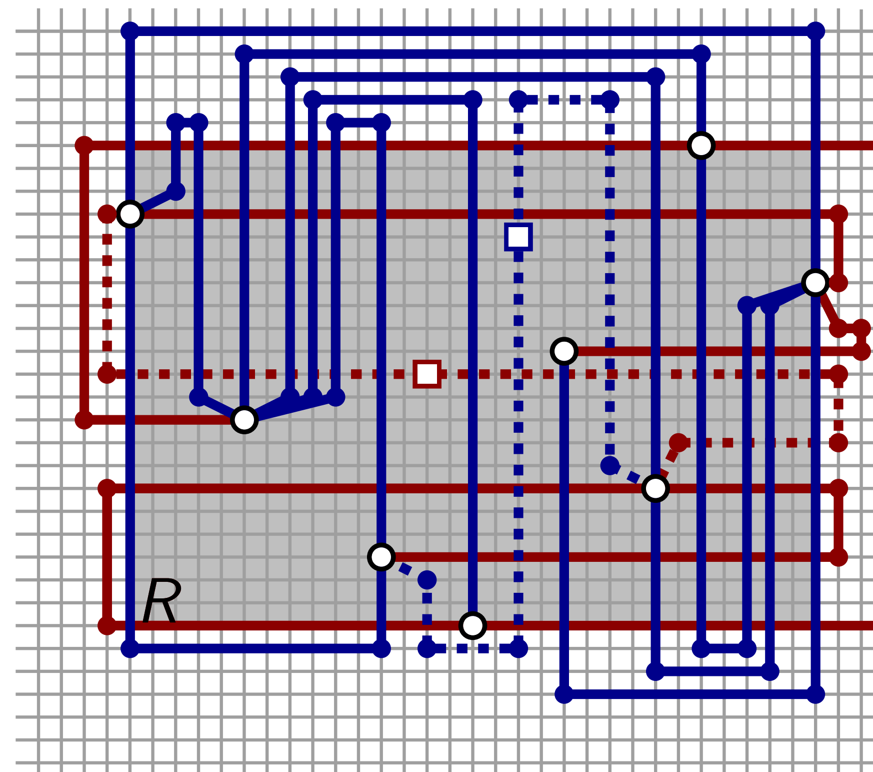


Bends: 6×6
Grid size: $(14n - 26)^2$

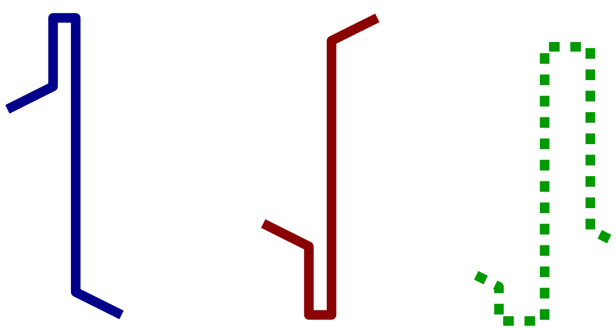
Graph 1: x-coordinates

In R: All segments vertical or slanted of y-length 1.

Graph 2: y-coordinates

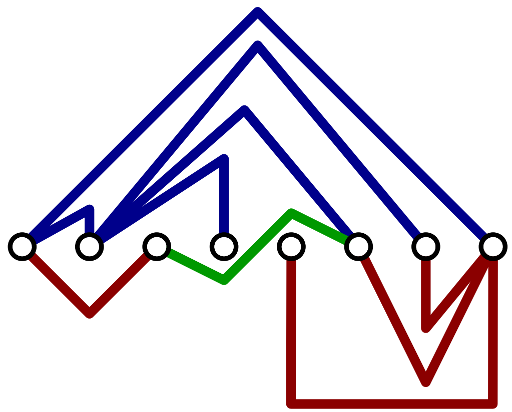


Edges:



4 4 6

2-Page Book Embed. × 2-Page Book Embed.



Graph 1: x-coordinates

In R: All segments vertical or slanted of y-length 1.

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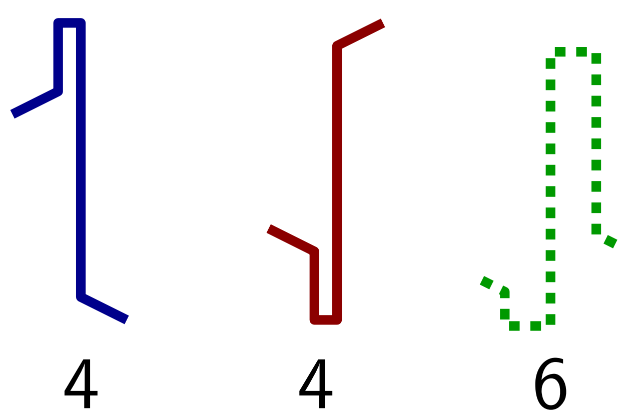
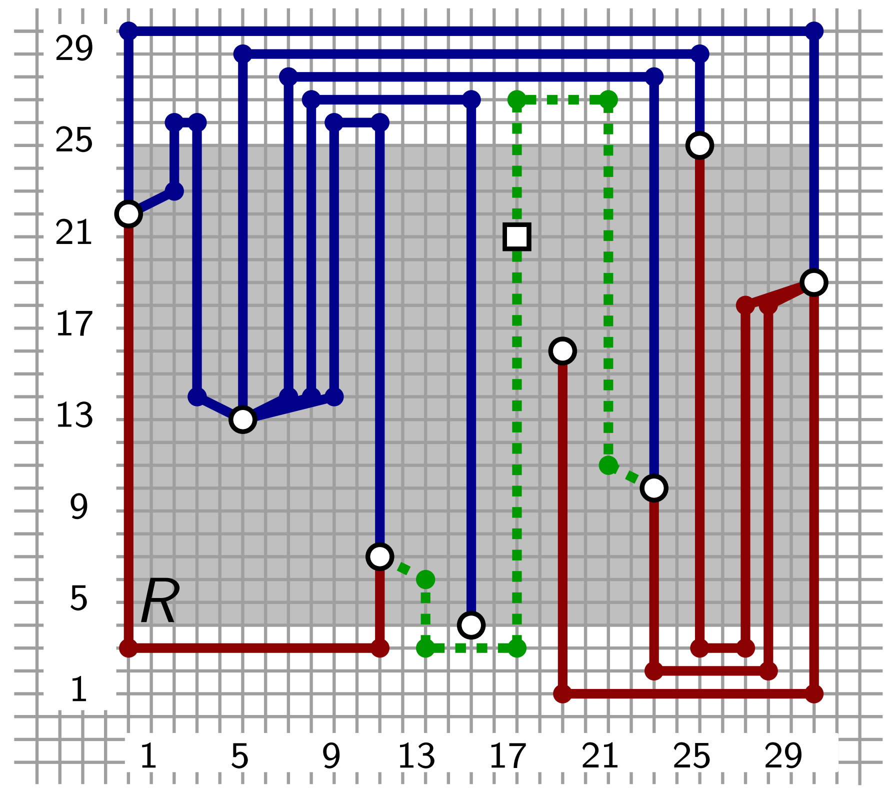
Edges:





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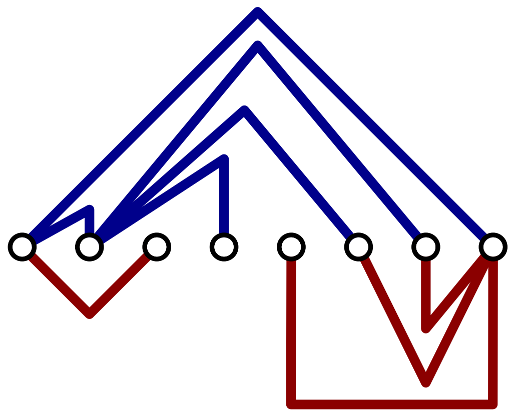

Graph 1: x-coordinates

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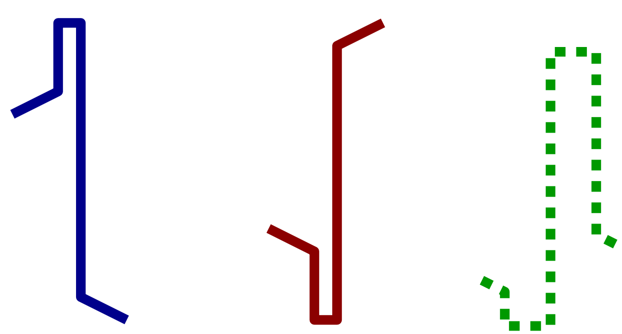
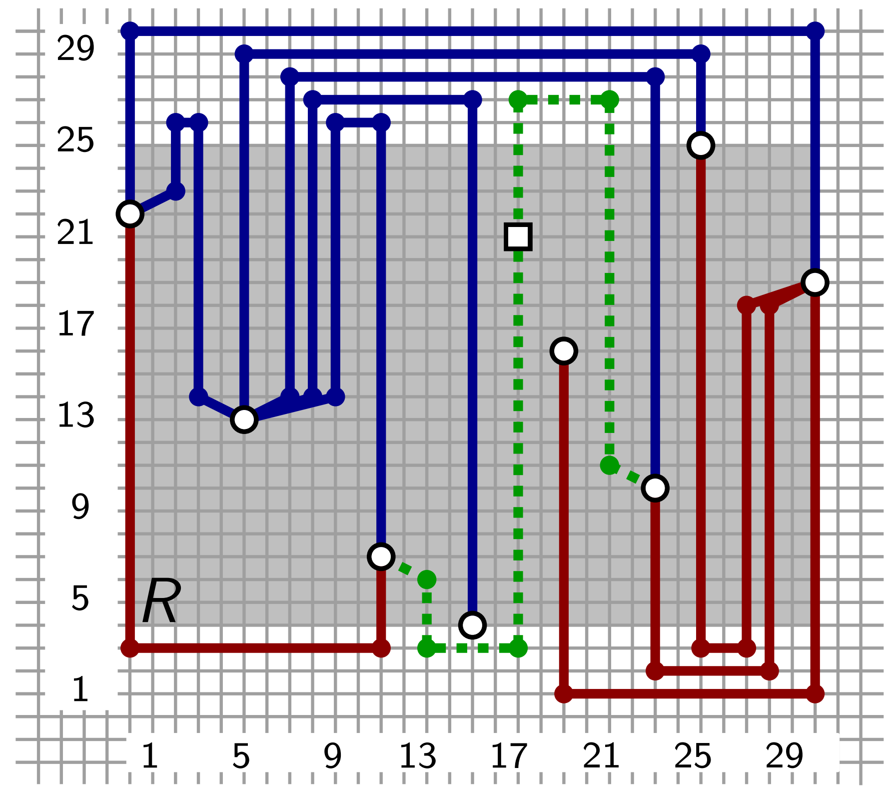
Edges:



4 4 6



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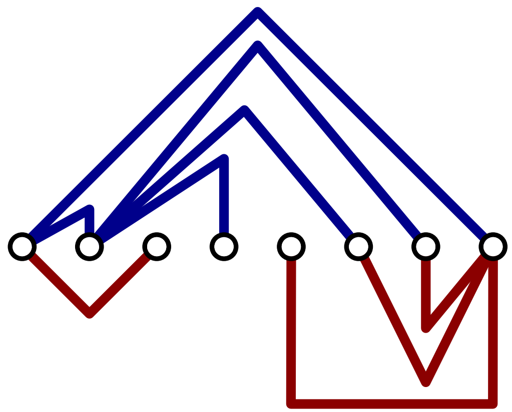

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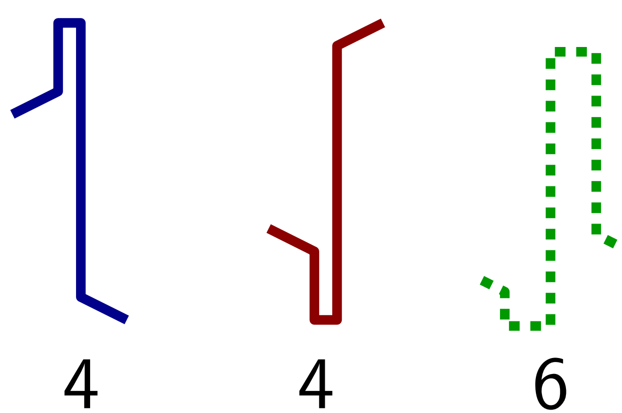

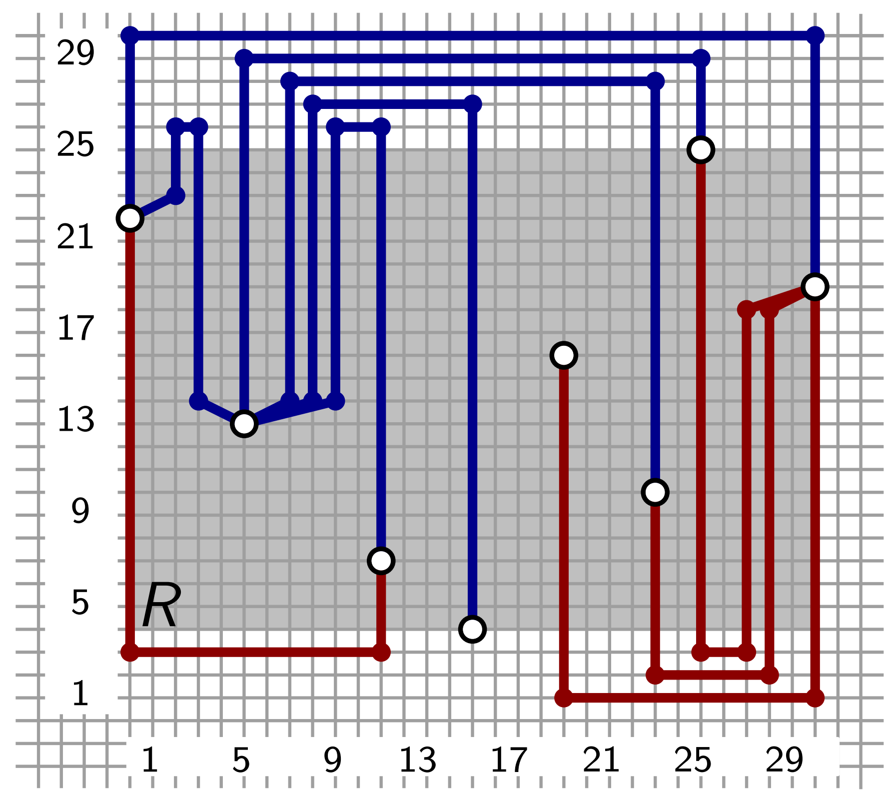

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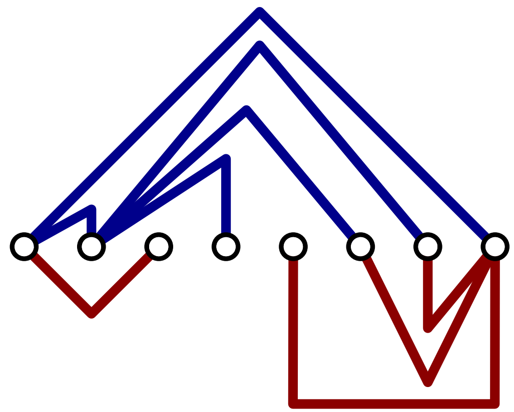

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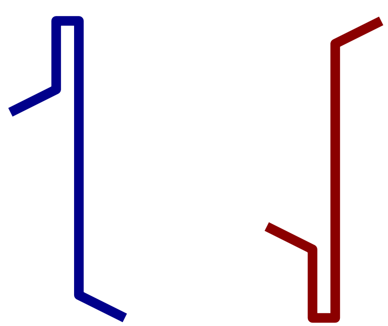
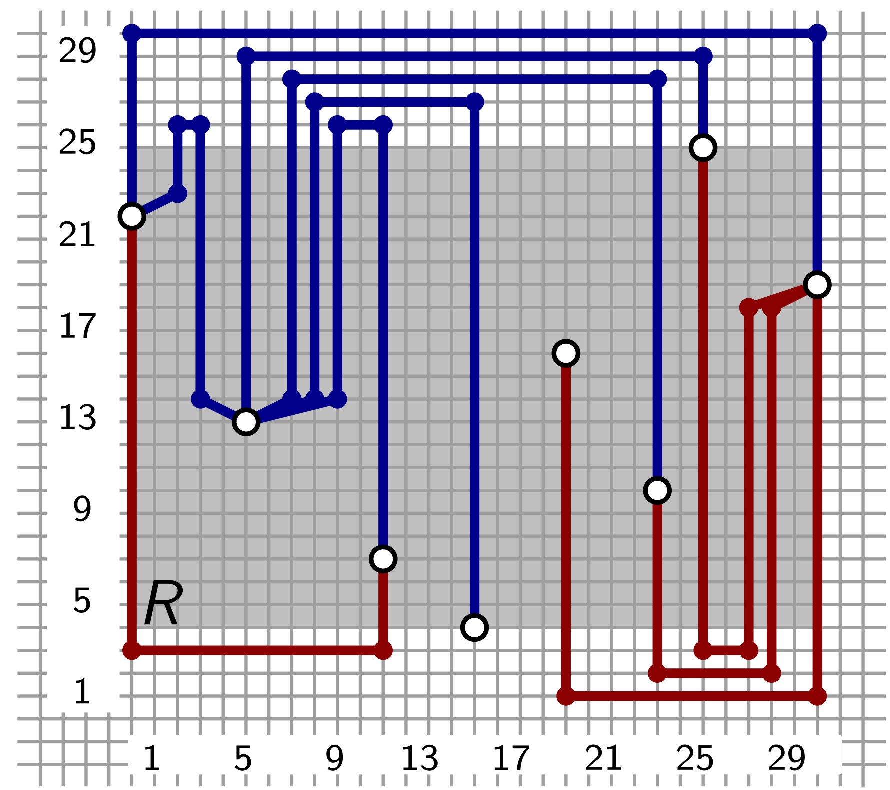
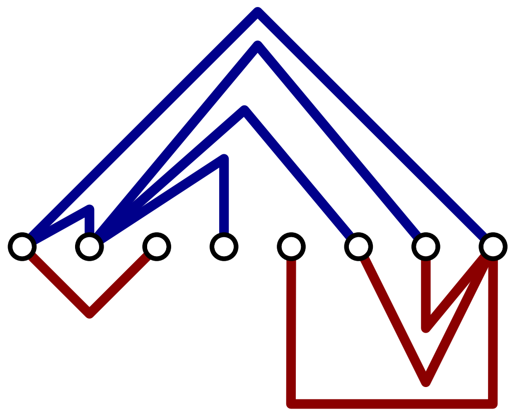


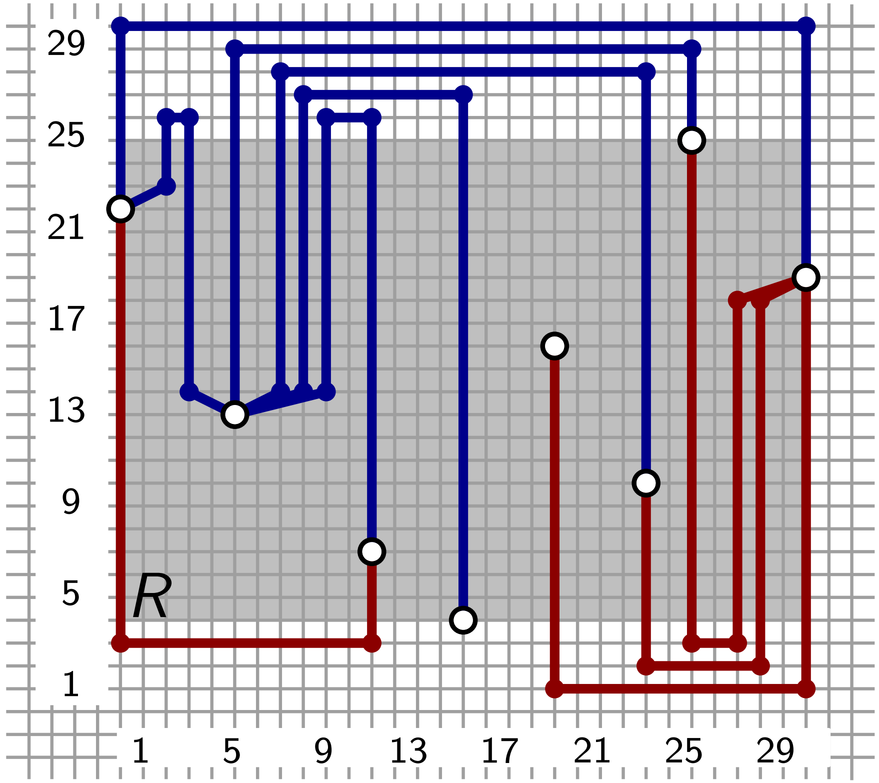
4 4



2-Page Book Embed. × 2-Page Book Embed.



Bends: 4 × 4
Grid size: $(11n - 32)^2$



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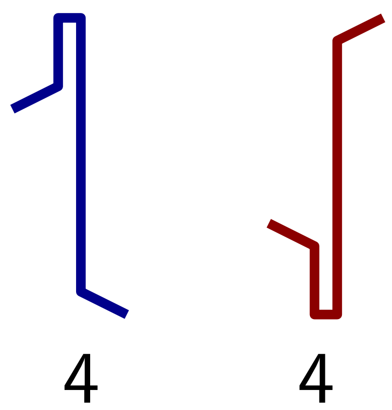
In *R*: All segments vertical or slanted of *y*-length 1.

Graph 2: *y*-coordinates

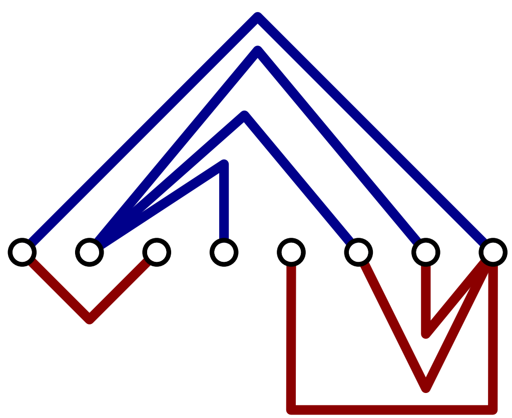
Edges:



~~2~~ 1-Page Book Embed. × ~~2~~ 1-Page Book Embed.

Bends: 4 × 4
Grid size: $(11n - 32)^2$

?



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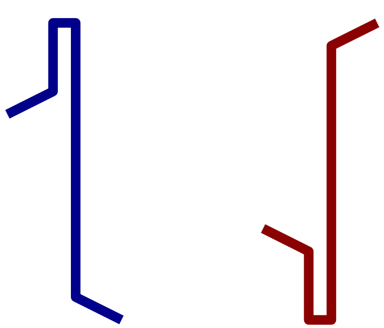
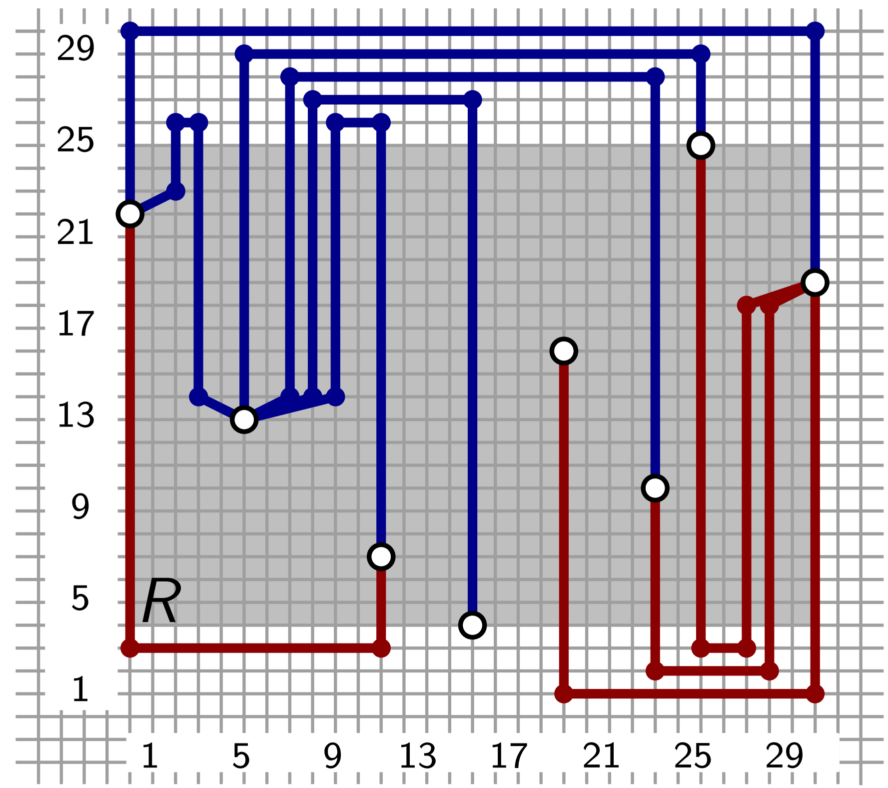
Edges:



4 4



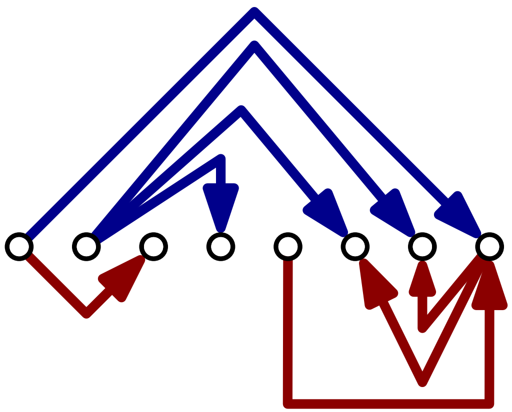
Outerplanar × Outerplanar

Bends: 3×3
Grid size: $(7n - 10)^2$

Decompose into two forests. . .

[Nash-Williams '64]

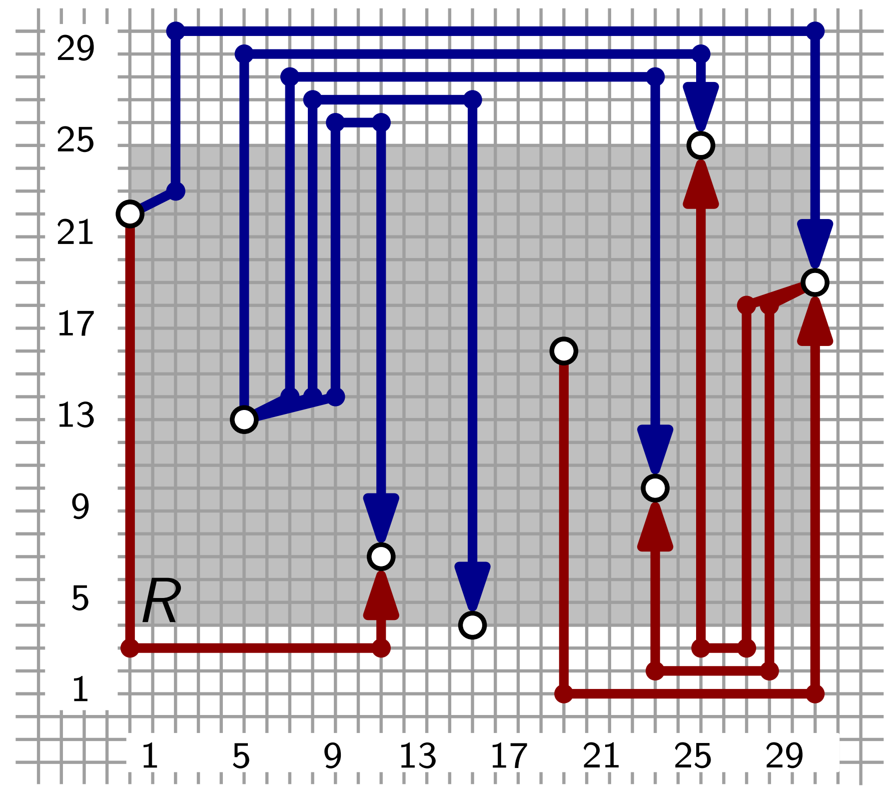
and direct them!



Graph 1: x-coordinates

In R: All segments vertical or slanted of y-length 1.

Graph 2: y-coordinates



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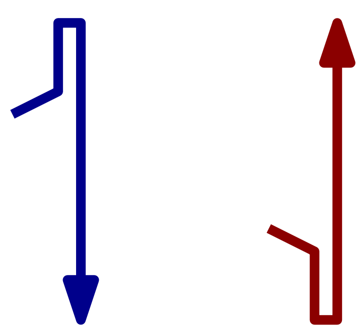

3 3

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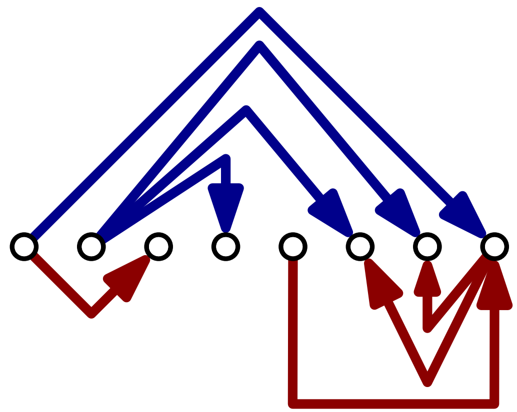

Graph 1: x-coordinates

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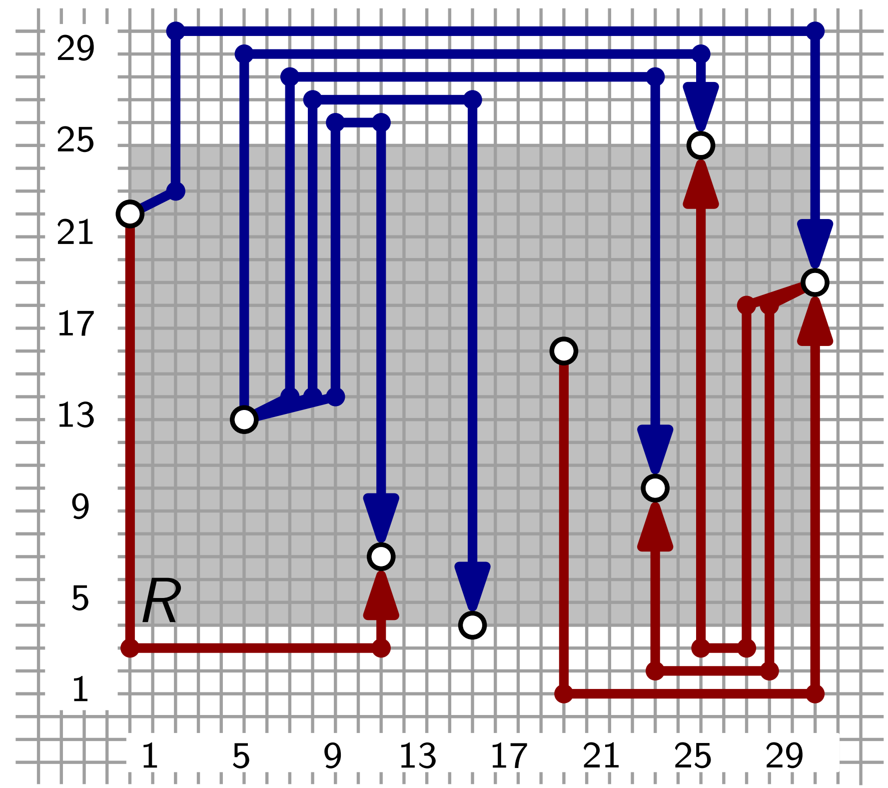
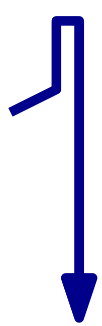
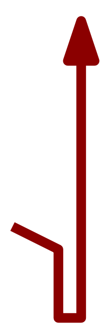


Graph 2: y-coordinates

Edges:



3



3

Every vertex has ≤ 1 incoming edge from above and ≤ 1 from below.

Conclusions

Graph classes			Number of bends	
Cycle	$\times$	Cycle	1×1	
Caterpillar	$\times$	Cycle	1×1	
Four Matchings			$1 \times 1 \times 1 \times 1$	
Tree	$\times$	Matching	1×0	
Wheel	$\times$	Matching	2×0	
Outerpath	$\times$	Matching	2×1	
Outerplanar	$\times$	Outerplanar	3×3	✓
2-page book emb.	$\times$	2-page book emb.	4×4	✓
Planar	$\times$	Planar	6×6	✓

Conclusions

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Outerplanar	$\times$	Outerplanar	3×3
2-page book emb.	$\times$	2-page book emb.	4×4
Planar	$\times$	Planar	6×6

- All graphs are drawn on the $O(n) \times O(n)$-grid.
- All algorithms run in $O(n)$ time.

Open Problems

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  $\Rightarrow$ LacSim
  (*Simultaneous Embedding with Large-Angle Crossings*)

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Our Results (Journal Version)

Graph classes			Number of bends	RAC-SEFE
Cycle	×	Cycle	1 × 1	✓
Caterpillar	×	Cycle	1 × 1	✓
Four Matchings			1 × 1 × 1 × 1	
Tree	×	Matching	1 × 0	✓
Wheel	×	Matching	2 × 0	✓
Outerpath	×	Matching	2 × 1	✓
Outerplanar	×	Outerplanar	3 × 3	
2-page book emb.	×	2-page book emb.	4 × 4	
Planar	×	Planar	6 × 6	

- All graphs are drawn on the $O(n) \times O(n)$-grid.
- All algorithms run in $O(n)$ time.