

Why Use Open-Ended Problems in Mathematics?

- Open-ended problems provide Multiple Entry Points for students
- Differentiate Instruction for many developmental levels
- Meet needs of both struggling learners and promising students who need challenges
- Maintains a level for all to succeed and ability to discuss mathematics

	Type of Open-Ended Problem	Example
1	Ask for more than one strategy	"Can you show two different ways to add 24 and 37?"
2	Show more than one representation	"Use numbers, pictures, and words to show your answer."
3	Require an explanation	"Please explain how you know you have the correct solution."
4	Ask for more than one solution	"Find three ways that the sisters can spend ten dollars."
5	Ask for a story problem and solution.	"Write a story problem using the numbers 3, 5, and 15. Solve the problem."
6	Begin with the answer and ask for possibilities.	"Find all the number combinations that make 16." Or "What are three different ways that Henry could spend \$15 on carnival rides?"
7	Create many possibilities by using dice, number cards, or spinner.	"Use two dice and three number cards to make an addition problem. Solve the problem."
8	Put a constraint on the problem.	"The 5 key on the calculator is broken. How could you solve the problem without using the 5 key?" or "How could you find the area of a trapezoid without using the formula?"
9	Use a range of numbers in the problem.	"Use the numbers 5, 6, 7, and 8 to make as many addition problems as you can. Solve."
10	Use a developmental progression in the problem.	"First draw a picture of the shape, identify the shape by name, and describe the shape using as many properties as you can."
11	Connect problem situation to real world and ask for expertise.	"Design three different sized boxes to hold 24 chocolate candies. Tell which box is the best design and why."

12	Ask for similarities and differences	"How are the numbers 75 and 100 alike? How are 75 and 100 different?" "How are the numbers 3.007 and 8.002 alike? Different?"
13	Replace a number with a blank	"Mr. Alvarado's class has ____ students. Ms. Kyoto's class has ____ students. How many students are there altogether?"
14	Ask for a number sentence	"Use the numbers 8 and 7 as well as the words 'product' and 'equal' to create a number sentence. Can you find more than one sentence?"
15	Change a textbook question	<i>Textbook:</i> What number has 4 hundreds, 5 tens, and 3 ones? <i>Change to:</i> "You can use 8 base ten blocks to model a number. What could the number be?"
16	Student choice in working with numbers	"The diameter of a sugar cookie is between 2 and 3 inches. Give the diameter as a fraction using two different ways."
		"You multiply two numbers and the product is almost 600. What could the numbers have been? Explain."
		"Draw a small rectangle. Draw a bigger rectangle that the smaller one is part of. Tell what fraction of the big rectangle the small one is."
		"Choose a fraction and a percent. Tell which is greater and how you know."
17	Use parallel tasks that explore same idea but fit different developmental levels.	"What coin combinations can you use to show your amount?" <i>Option 1:</i> 12 cents <i>Option 2:</i> 60 cents
		"Choose one of the measurements below. Estimate how many years old a person is who has lived: 1000 days 10,000 hours 1 million seconds

Resources

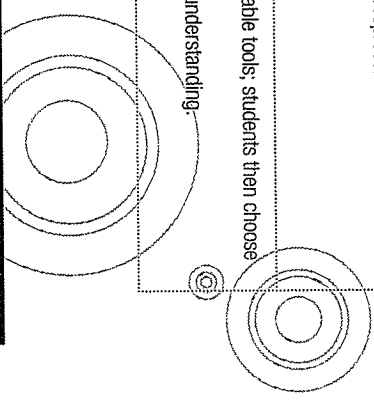
Small, Marian. (2009). *Good questions: Great ways to differentiate mathematics instruction*. Teachers College Press: NY.
ISBN: 978-0-8077-4978 - 4

Kiberi, M. S., & Smith, N. L. (2003). Turning traditional textbook problems into open-ended problems. *Mathematics teaching in the middle school*, 9(3), 186 - 192.

MATH

Standards for Mathematical Practice in Action

Practice	Sample Student Evidence	Sample Teacher Actions
1. Make sense of problems and persevere in solving them.	<ul style="list-style-type: none"> <input type="checkbox"/> Display sense-making behaviors. <input type="checkbox"/> Show patience and listen to others. <input type="checkbox"/> Turn and talk for first steps or generate a solution plan. <input type="checkbox"/> Analyze information in problems. <input type="checkbox"/> Use and recall multiple strategies. <input type="checkbox"/> Self-evaluate and redirect. <input type="checkbox"/> Assess the reasonableness of process and answer. 	<ul style="list-style-type: none"> <input type="checkbox"/> Provide open-ended problems. <input type="checkbox"/> Ask probing questions. <input type="checkbox"/> Probe student responses. <input type="checkbox"/> Promote and value discourse. <input type="checkbox"/> Promote collaboration. <input type="checkbox"/> Model and accept multiple approaches.
2. Reason abstractly and quantitatively.	<ul style="list-style-type: none"> <input type="checkbox"/> Represent abstract and contextual situations symbolically. <input type="checkbox"/> Interpret problems logically in context. <input type="checkbox"/> Estimate for reasonableness. <input type="checkbox"/> Make connections, including real-life situations. <input type="checkbox"/> Create and use multiple representations. <input type="checkbox"/> Visualize problems. <input type="checkbox"/> Put symbolic problems into context. 	<ul style="list-style-type: none"> <input type="checkbox"/> Model context to symbol and symbol to context. <input type="checkbox"/> Create problems such as, "What word problem will this equation solve?" <input type="checkbox"/> Give real-world situations. <input type="checkbox"/> Offer authentic performance tasks. <input type="checkbox"/> Place less emphasis on the answer. <input type="checkbox"/> Value invented strategies. <input type="checkbox"/> Think aloud.
3. Construct viable arguments and critique the reasoning of others.	<ul style="list-style-type: none"> <input type="checkbox"/> Question others. <input type="checkbox"/> Use examples and nonexamples. <input type="checkbox"/> Support beliefs and challenges with mathematical evidence. <input type="checkbox"/> Form logical arguments with conjectures and counterexamples. <input type="checkbox"/> Use multiple representations for evidence. <input type="checkbox"/> Listen and respond to others well. <input type="checkbox"/> Use precise mathematical vocabulary. 	<ul style="list-style-type: none"> <input type="checkbox"/> Create a safe and collaborative environment. <input type="checkbox"/> Model respectful discourse behaviors. <input type="checkbox"/> Provide find-the-error problems. <input type="checkbox"/> Promote student-to-student discourse (do not mediate discussion). <input type="checkbox"/> Plan effective questions or Socratic formats. <input type="checkbox"/> Provide time and value discourse.
4. Model with mathematics.	<ul style="list-style-type: none"> <input type="checkbox"/> Connect math (numbers and symbols) to real-life situations. <input type="checkbox"/> Symbolize real-world problems with math. <input type="checkbox"/> Make sense of mathematics. <input type="checkbox"/> Apply prior knowledge to solve problems. <input type="checkbox"/> Choose and apply representations, manipulatives, and other models to solve problems. <input type="checkbox"/> Use strategies to make problems simpler. <input type="checkbox"/> Use estimation and logic to check the reasonableness of an answer. 	<ul style="list-style-type: none"> <input type="checkbox"/> Model reasoning skills. <input type="checkbox"/> Provide meaningful, real-world, authentic, performance-based tasks. <input type="checkbox"/> Make appropriate tools available. <input type="checkbox"/> Model various modeling techniques. <input type="checkbox"/> Accept and value multiple approaches and representations.
5. Use appropriate tools strategically.	<ul style="list-style-type: none"> <input type="checkbox"/> Choose appropriate tool(s) for a given problem. <input type="checkbox"/> Use technology to deepen understanding. <input type="checkbox"/> Identify and locate resources. <input type="checkbox"/> Defend mathematically the choice of a tool. 	<ul style="list-style-type: none"> <input type="checkbox"/> Provide a toolbox at all times with all available tools; students then choose as needed. <input type="checkbox"/> Model tool use, especially technology for understanding.

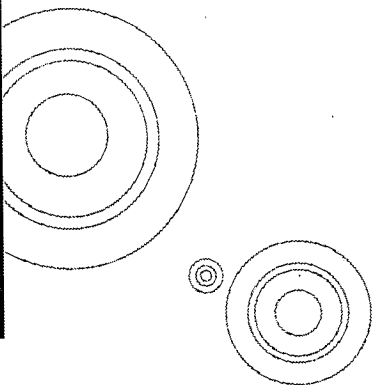


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6. Attend to precision.	<ul style="list-style-type: none"> <input type="checkbox"/> Communicate (orally and in writing) with precise vocabulary. <input type="checkbox"/> Carefully formulate questions and explanations (not retelling steps). <input type="checkbox"/> Decode and interpret the meaning of symbols. <input type="checkbox"/> Pay attention to units, labeling, scale, and so forth. <input type="checkbox"/> Calculate accurately and effectively. <input type="checkbox"/> Express answers within context when appropriate. 	<ul style="list-style-type: none"> <input type="checkbox"/> Model problem-solving strategies. <input type="checkbox"/> Give explicit and precise instruction. <input type="checkbox"/> Ask probing questions. <input type="checkbox"/> Use English language arts strategies of decoding, comprehending, and text-to-self connections for interpreting symbolic and contextual math problems. <input type="checkbox"/> Guided inquiry.
7. Look for and make use of structure.	<ul style="list-style-type: none"> <input type="checkbox"/> Look for, identify, and interpret patterns and structures. <input type="checkbox"/> Make connections to skills and strategies previously learned to solve new problems and tasks. <input type="checkbox"/> Breakdown complex problems into simpler and more manageable chunks. <input type="checkbox"/> Use multiple representations for quantities. <input type="checkbox"/> View complicated quantities as both a single object and a composition of objects. 	<ul style="list-style-type: none"> <input type="checkbox"/> Let students explore and explain patterns. <input type="checkbox"/> Use open-ended questioning. <input type="checkbox"/> Prompt students to make connections and choose problems that foster connections. <input type="checkbox"/> Ask for multiple interpretations of quantities.
8. Look for and express regularity in repeated reasoning.	<ul style="list-style-type: none"> <input type="checkbox"/> Design and state shortcuts. <input type="checkbox"/> Generate rules from repeated reasoning or practice (e.g., integer operations). <input type="checkbox"/> Evaluate the reasonableness of intermediate steps. <input type="checkbox"/> Make generalizations. 	<ul style="list-style-type: none"> <input type="checkbox"/> Provide tasks that allow students to generalize. <input type="checkbox"/> Don't teach steps or rules, but allow students to explore and generalize to discover and formalize. <input type="checkbox"/> Ask deliberate questions. <input type="checkbox"/> Create strategic and purposeful check-in points.

Source: Adapted from "Common Core Look Fors (CCL4s)" (iPad App). Adapted from NCSM Summer Leadership Academy, June, 2011, Atlanta, Ga.



Preparing for Common Core State Standards in Math

Resource List

What Really Works 2013 Conference

Robert Kaplinsky, Downey USD, rkaplinsky@dusd.net

Nancy O'Rode, Ph.D. CSUN nancyo@csun.edu

Common Core Standards Information

- This website introduces the Common Core and lists the standards in Mathematics and English Language Arts.
 - www.corestandards.org
- The Illustrative Mathematics Project currently has from 25 to 70 downloadable PDFs for the Common Core Math Standards at each grade level. These activities illustrate the range of mathematical work that students should be experiencing when implementing Common Core.
 - www.illustrativemathematics.org

Problem-Based Lessons

- Intriguing Math lessons using real-life applications – a sinkhole in Guatemala, the largest deliverable pizza, a \$100,000 speeding ticket—all downloadable materials
 - www.robertkaplinsky.com/lessons
- Over 600 lessons carefully designed by the National Council of Teachers of Mathematics for PreK -12 teachers with Learning Objectives, Materials, Lesson Plan, Questions for students and Assessment.
 - <http://illuminations.nctm.org>
- NCTM's *Illustrations* page also has over 100 online activities for students organized by grade level and math topic. preK – 12
 - <http://illuminations.nctm.org>
- Handout: *Using Open-Ended Problems* compiled by Nancy O'Rode lists 17 ways to change low-level demand textbook problems to demand higher-level cognition—the type required by the new computer-adaptive assessment system.