Why Use Open-Ended Problems in Mathematics?

- Open-ended problems provide Multiple Entry Points for students
- Differentiate Instruction for many developmental levels
- Meet needs of both struggling learners and promising students who need challenges
- Maintains a level for all to succeed and ability to discuss mathematics

	Type of Open-Ended Problem	Example
1	Ask for more than one strategy	"Can you show two different ways to add 24 and 37?"
2	Show more than one representation	"Use numbers, pictures, and words to show your answer."
3	Require an explanation	"Please explain how you know you have the correct solution."
4	Ask for more than one solution	"Find three ways that the sisters can spend ten dollars."
5	Ask for a story problem and solution.	"Write a story problem using the numbers 3, 5, and 15. Solve the problem."
6	Begin with the answer and ask for possibilities.	"Find all the number combinations that make 16." Or "What are three different ways that Henry could spend $15 on carnival rides?"
7	Create many possibilities by using dice, number cards, or spinner.	"Use two dice and three number cards to make an addition problem. Solve the problem."
8	Put a constraint on the problem.	"The 5 key on the calculator is broken. How could you solve the problem without using the 5 key?" or "How could you find the area of a trapezoid without using the formula?"
9	Use a range of numbers in the problem.	"Use the numbers 5, 6, 7, and 8 to make as many addition problems as you can. Solve."
10	Use a developmental progression in the problem.	"First draw a picture of the shape, identify the shape by name, and describe the shape using as many properties as you can."
11	Connect problem situation to real world and ask for expertise.	"Design three different sized boxes to hold 24 chocolate candies. Tell which box is the best design and why."

12	Ask for similarities and differences	"How are the numbers 75 and 100 alike? How are 75 and 100 different?" "How are the numbers 3.007 and 8.002 alike? Different?"
13	Replace a number with a blank	"Mr. Alvarado's class has _____ students. Ms. Kyoto's class has _____ students. How many students are there altogether?"
14	Ask for a number sentence	"Use the numbers 8 and 7 as well as the words 'product' and 'equal' to create a number sentence. Can you find more than one sentence?"
15	Change a textbook question	*Textbook:* What number has 4 hundreds, 5 tens, and 3 ones? *Change to:* "You can use 8 base ten blocks to model a number. What could the number be?"
16	Student choice in working with numbers	"The diameter of a sugar cookie is between 2 and 3 inches. Give the diameter as a fraction using two different ways."
		"You multiply two numbers and the product is almost 600. What could the numbers have been? Explain."
		"Draw a small rectangle. Draw a bigger rectangle that the smaller one is part of. Tell what fraction of the big rectangle the small one is."
		"Choose a fraction and a percent. Tell which is greater and how you know."
17	Use parallel tasks that explore same idea but fit different developmental levels	" What coin combinations can you use to show your amount?" *Option 1:* 12 cents *Option 2*: 60 cents
		"Choose one of the measurements below. Estimate how many years old a person is who has lived: 1000 days 10,000 hours 1 million seconds

Resources

Small, Marian. (2009). *Good questions: Great ways to differentiate mathematics instruction.* Teachers College Press: NY.
ISBN: 978-0-8077-4978 - 4

Kiberi, M. S., & Smith, N. L. (2003). Turning traditional textbook problems into open-ended problems. *Mathematics teaching in the middle school,* 9(3), 186 - 192.

MATH | Standards for Mathematical Practice in Action

Practice	Sample Student Evidence	Sample Teacher Actions
1. Make sense of problems and persevere in solving them.	☐ Display sense-making behaviors. ☐ Show patience and listen to others. ☐ Turn and talk for first steps or generate a solution plan. ☐ Analyze information in problems. ☐ Use and recall multiple strategies. ☐ Self-evaluate and redirect. ☐ Assess the reasonableness of process and answer.	☐ Provide open-ended problems. ☐ Ask probing questions. ☐ Probe student responses. ☐ Promote and value discourse. ☐ Promote collaboration. ☐ Model and accept multiple approaches.
2. Reason abstractly and quantitatively.	☐ Represent abstract and contextual situations symbolically. ☐ Interpret problems logically in context. ☐ Estimate for reasonableness. ☐ Make connections, including real-life situations. ☐ Create and use multiple representations. ☐ Visualize problems. ☐ Put symbolic problems into context.	☐ Model context to symbol and symbol to context. ☐ Create problems such as, "What word problem will this equation solve?" ☐ Give real-world situations. ☐ Offer authentic performance tasks. ☐ Place less emphasis on the answer. ☐ Value invented strategies. ☐ Think aloud.
3. Construct viable arguments and critique the reasoning of others.	☐ Question others. ☐ Use examples and nonexamples. ☐ Support beliefs and challenges with mathematical evidence. ☐ Form logical arguments with conjectures and counterexamples. ☐ Use multiple representations for evidence. ☐ Listen and respond to others well. ☐ Use precise mathematical vocabulary.	☐ Create a safe and collaborative environment. ☐ Model respectful discourse behaviors. ☐ Provide find-the-error problems. ☐ Promote student-to-student discourse (do not mediate discussion). ☐ Plan effective questions or Socratic formats. ☐ Provide time and value discourse.
4. Model with mathematics.	☐ Connect math (numbers and symbols) to real-life situations. ☐ Symbolize real-world problems with math. ☐ Make sense of mathematics. ☐ Apply prior knowledge to solve problems. ☐ Choose and apply representations, manipulatives, and other models to solve problems. ☐ Use strategies to make problems simpler. ☐ Use estimation and logic to check the reasonableness of an answer.	☐ Model reasoning skills. ☐ Provide meaningful, real-world, authentic, performance-based tasks. ☐ Make appropriate tools available. ☐ Model various modeling techniques. ☐ Accept and value multiple approaches and representations.
5. Use appropriate tools strategically.	☐ Choose appropriate tool(s) for a given problem. ☐ Use technology to deepen understanding. ☐ Identify and locate resources. ☐ Defend mathematically the choice of a tool.	☐ Provide a toolbox at all times with all available tools; students then choose as needed. ☐ Model tool use, especially technology for understanding.

MATH | Standards for Mathematical Practice in Action

Practice	Sample Student Evidence	Sample Teacher Actions
6. Attend to precision.	☐ Communicate (orally and in writing) with precise vocabulary. ☐ Carefully formulate questions and explanations (not retelling steps). ☐ Decode and interpret the meaning of symbols. ☐ Pay attention to units, labeling, scale, and so forth. ☐ Calculate accurately and effectively. ☐ Express answers within context when appropriate.	☐ Model problem-solving strategies. ☐ Give explicit and precise instruction. ☐ Ask probing questions. ☐ Use English language arts strategies of decoding, comprehending, and text-to-self connections for interpreting symbolic and contextual math problems. ☐ Guided inquiry.
7. Look for and make use of structure.	☐ Look for, identify, and interpret patterns and structures. ☐ Make connections to skills and strategies previously learned to solve new problems and tasks. ☐ Breakdown complex problems into simpler and more manageable chunks. ☐ Use multiple representations for quantities. ☐ View complicated quantities as both a single object and a composition of objects.	☐ Let students explore and explain patterns. ☐ Use open-ended questioning. ☐ Prompt students to make connections and choose problems that foster connections. ☐ Ask for multiple interpretations of quantities.
8. Look for and express regularity in repeated reasoning.	☐ Design and state shortcuts. ☐ Generate rules from repeated reasoning or practice (e.g., integer operations). ☐ Evaluate the reasonableness of intermediate steps. ☐ Make generalizations.	☐ Provide tasks that allow students to generalize. ☐ Don't teach steps or rules, but allow students to explore and generalize to discover and formalize. ☐ Ask deliberate questions. ☐ Create strategic and purposeful check-in points.

Source: Adapted from "Common Core Look Fors (CCL4s)" (iPad App). Adapted from NCSM Summer Leadership Academy, June, 2011, Atlanta, Ga.

Preparing for Common Core State Standards in Math

Resource List

What Really Works 2013 Conference
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Nancy O'Rode, Ph.D. CSUN nancyo@csun.edu

Common Core Standards Information

- This website introduces the Common Core and lists the standards in Mathematics and English Language Arts.
    - www.corestandards.org
- The Illustrative Mathematics Project currently has from 25 to 70 downloadable PDFs for the Common Core Math Standards at each grade level. These activities illustrate the range of mathematical work that students should be experiencing when implementing Common Core.
    - www.illustrativemathematics.org

Problem-Based Lessons

- Intriguing Math lessons using real-life applications – a sinkhole in Guatemala, the largest deliverable pizza, a $100,000 speeding ticket—all downloadable materials
    - www.robertkaplinsky.com/lessons
- Over 600 lessons carefully designed by the National Council of Teachers of Mathematics for PreK -12 teachers with Learning Objectives, Materials, Lesson Plan, Questions for students and Assessment.
    - http://illuminations.nctm.org
- NCTM's *Illuminations* page also has over 100 online activities for students organized by grade level and math topic. preK – 12
    - http://illuminations.nctm.org
- Handout: *Using Open-Ended Problems* compiled by Nancy O'Rode lists 17 ways to change low-level demand textbook problems to demand higher-level cognition—the type required by the new computer-adaptive assessment system.